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Mode Converted Electrostatic Nonlinear Ion-Ion Hybrid Mode

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Mode conversion (MC) process proves to be of prime importance in fusion as well as in magnetospheric plasma. However, the presence of multiple ion species, even with small concentrations, can lead to the appearance of new and modified resonance, cutoff, and crossover frequencies. Nonlinear effects such as pump self induced filamentation and parametric decays further complicate the MC physics and associated heating processes. It is generally seen that intense localized electric fields of the soliton form are generated due to the density changes (density cavities) caused by the dominant ponderomotive forces acting on the charged species. It turns out that the nonlinear fate of the mode converted electrostatic wave beyond the MC layer is still an open question. With this motivation, a nonlinear state of the mode converted electrostatic ion-ion hybrid wave in the vicinity of the MC layer, is investigated analytically. In context with it, an exact nonlinear solution of the ion-ion hybrid mode is estimated under the influence of adiabatic perturbations in a Two ion species magnetized plasma. The dominant nonlinearity arises through the ion ponderomotive force term thereby modulating the plasma density profile. The nonlinear equation which has KorteVeg De Vries [KdV] soliton as its solution, represents the nonlinear stage of a purely growing mode. It turns out that these solitons exists only if the wave frequency is lower than the Buschbaum frequency and if the concentration of the lighter ions is less than the heavier one. The resultant ponderomotive expulsion of plasma is discussed in terms of intense localized electric fields and associated density cavities. The application of the analytical model is discussed in terms of Proton and Tritium minority concentration ratios in Deuterium plasma.

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I. Introduction

Many efficient heating methods have been proposed to heat Tokamak plasma upto thermonuclear temperatures. It includes various techniques such as compressional heating [1] (magnetic field, electric field, shock wave, beam pressure), wave heating [2] (radio waves, microwaves, laser beams), particle beam injection (electron beams, ion beams, neutral beams) as well as alpha particle heating [3]. Particularly for wave heating processes, it seems that ICRF heating scheme turns out to be highly successful and has been actively explored by various authors over many decades in the past and thus these works serve as excellent reviews for the readers [4-10].

Although RF heating utilizes electromagnetic waves as the principal energy carrier, a mode conversion (MC) is expected to occur from an electromagnetic wave into electrostatic or electromagnetic waves near the ion-ion hybrid resonance layer [11-13]. Advanced visualisation techniques such as phase contrast imaging, sophisticated plasma diagnostic tools like ion-ion hybrid reflectometry and two-dimensional wave simulations codes such as TORIC have made better understanding of the MC physics. Further, these techniques have simplified the in-board and out-board operational difficulties associated with the RF launching antenna [14-18]. These dominant mode conversion processes are of prime importance not only in fusion plasma but even in the investigation of magnetospheric and auroral plasma dynamics [19].

In a typical D (H) minority heating case, power is transmitted from the antenna through the plasma to an absorbing region as a compressional-Alfven wave (also called the fast magnetosonic wave). However, absorption can be via cyclotron damping on minority ions or through electron Landau damping of the incoming fast wave or short-wavelength, mode converted waves. Standard Tokamak plasma heating models suggests that 70% of absorbed power is coupled to a fast minority ion tail, 20% to majority ions via second harmonic deuterium cyclotron damping, and 10% of power directly to electrons via Landau damping. Heating efficiency is found to be optimum with a few percent minority concentration. More precisely, when the electromagnetic long wavelength magnetosonic fast wave (FW) encounters a mode conversion (MC) layer in a multi-ion-species plasma, both the shorter wavelength electrostatic ion-Bernstein (IBW) slow wave (SW) or the electromagnetic ion cyclotron (ICW) slow wave (SW) can result as MC products. [20-25].

Various models related to ion-ion hybrid mode has indicated the dominant role of minority species concentration in plasma heating [26-35]. Nonlinear behaviour such as pump self induced filamentation and parametric decays prove to be critical near the MC layer [36-42]. It is seen that intense localized electric fields of the soliton form are generated due to the density changes (density cavities) caused by the dominant ponderomotive forces acting on the charged species. The interaction of the plasma particles with these fields seriously hampers the transport of RF energy beyond the MC layer. On the same lines, the existence of upper hybrid and lower hybrid solitons have been investigated and reviewed earlier [4][11][37][42].

Although it is generally believed that during the ICRH propagation, the absorbed power gets coupled to the fast minority ion tail, majority ions and electrons as per standard Tokamak plasma heating models, but the nonlinear fate of the mode converted electrostatic wave beyond the MC layer is still an open question. With our prime motivation in the existence of nonlinear density cavities and localised electric field structures beyond the MC layer, theoretical analysis is being carried for the situations when ion-ion hybrid soliton occurs in a homogeneously magnetized plasma system. The emphasis is more on parametric and nonlinear effects in the vicinity of ion-ion hybrid resonance layer in slab geometry with the Bi-ion scenario. The outline for the paper is as follows. In section 2, we describe the basic fluid equations under the influence of adiabatic perturbations invoked via ponderomotive force. The nonlinear equation which has KorteVeg De Vries (KdV) soliton as its solution, is explained in section 3. Finally, conclusion and the limitation of the model is delivered in section 4.

II. Mode converted Ion – Ion hybrid wave

Let us consider a magnetized plasma in a slab field geometry consisting of electrons (mass $\mathbf{m}$, charge $-\mathbf{e}$) and ions of two species (mass $\mathbf{m}_1, \mathbf{m}_2$ charge $\mathbf{e}$) embedded in a uniformmagnetic field $\mathbf{B}_0 = \mathbf{B}_0\hat{\mathbf{z}}$. The two species ion dynamics is described as

$$\left(\frac{\partial}{\partial \mathbf{t}} + \mathbf{V}_\alpha.\nabla\right)\mathbf{V}_\alpha = \frac{\mathbf{e}}{\mathbf{m}_\alpha}\mathbf{E} + \mathbf{V}_\alpha \times \Omega_\alpha - \frac{\mathbf{T}}{\mathbf{m}_\alpha \mathbf{n}_\alpha}\nabla \mathbf{n}_\alpha \quad [1]$$

where $\alpha = 1,2$ denotes the two ion species. Here, it is assumed that both the ion species are at temperatures $\mathbf{T}_1 = \mathbf{T}_2 = \mathbf{T}$. With prime interest in the post mode conversion process in the vicinity of the mode conversion layer, it is supposed that an electrostatic ion-ion hybrid mode is formed after the mode conversion process and is propagating along the x-axis with the frequency of oscillation ω. Now the ion dynamics along x-axis is given by

$$\mathbf{V}_{x\alpha} = \frac{\mathbf{ie}}{\mathbf{m}_\alpha}\frac{\omega \mathbf{E}}{\omega^2 - \Omega_\alpha^2} - \frac{\mathbf{i}\mathbf{V}_\alpha^2 \omega}{\omega^2 - \Omega_\alpha^2}\frac{1}{\mathbf{n}_\alpha}\frac{\mathbf{d}}{\mathbf{dx}}\mathbf{n}_\alpha \quad [2]$$

This induces the density perturbation

$$\mathbf{n}_\alpha = -\frac{\mathbf{i}}{\omega}\frac{\mathbf{d}}{\mathbf{dx}}\left(\mathbf{n}_\alpha \mathbf{V}_\alpha\right) \quad [3]$$

Combining equation (2) and (3) we get an equation for the density fluctuations as

$$\mathbf{O}_\alpha \mathbf{n}_\alpha = \frac{\mathbf{e}}{\mathbf{m}_\alpha}\frac{\mathbf{d}}{\mathbf{dx}}\left(\mathbf{n}_\alpha \mathbf{E}\right) \quad [4]$$

where the operator is defined by $\mathbf{O}_\alpha = \omega^2 - \Omega_\alpha^2 + \mathbf{T}/\mathbf{m}_\alpha\, \mathbf{d}^2/\mathbf{dx}^2$

For post mode converted ion-ion hybrid mode, propagating perpendicular to the magnetic field, the quas-ineutrality is maintained by appropriately balancing the two ions density perturbations. The electrons do not participate in this process, electrons maintain a uniform negatively charged background in which the ions oscillate to maintain the quas-ineutrality and thereby support the ion – ion hybrid mode. The perturbation of each ion species is described by equation (4). By adopting the charge neutrality along with equation (4), we get the ion-ion hybrid mode equation as

$$\mathbf{m}_2\mathbf{O}_2\left(\mathbf{n}_1\mathbf{E}\right) + \mathbf{m}_1\mathbf{O}_1\left(\mathbf{n}_2\mathbf{E}\right) = 0 \quad [5]$$

The electric field $\mathbf{E}$ in this equation is the ion –ion hybrid field while the ion density $\mathbf{n}_\alpha$ is obtained by averaging over the ion –ion hybrid period. In the linear mode $\mathbf{n}_\alpha$ is the uniform backgrounddensity.Under the influence of the adiabatic perturbations, which is the case under consideration, we have $\mathbf{n}_\alpha = \mathbf{n}_{0\alpha} + \mathbf{n}_\alpha^{\mathbf{s}}\left(\mathbf{x}\right)$ where we denote $\mathbf{n}_\alpha^{\mathbf{s}}\left(\mathbf{x}\right)$ as the density fluctuations of the ion species due to the adiabatic perturbation and $\mathbf{n}_{0\alpha}$ as the corresponding equilibrium density. Equation (5) now can be separated into the linear and nonlinear parts.The nonlinear part being the contribution from the adiabatic perturbations. Using the explicit expressions for the operators and simplifying the algebra, we get the nonlinear ion – ion hybrid equation as

$$\left(\omega^2 - \omega_B^2 + V_B^2 \frac{d^2}{dx^2}\right) E = \frac{m_1 m_2 \left(\Omega_1^2 - \Omega_2^2\right)}{\left(n_{01} m_2 + n_{02} m_1\right)^2} \left(n_{01} n_2^s - n_{02} n_1^s\right) E \quad [6]$$

where

$$\omega_B^2 = \left(n_{01} m_2 \Omega_2^2 + n_{02} m_1 \Omega_1^2\right) \Big/ \left(n_{01} m_2 + n_{02} m_1\right) \quad [7]$$

$$V_B^2 = n_0 T \Big/ \left(n_{01} m_2 + n_{02} m_1\right); n_0 = n_{01} + n_{02} \quad [8]$$

Note that ω_B is Buschbaum frequency, V_B the effective ion thermal speed and n_0 the total plasma density. The left hand side of equation (6) describes the linear ion – ion hybrid mode, the last term being a thermal correction. Right hand side is the nonlinear contribution through the adiabatic perturbation. We assumed the thermal correction to be small and ignore it in the nonlinear terms.

III. Ponderomotive Force

The adiabatic density perturbation n_α^s is caused by the ponderomotive force due to the ion – ion hybrid mode. The ion velocity under the influence of ion –ion hybrid mode is described by equation (1). Since the ponderomotive force in nonlinear, we neglect the small contributions from the thermal term. Thus it is sufficient to take the ion velocity as

$$V_{\alpha x} = \frac{ie}{m_\alpha} \frac{\omega}{\omega^2 - \Omega_\alpha^2} E \quad [9]$$

and

$$V_{\alpha y} = \frac{e}{m_\alpha} \frac{\Omega_\alpha}{\omega^2 - \Omega_\alpha^2} E \quad [10]$$

The ponderomotive force is given by

$$F = -m_\alpha \left\langle \underset{\sim}{V_\alpha} . \nabla \underset{\sim}{V_\alpha} \right\rangle \quad [11]$$

where the average is taken over the ion – ion hybrid periods. It turns out that

$$F_\alpha = F_{\alpha x} = -m_\alpha q_\alpha \frac{d}{dx} E^2 \quad [12]$$

where

$$q_\alpha = \frac{e^2}{2 m_\alpha^2} \frac{\omega^2}{\left(\omega^2 - \Omega_\alpha^2\right)^2} \quad [13]$$

It has been assumed in the above calculations that E is real and is independent of the y co-ordinate.This ponderomotive force will cause the adiabatic density perturbation as

$$\frac{n_\alpha^s}{n_{0\alpha}} = -\frac{e\phi^s}{T_i} - \frac{q_\alpha E^2}{T_i} \quad [14]$$

The ϕ^s is the adiabatic potential which will be neutralized by the electron density perturbation,

$$\frac{n_e^s}{n_0} = +\frac{e\phi^s}{T_e} \quad [15]$$

Since electrons do not participate in the ion – ion hybrid mode, they do not experience the ponderomotive force. Eliminating ϕ^s from (14) and (15), after maintaining quasi-neutrality, the ion density perturbations are given as

$$n_1^s = -n_{01} \frac{E^2}{T_i} \left(q_1 - \frac{n_{01} q_1 + n_{02} q_2}{n_0} \frac{T_e}{T_e + T_i} \right) \quad [16]$$

and

$$\mathbf{n}_2^{\mathbf{s}} = -\mathbf{n}_{02}\frac{\mathbf{E}^2}{\mathbf{T_i}}\left(\mathbf{q}_2 - \frac{\mathbf{n}_{01}\mathbf{q}_1 + \mathbf{n}_{02}\mathbf{q}_2}{\mathbf{n}_0}\frac{\mathbf{T_e}}{\mathbf{T_e}+\mathbf{T_i}}\right) [17]$$

Note that there is no density perturbations in the absence of the ponderomotive force. This is because the adiabatic perturbations are not an eigen mode of the uniform magnetized plasma in the absence of the high frequency fields. The ponderomotive force in our case, is perpendicular to the magnetic field. This could cause an motion which is divergence free and does not support a density perturbation. Similarly, a small potential fluctuation across the magnetic field causes drift. This also is divergence free and cannot be supported self-consistently. However, the ponderomotive force could induce the density perturbations such that the electrostatic force thus created by the density imbalance can balance the ponderomotive itself self consistently. Equations (16) and (17) give us the required ion density perturbations. The electrostatic potential due to them will be neutralized by the electron density perturbations, equation (16).

The adiabatic density perturbation from equations (16) and (17) can be substituted back into the equation (7). This results into

$$\left(\boldsymbol{\omega}^2 - \boldsymbol{\omega}_{\mathbf{B}}^2 + \mathbf{V}_{\mathbf{B}}^2\frac{\mathbf{d}^2}{\mathbf{dx}^2}\right)\mathbf{E} = -\frac{\mathbf{m}_1\mathbf{m}_2\left(\Omega_1^2 - \Omega_2^2\right)}{\left(\mathbf{n}_{01}\mathbf{m}_2 + \mathbf{n}_{02}\mathbf{m}_1\right)^2}\frac{\mathbf{n}_{01}\mathbf{n}_{02}\left(\mathbf{q}_1 - \mathbf{q}_2\right)}{\mathbf{T}}\mathbf{E}^3 [18]$$

This is the equation, (Eq 19) describing the nonlinear ion – ion hybrid mode. The left hand side describes the usual linear propagation across the magnetic field. The right hand side is the contribution of the nonlinearity due to the adiabatic perturbation.

3. The solution –

Equation (18) can be rewritten as

$$\frac{\mathbf{d}^2\mathbf{E}}{\mathbf{dx}^2} - \mathbf{aE} + \mathbf{bE}^3 = 0 [19]$$

where

$$\mathbf{a} = \frac{\boldsymbol{\omega}_{\mathbf{B}}^2 - \boldsymbol{\omega}^2}{\mathbf{V}_{\mathbf{B}}^2}; \mathbf{b} = \frac{\mathbf{m}_1\mathbf{m}_2\mathbf{n}_{01}\mathbf{n}_{02}\left(\mathbf{q}_1 - \mathbf{q}_2\right)\left(\Omega_1^2 - \Omega_2^2\right)}{\mathbf{TV}_{\mathbf{B}}^2\left(\mathbf{n}_{01}\mathbf{m}_2 + \mathbf{n}_{02}\mathbf{m}_1\right)^2} [20]$$

Multiply (19) by $\frac{\mathbf{dE}}{\mathbf{dx}}$ and integrate, we get an intermediate integral for $\left(\mathbf{E}\right)_{|\mathbf{x}|=\infty} = 0 = \left(\frac{\partial\mathbf{E}}{\partial\mathbf{X}}\right)_{|\mathbf{x}|=\infty}$

$$\left(\frac{\mathbf{dE}}{\mathbf{dx}}\right)^2 = \mathbf{aE}^2\left(1 - \frac{\mathbf{b}}{2\mathbf{a}}\mathbf{E}^2\right) [21]$$

The solution/soliton of equation (21) is

$$\mathbf{E} = 2\sqrt{\frac{\mathbf{a}}{\mathbf{b}}}\ \mathbf{Sech}\left(\frac{\mathbf{x}}{1/\sqrt{\mathbf{a}}}\right) [22]$$

Hence soliton solution is possible if and only if $\left(\mathbf{a} > 0\right)$ and $\left(\mathbf{b} > 0\right)$.

Thus ion – ion hybrid soliton exist if (i) $\left(\boldsymbol{\omega} < \boldsymbol{\omega}_{\mathbf{B}}\right)$ (i.e. for purely growing mode) and (ii) $\left(\mathbf{q}_1 > \mathbf{q}_2\right), \left(\Omega_1^2 > \Omega_2^2\right)$ or $\left(\mathbf{q}_1 < \mathbf{q}_2\right)\left(\Omega_1^2 < \Omega_2^2\right)$.The condition $\left(\mathbf{a} > 0\right)$ implies that the wave frequency is less than the Buschbaum resonance frequency $\left(\boldsymbol{\omega}_{\mathbf{B}}\right)$. Since we are discussing the ion – ion hybrid mode, $\left(\boldsymbol{\omega} \sim \boldsymbol{\omega}_{\mathbf{B}}\right)$. For a small deviation $\boldsymbol{\delta} = \boldsymbol{\omega}_{\mathbf{B}} - \boldsymbol{\omega} << \boldsymbol{\omega}_{\mathbf{B}}$, we can write

$$\mathbf{E} = \frac{2}{\mathbf{V}_{\mathbf{B}}}\sqrt{\frac{2\boldsymbol{\omega}_{\mathbf{B}}}{\mathbf{b}}}\sqrt{\boldsymbol{\delta}}\ \mathbf{Sech}\left(\frac{\mathbf{x}}{\mathbf{V}_{\mathbf{B}}/\sqrt{2\boldsymbol{\omega}_{\mathbf{B}}}\sqrt{\boldsymbol{\delta}}}\right) [23]$$

Thus if the deviation of $\boldsymbol{\omega}$ from $\boldsymbol{\omega}_{\mathbf{B}}$ is larger, we get bigger but thinner solitons. Closer is $\boldsymbol{\omega}$ to $\boldsymbol{\omega}_{\mathbf{B}}$, smaller and flatter is the soliton. Note that the product of the amplitude and the width is $\mathbf{b}^{-\frac{1}{2}}$, which is almost independent of the deviation $\boldsymbol{\delta}$.

Since we have taken the amplitude $\mathbf{E}$ as real, the parameter $\mathbf{b}$ must be positive. From equations (8), (14) and (21) we see that $\mathbf{b} > 0$ amplies, for $\boldsymbol{\omega} \sim \boldsymbol{\omega}_{\mathbf{B}}$ as

$$\left(\frac{\mathbf{n}_{02}}{\mathbf{n}_{01}} - \sqrt{\frac{\mathbf{m}_2}{\mathbf{m}_1}}\right)(\mathbf{m}_2 - \mathbf{m}_1) > 0 \quad [24]$$

For $\mathbf{m}_2 > \mathbf{m}_1$, $\mathbf{n}_{02} > \mathbf{n}_{01}$. The heavier species must be denser. For a deuterium-proton plasma [Perkins, F.W.,NF(1977),17,1197] $\frac{\mathbf{m}_2}{\mathbf{m}_1} \sim 2$. Therefore $\frac{\mathbf{n}_{01}}{\mathbf{n}_{02}} \leq \frac{1}{\sqrt{2}} \sim 0.71$. This is not so difficult to be satisfied.

In the case of the heavier species being less dense, for example a deuterium-tritium plasma as discussed by Perkins [Nuclear Fusion (1977),17,1197] or for $\boldsymbol{\omega} > \boldsymbol{\omega}_{\mathbf{B}}$ the amplitude $\mathbf{E}$ of the nonlinear ion – ion hybrid mode, according to equation (23) , becomes complex. Under such conditions our formulation breaks down.

IV.Conclusion

Standard plasma heating models suggest the mode conversion of an electromagnetic wave into electrostatic waves during the ICRH propagation. This mode conversion process couples power to the fast minority ion tail, majority ions and electrons. The mode converted electrostatic wave which evolves as a MC bi-product in the vicinity of ion-ion hybrid resonance layer contain an interesting nonlinear physics. In this paper, nonlinear fate of the mode converted electrostatic wave in the vicinity of the MC layer has been taken into account. We analyse the mode converted electrostatic wave and the associated plasma dynamics in the vicinity of the ion-ion hybrid layer and the demonstrate the presence of nonlinear localised electric-field structures in the form of KdV soliton.

Further, the low frequency modes considered for this purpose are adiabatic perturbations. The nonlinear equation, which has ion-ion hybrid soliton as its solution, represents the nonlinear stage of the purely growing mode $(\boldsymbol{\omega} \sim \boldsymbol{\omega}_{\mathbf{B}})$. It turns out that ion-ion hybrid soliton exists if the wave frequency is less than the Buschbaum resonance frequency $\boldsymbol{\omega}_{\mathbf{B}}$ and if the concentration of lighter ions is less than the heavier ones, $\frac{\mathbf{n}_{01}}{\mathbf{n}_{02}} < \sqrt{\frac{\mathbf{m}_1}{\mathbf{m}_2}}$ for $\mathbf{m}_2 > \mathbf{m}_1$. It is noticed that the square of the amplitude of the soliton under discussion goes as $\mathbf{Sech}^2\mathbf{x}\sqrt{\mathbf{a}}$ which is similar to KdV soliton. Thus the energy in the ion-ion hybrid mode, under the conditions mentioned above, will be distributed as the KdV soliton. Physically, density cavities are generated due to the density modulations invoked by the ponderomotive expulsion of plasma. Further, these density cavities grow with trapped electric fields and leads to intense localized electric fields of the soliton form. Heating efficiency have been found to be dependent on the relative concentration of the minority ionic species.

It is important to discuss the limitations of the discussed analytical model. In this paper, the effect of shear in the geometry as well as the effect of inhomogeneity in the magnetic field is not taken into account in the treatment. The inclusion of these physical effects will certainly improve the model to a much realistic geometrical configuration as applicable in tokamaks. These works are in progress and will be dealt in the future publications. Recently, standard ICRF-simulation codes have begun incorporating nonlinear treatments and saturation mechanisms for the investigation of problems like wave-particle interaction and power transmission across various mode conversion layers. Quantitative analysis of these discussed dominant nonlinear phenomenon pre and post mode conversion processes seems to be important in the full wave simulation techniques which are being used for the estimation of local and global electric field profiles across the various mode conversion layers (41and references therein,44) and in plasma diagnostic techniques such as Ion-Ion hybrid reflectometry used for tracing minority species.

Appendix

A.Velocity field component calculation from ion equation of motion

Assume that $\mathbf{V}_\alpha = \left(V_{\alpha x}, V_{\alpha y}, V_{\alpha z}\right)$, $\mathbf{\Omega}_\alpha = \left(0,0,\Omega_\alpha\right)$, $\mathbf{E} = \left(E,0,0\right)$, $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ So,

$$\mathbf{V}_\alpha \times \mathbf{\Omega}_\alpha = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ V_{\alpha x} & V_{\alpha y} & V_{\alpha z} \\ 0 & 0 & \Omega_\alpha \end{vmatrix} = \mathbf{i}\begin{vmatrix} V_{\alpha y} & V_{\alpha z} \\ 0 & \Omega_\alpha \end{vmatrix} - \mathbf{j}\begin{vmatrix} V_{\alpha x} & V_{\alpha z} \\ 0 & \Omega_\alpha \end{vmatrix} + \mathbf{k}\begin{vmatrix} V_{\alpha x} & V_{\alpha y} \\ 0 & 0 \end{vmatrix} =$$

$$= \Omega_\alpha V_{\alpha y}\, \mathbf{i} - \Omega_\alpha V_{\alpha x}\, \mathbf{j} + 0\mathbf{k}$$

Consider perturbation case along x-axis $\mathbf{V}_\alpha = \mathbf{V}_\alpha \exp\left(\mathbf{i}\left(\mathbf{kx} - \boldsymbol{\omega}\mathbf{t}\right)\right)$

and transform time derivative only $\frac{\partial \mathbf{V}_\alpha}{\partial \mathbf{t}} \to -\mathbf{i}\boldsymbol{\omega}\mathbf{V}_\alpha$ so that $-\mathbf{i}\boldsymbol{\omega}\mathbf{V}_\alpha = \frac{\mathbf{eE}}{\mathbf{m}_\alpha} + \mathbf{V}_\alpha \times \Omega_\alpha - \frac{\mathbf{T}}{\mathbf{m}_\alpha}\frac{\nabla \mathbf{n}_\alpha}{\mathbf{n}_\alpha}$

yields $\mathbf{V}_\alpha = \frac{\mathbf{i}}{\boldsymbol{\omega}}\left(\frac{\mathbf{eE}}{\mathbf{m}_\alpha} + \mathbf{V}_\alpha \times \Omega_\alpha - \frac{\mathbf{T}}{\mathbf{m}_\alpha}\frac{\nabla \mathbf{n}_\alpha}{\mathbf{n}_\alpha}\right)$ in vector form, the expression yields as follows

$$V_{\alpha x}\mathbf{i} + V_{\alpha y}\mathbf{j} + V_{\alpha z}\mathbf{k} = \frac{\mathbf{i}}{\boldsymbol{\omega}}\left(\frac{\mathbf{e}}{\mathbf{m}_\alpha}\left(E\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}\right) + V_{\alpha y}\Omega_\alpha \mathbf{i} - V_{\alpha x}\Omega_\alpha \mathbf{j} + 0\mathbf{k} - \frac{\mathbf{T}}{\mathbf{m}_\alpha}\frac{1}{\mathbf{n}_\alpha}\left(\frac{\partial \mathbf{n}_\alpha}{\partial x}\mathbf{i} + \frac{\partial \mathbf{n}_\alpha}{\partial x}\mathbf{j} + \frac{\partial \mathbf{n}_\alpha}{\partial x}\mathbf{k}\right)\right)$$

For vector equation,a set of scalar equation by coordinates comes out to be

$$V_{\alpha x} = \frac{\mathbf{i}}{\boldsymbol{\omega}}\left(\frac{\mathbf{e}E}{\mathbf{m}_\alpha} + V_{\alpha y}\Omega_\alpha - \frac{\mathbf{T}}{\mathbf{m}_\alpha \mathbf{n}_\alpha}\frac{\partial \mathbf{n}_\alpha}{\partial x}\right)$$

$$V_{\alpha y} = -\frac{\mathbf{i}}{\boldsymbol{\omega}}\left(V_{\alpha x}\Omega_\alpha + \frac{\mathbf{T}}{\mathbf{m}_\alpha \mathbf{n}_\alpha}\frac{\partial \mathbf{n}_\alpha}{\partial y}\right)$$

$$V_{\alpha z} = \frac{\mathbf{i}}{\boldsymbol{\omega}}\left(0 + 0 - \frac{\mathbf{T}}{\mathbf{m}_\alpha \mathbf{n}_\alpha}\frac{\partial \mathbf{n}_\alpha}{\partial z}\right)$$

Since we are interested in x-direction only we are able to drop partial derivatives along y- and z- directions and neglect z- coordinate of velocity, i.e.

$$V_{\alpha x} = \frac{\mathbf{i}}{\boldsymbol{\omega}}\left(\frac{\mathbf{e}E}{\mathbf{m}_\alpha} + V_{\alpha y}\Omega_\alpha - \frac{\mathbf{T}}{\mathbf{m}_\alpha \mathbf{n}_\alpha}\frac{d\mathbf{n}_\alpha}{dx}\right)$$

$$V_{\alpha y} = -\frac{\mathbf{i}}{\boldsymbol{\omega}}V_{\alpha x}\Omega_\alpha$$

$$V_{\alpha z} = 0$$

B. Quasi-neutrality Conditions and corresponding operator estimations

$$\mathbf{n}_1 + \mathbf{n}_2 = 0$$

which yields

$$\mathbf{O}_1^{-1}\left[\frac{\mathbf{e}}{\mathbf{m}_1}\frac{\mathbf{d}}{\mathbf{dx}}\left(\mathbf{n}_1\mathbf{E}\right)\right] + \mathbf{O}_2^{-1}\left[\frac{\mathbf{e}}{\mathbf{m}_2}\frac{\mathbf{d}}{\mathbf{dx}}\left(\mathbf{n}_2\mathbf{E}\right)\right] = 0$$

$$\mathbf{m}_2\mathbf{O}_2\left(\mathbf{n}_1\mathbf{E}\right) + \mathbf{m}_1\mathbf{O}_1\left(\mathbf{n}_2\mathbf{E}\right) = 0$$

C. Nonlinear Ponderomotive force Estimations

$$\mathbf{F} = -\mathbf{m}_\alpha\left\langle \underset{\sim}{\mathbf{V}_\alpha}.\nabla \underset{\sim}{\mathbf{V}_\alpha}\right\rangle$$

Considering x component, one could rewrite in the following form

$$\left\langle \underset{\sim}{\mathbf{V}_\alpha}.\nabla \underset{\sim}{\mathbf{V}_\alpha}\right\rangle = \mathbf{V}_{\alpha x}\frac{\mathbf{d}}{\mathbf{dx}}\mathbf{V}_{\alpha x} = \frac{\mathbf{ie}}{\mathbf{m}_\alpha}\frac{\boldsymbol{\omega}}{\boldsymbol{\omega}^2 - \Omega_\alpha^2}E\frac{\mathbf{d}}{\mathbf{dx}}\left(\frac{\mathbf{ie}}{\mathbf{m}_\alpha}\frac{\boldsymbol{\omega}}{\boldsymbol{\omega}^2 - \Omega_\alpha^2}E\right) = -\frac{\mathbf{e}^2}{2\mathbf{m}_\alpha^2}\frac{\boldsymbol{\omega}^2}{\left(\boldsymbol{\omega}^2 - \Omega_\alpha^2\right)^2}\frac{\mathbf{d}}{\mathbf{dx}}\left(E^2\right)$$

where the average is taken over the ion – ion hybrid periods. It turns out that

$$\mathbf{F}_\alpha = \mathbf{F}_{\alpha x} = -\mathbf{m}_\alpha \mathbf{q}_\alpha \frac{\mathbf{d}}{\mathbf{dx}} \mathbf{E}^2$$

so that, $\mathbf{F}_\alpha = \mathbf{F}_{\alpha x} = -\mathbf{m}_\alpha \left\langle \underset{\sim}{\mathbf{V}_\alpha} . \nabla \underset{\sim}{\mathbf{V}_\alpha} \right\rangle = -\mathbf{m}_\alpha \left(-\frac{\mathbf{e}^2}{2\mathbf{m}_\alpha^2} \frac{\boldsymbol{\omega}^2}{\left(\boldsymbol{\omega}^2 - \Omega_\alpha^2\right)^2} \right) \frac{\mathbf{d}}{\mathbf{dx}} \left(\mathrm{E}^2 \right)$

where $\mathbf{q}_\alpha = \frac{\mathbf{e}^2}{2\mathbf{m}_\alpha^2} \frac{\boldsymbol{\omega}^2}{\left(\boldsymbol{\omega}^2 - \Omega_\alpha^2\right)^2}$

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