

Effect of apodization on two point resolution with truncated aperture

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Abstract

Two point resolution due to high intensity side lobes caused by optical aberrations when the intensity ratio between the two-object points is $\alpha=0.2$. Truncation of circular aperture is greatly reducing the effect of aberrations. Further suppression of side lobes is needed to improve resolving power of optical system. Keeping this point in focus we have adopted apodization technique to improve the two-point resolution. For this process we have selected Hanning amplitude filter. In this work we are reporting two point resolution of an optical imaging system with truncated apertures by apodization for intensity ratios $\alpha=0.2, 0.4, 0.6, 0.8$ and 1.0 under various coherence illumination. The resolution of imaging system was confirmed by taking the Sparrow limit as a standard.

Key words:- Two-point resolution, truncated aperture, defocus, spherical aberration and Sparrow limit.

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I. Introduction

Apodization is the simplest and widely used technique to suppress the diffraction side lobes. T. Asakura and T. Ueno formulated an apodization scheme via calculus of variations [1]. Earlier, Asakura, apodization found in the article of Jacquinot and Roizen-Dossier [2]. Patra W. Peng and J. Ernest Wilkins, Jr published a paper on Apodization for maximum Strehl criterion and specified Sparrow limit of resolution for incoherent illumination [3]. Keshavulu, R. Sayanna, D. Karuna Sagar, S. L. Goud, studied on Effects of defocusing on the Sparrow limits for apodized optical systems with circular aperture [4]. D. Karunasagar et al investigated on Defect of focus in two-line resolution with Hanning amplitude filters [5]. A. N. K. Reddy and Dasari Karunasagar studied Two-Point resolution of Asymmetrically apodized optical systems [6]. A. N. K. Reddy et al investigated in two point resolution with primary aberration [7-10]. In this paper we are studying the resolution of optical system with various truncated apertures under apodization.

II. Theory

The mathematical expression for resultant intensity distribution of two point objects at a distance of Z_0 apart is given as

$$I(Z) = \left| G\left(Z - \frac{Z_0}{2}\right) \right|^2 + \alpha \left| G\left(Z + \frac{Z_0}{2}\right) \right|^2 + 2\gamma\sqrt{\alpha} \left| G\left(Z - \frac{Z_0}{2}\right) \right| \left| G\left(Z + \frac{Z_0}{2}\right) \right| \quad (1)$$

Where Z is the reduced dimensionless diffraction coordinate, $\frac{Z_0}{2}$ is the position of two point objects on both sides of the optical axis. $G\left(Z - \frac{Z_0}{2}\right)$ and $G\left(Z + \frac{Z_0}{2}\right)$ are the amplitude response of the optical system, α is the ratio of intensity taken as $0.2, 0.4, 0.6, 0.8, 1.0$, γ is the degree of coherence

The normalized amplitude PSF function of two point objects for Truncated aperture under optical apodization can be written as

$$G\left(Z \pm \frac{Z_0}{2}\right) = 2 \int_0^{1-\epsilon} P(r) J_0 \left[\left(Z \pm \frac{Z_0}{2} \right) r \right] r dr \quad (2)$$

Where J_0 is zero order first kind Bessel function, $P(r)$ is the pupil optical transmission function.

We are using Hanning amplitude filter for optical apodization. Mathematical expression of Hanning filter is $P(r) = \cos(\pi\beta r)$. Where β is the apodization parameter.

In our study two wave front aberrations included, but equation 2 give the amplitude response of imaging system in the aberration free case. The amplitude response function in the case of defocus and spherical aberration can be expressed as

$$G\left(\phi_d, \phi_s, Z \pm \frac{Z_0}{2}\right) = 2 \int_0^1 P(r) e^{-i(\phi_d \frac{r^2}{2} + \phi_s \frac{r^4}{4})} \left[G\left(Z + \frac{Z_0}{2}\right) + \alpha G\left(Z - \frac{Z_0}{2}\right) + 2\sqrt{\alpha}\gamma G\left(Z + \frac{Z_0}{2}\right) G\left(Z - \frac{Z_0}{2}\right) \right] \quad (3)$$

Where ϕ_d, ϕ_s are the defocus and spherical aberration parameter.

In the presence of aberrations, the mathematical expression for image intensity distribution is given by

$$I(Z) = \left| G\left(Y, \left(Z - \frac{Z_0}{2}\right)\right) \right|^2 + \alpha \left| G\left(Y, \left(Z + \frac{Z_0}{2}\right)\right) \right|^2 + 2\gamma\sqrt{\alpha} \left| G\left(Y, \left(Z - \frac{Z_0}{2}\right)\right) \right| \left| G\left(Y, \left(Z + \frac{Z_0}{2}\right)\right) \right|^2$$

Where $Y = e^{-i(\phi_d \frac{r^2}{2} + \phi_s \frac{r^4}{4})}$

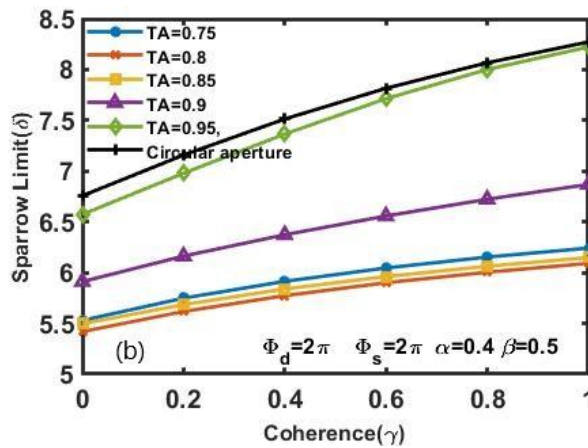
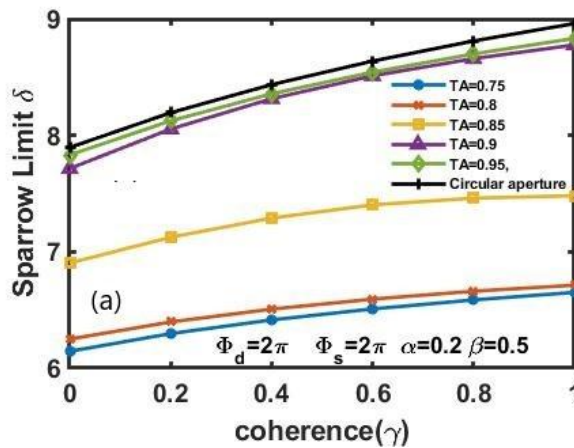
III. Results and Discussions

We calculated the sparrow limits for various truncated apertures at $\beta=0.5$ apodization parameter , $\phi_d =$
 $\phi_s = 2\pi$ with help of MATLAB R2019b tool and tabulated in following Table.

α	γ	TA=0.75	TA=0.8	TA=0.85	TA=0.9	TA=0.95	Circular aperture
0.2	0	6.14488	6.2498	6.9037	7.71501	7.8317	7.89574
	0.2	6.29701	6.39698	7.1243	8.05426	8.1247	8.19481
	0.4	6.41465	6.50735	7.2892	8.31475	8.3572	8.4359
	0.6	6.50887	6.59332	7.4031	8.51105	8.5431	8.63808
	0.8	6.58634	6.66122	7.4608	8.65913	8.6993	8.81108
	1	6.65138	6.71278	7.4813	8.77692	8.8332	8.96062
0.4	0	5.52964	5.419	5.4956	5.91157	6.5732	6.75684
	0.2	5.74853	5.61697	5.6852	6.16296	6.9793	7.15869
	0.4	5.91411	5.77336	5.8393	6.37319	7.3658	7.51361
	0.6	6.04593	5.90017	5.9616	6.55818	7.7138	7.81631
	0.8	6.15346	6.00403	6.063	6.7232	8.0017	8.06788
	1	6.24163	6.09149	6.1489	6.86891	8.2232	8.27259
0.6	0	5.12286	4.95672	4.9162	5.08044	5.4899	5.72744
	0.2	5.3971	5.19558	5.135	5.30834	5.7915	6.08347
	0.4	5.60769	5.38528	5.3108	5.49573	6.0683	6.42032
	0.6	5.77215	5.5393	5.4541	5.6516	6.3287	6.75185
	0.8	5.90391	5.66624	5.5741	5.78689	6.5839	7.09109
	1	6.011	5.77249	5.6761	5.89999	6.8541	7.44427
0.8	0	4.74988	4.56258	4.472	4.52855	4.7719	4.95462
	0.2	5.07167	4.84241	4.7204	4.76902	5.0473	5.26359
	0.4	5.33105	5.06713	4.9219	4.9673	5.2812	5.53908
	0.6	5.5382	5.25155	5.088	5.13128	5.4871	5.78923
	0.8	5.70199	5.40322	5.2273	5.27102	5.673	6.02294
	1	5.83258	5.53026	5.3457	5.39184	5.8435	6.24957
1	0	4.21317	4.02219	3.9019	3.8865	4.0077	4.12609
	0.2	4.58443	4.34988	4.1928	4.15668	4.2872	4.42466
	0.4	4.90849	4.62931	4.4362	4.38135	4.5235	4.68168
	0.6	5.19133	4.86825	4.6414	4.57052	4.7269	4.9071
	0.8	5.43779	5.07305	4.8155	4.73161	4.9042	5.10805
	1	5.64755	5.24912	4.9645	4.87018	5.0609	5.28981

Figure.1a to Figure.1e shows the variation of sparrow limits for different apertures as a function of coherence for extreme defocus and primary spherical aberration. Figure.1a is the sparrow limit of intensity ratio $\alpha=0.2$. Irrespective of aperture, sparrow limit increases with increase in coherence parameter. Higher sparrow limits are recorded in the case of circular aperture. With increasing in the truncation parameter, Sparrow limits show a decreasing trend and lower sparrow limits are recorded in for truncated aperture with TA = 0.75. Figure.1b show sparrow limit for intensity ratio $\alpha=0.4$; for this intensity ratio highest Sparrow limit recorded for circular aperture and lowest Sparrow limit is noted for truncated aperture TA = 0.85. Highest resolution with less limit is noted for truncated aperture TA = 0.85.

For intensity ratios with $\alpha=0.6$, and 0.8 exhibits highest Sparrow limits for circular aperture and lowest sparrow limits found for truncated aperture with TA = 0.85. In the case of $\alpha=0.6$ sparrow limit changing linearly only for circular and truncated aperture 0.95 and the linearity collapsed in remaining apertures. At equal intensity ratio higher sparrow limit recorded in 0.75 truncated aperture and lowest values recorded in 0.9 truncated aperture.



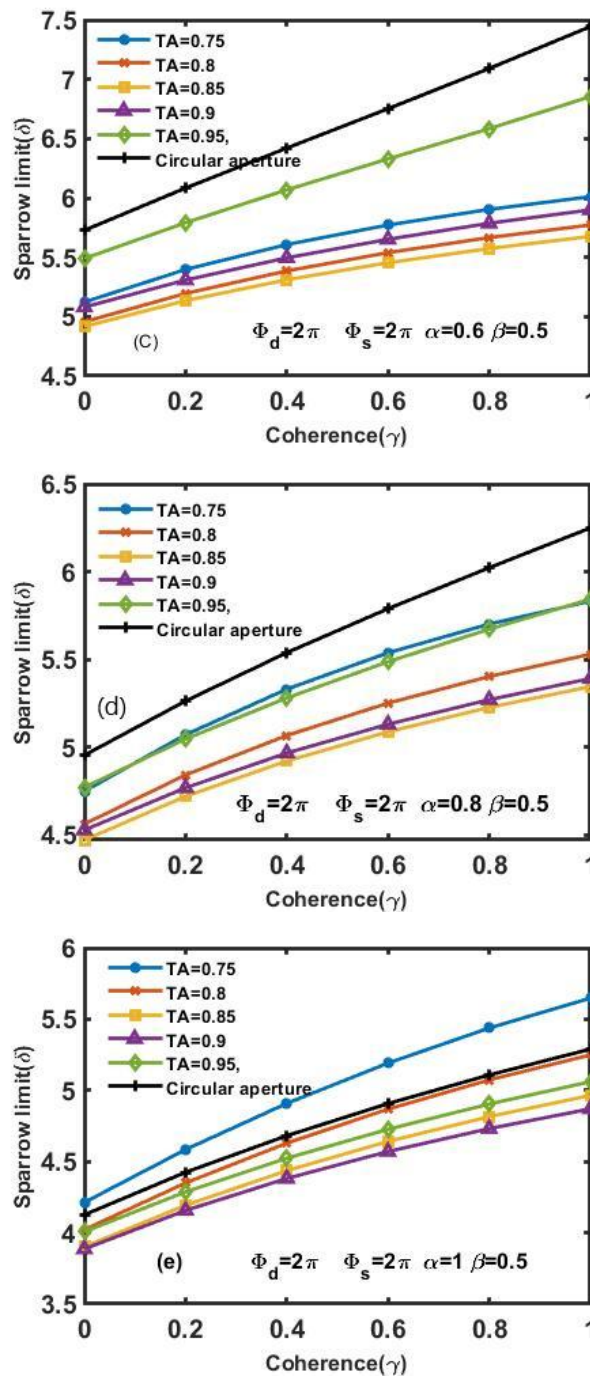
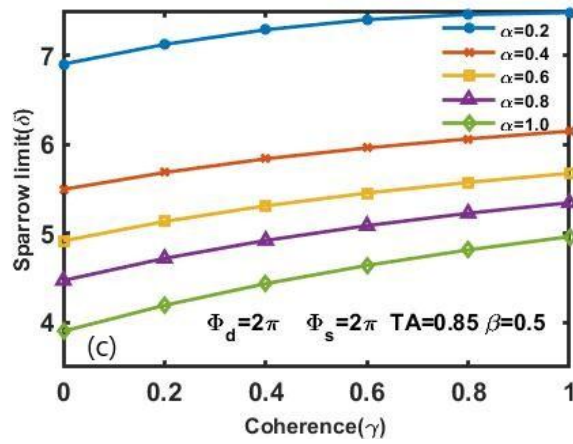
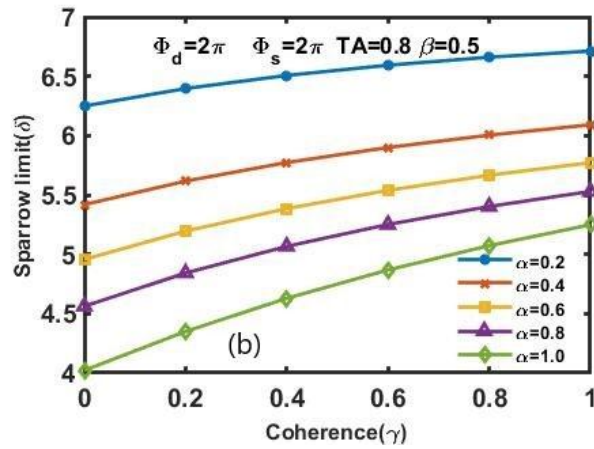
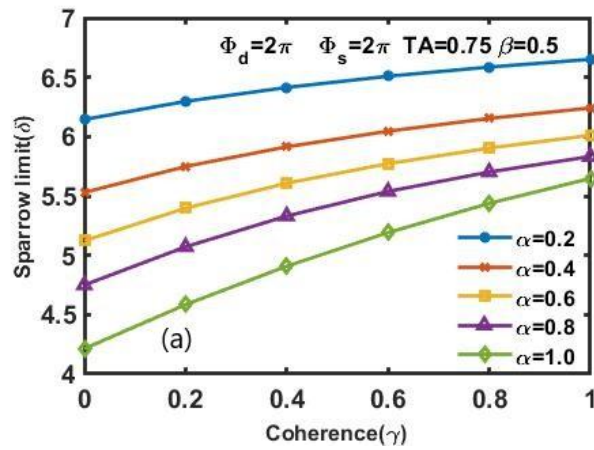


Figure.1 depicts the variation of sparrow limit as a function of coherence for various apertures and intensity ratios under apodization parameter $\beta=0.5$ in the presence of high defocus and spherical aberration.

Figure.2a to 2.f illustrates the aperture wise variation of sparrow limit as a function of coherence for various intensity ratio. Irrespective of aperture, resolution limits increasing with increase of coherence and higher limits recorded at higher intensity difference side and with decrease in the intensity ratio, resolution limit decreases.



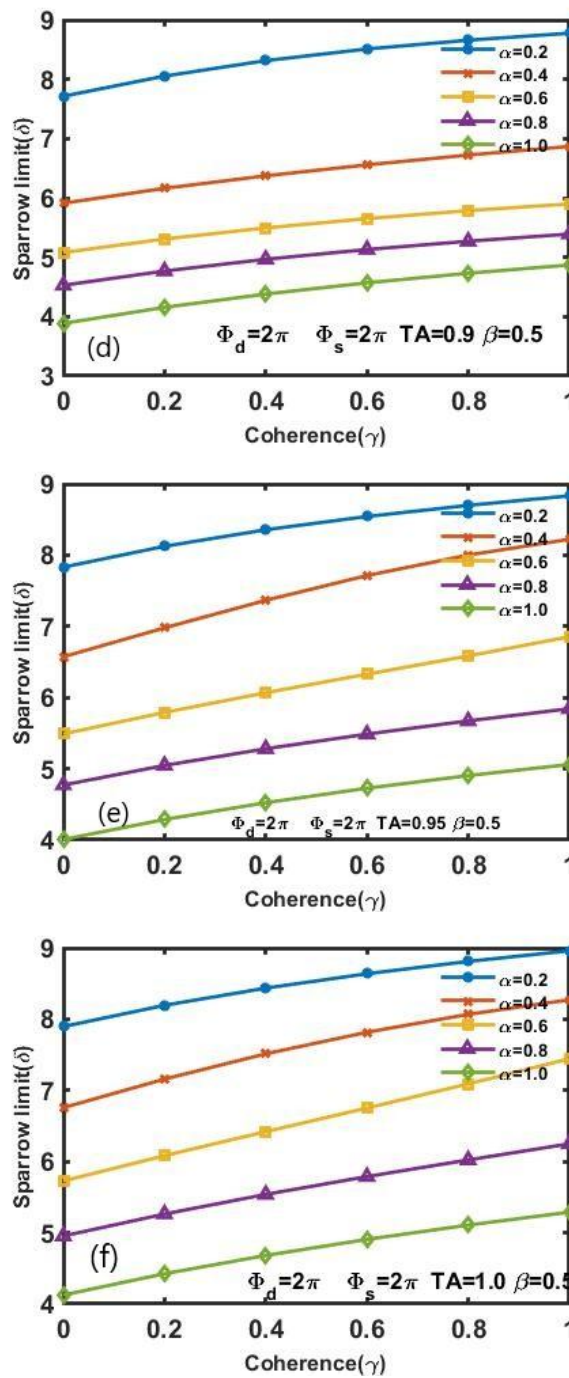


Figure.2 Aperture wise variation of sparrow limit as a function of coherence.

IV. Conclusion

In this paper we studied the resolution ability of optical imaging system with Truncated apertures and circular aperture. In my investigation we observed that no single aperture can resolve two different intensive point objects effectively. For $\alpha=0.2$, 0.75 Truncated aperture ; $\alpha=0.4$, 0.8 Truncated aperture; $\alpha=0.6$ and 0.8 intensity ratios, 0.85 Truncated aperture and for $\alpha=1.0$, 0.9 Truncated aperture Resolution limits are small.

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