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On The Intrinsic Mathematical Limitations Of The Lorentz Transformation Equations

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Abstract:
The fundamental premise of both the GZK and TeV-γ photon theories relies on the concept of Lorentz invariance. In our previous work[1], we introduced a novel perspective on the spatial orientation of inertial frames and highlighted a fundamental mathematical limitation of the Lorentz transformation equations in relating the space and time coordinates of two inertial frames in relative motion. This insight successfully explained the presence of ultra-high-energy cosmic ray particles and multi-TeV gamma photons on Earth — an observation that contradicts the predictions of the GZK and TeV-γ photon theories. In this paper, we provide further evidence and clarification to support these insights.
Keywords: *GZK Theory, TeV- γ photon theory, Orientation of inertial frames, Lorentz transformation.*

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I. Introduction

The scientific community has encountered significant challenges in reconciling observational data with the predicted limits imposed by the Greisen–Zatsepin–Kuzmin (GZK) cutoff and TeV-scale gamma photon attenuation theories[2,3,4,5,6,7], which are based on the current understanding of the theoretical framework of the Lorentz transformations and thus, of Lorentz invariance. This challenge arises due to the experimental detection of ultra-high-energy cosmic ray (UHECR) particles and multi-TeV gamma photons on Earth[8,9,10], whose observed energies exceed the theoretical cutoffs imposed by these theories. Despite extensive investigative efforts[11,12], no satisfactory explanation has been found for the existence of such energetic particles. This discrepancy has prompted a critical re-examination of the fundamental principles underlying Lorentz invariance, ultimately leading to novel theoretical insights that resolved the issue, as outlined in our previous research[1].

A novel perspective on the spatial orientation of inertial frames and the inherent limitations of the Lorentz transformation equations in relating the space and time coordinates of two inertial frames in motion relative to each other has garnered significant interest among researchers. In this paper, we present additional evidence supporting these insights. To achieve this, it is essential to revisit and reproduce some of the key material and arguments presented in our previous work[1].

II. Spatial Orientation of The Inertial Frames in Lorentz Transformation Derivation

The position coordinates of an event, as measured from the inertial frames in 'uniform translational motion relative to one another, as illustrated in (Fig.1), can be related using the Lorentz transformation equations. These transformations are a cornerstone of special relativity, forming the basis for deriving many of its key predictions. In this analysis, we first note that modern applications of the Lorentz transformation often overlook the *relative spatial orientation* of inertial frames (or coordinate systems) in motion relative to one another. To address this issue, we revisit the spatial orientation of the frames in motion relative to one another as originally described by Einstein[13], to derive the Lorentz transformation equations.

Einstein starts with two coordinate systems S and S' (Fig.1) positioned in standard configuration at time $t = t' = 0$, when their origins and all three axes coincide. He then states[13] that *now to the origin of one of the two systems let a constant velocity be imparted in the direction of the increasing X of the other stationary system and let this velocity be communicated to the axes of the coordinates, the relevant measuring rods, and the clocks.* If a point event is described by the coordinates (x, y, z, t) and (x', y', z', t') relative to S and S' respectively, then, to find a mathematical relation between these sets, Einstein introduced two fundamental postulates and based on them, arrived at the following transformation equations

$$x' = \gamma(x - vt),$$
$$y' = y, z' = z,$$
$$t' = \gamma(t - vx / c^2).$$

Lorentz, using the same orientation of coordinate systems, independently arrived at the results now known as the Lorentz transformation, as noted by Poincaré[14], who gave them this name. It is worth noting that the Galilean transformation was also derived using the same orientation of coordinate systems. This orientation is, thus, critical for both the derivation and application of the Lorentz transformation.

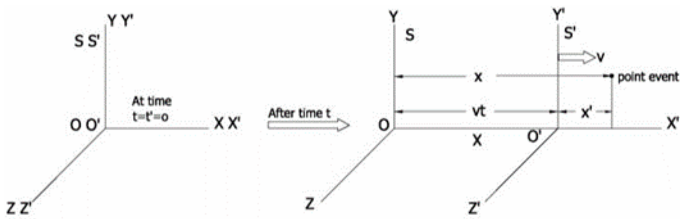

Fig. 1. The inertial frames *S* and s' are initially in standard configuration at time $t = t' = 0$, prior to the commencement of uniform translational motion. After a time *t*, s' has moved a distance *vt* along the positive X-axis of *S*. In the geometric relation, $x' = \gamma(x - vt)$, *vt* (a vector quantity) also describes the relative location in space of the two inertial frames used to measure the coordinates of the given point event.

III. Standard Configuration and Its Role in Lorentz Transformation Applicability

(i) It is essential to recognize that the pioneers of relativity theories we quoted here — Galileo, Einstein, Lorentz, and Poincaré — consistently adopted the same orientation of inertial frames when dealing with the derivation of the transformation equations. Another key aspect of their approach is that inertial frames are always in standard configuration before the onset of uniform translational motion relative to one another. We find that this prerequisite is essential, as it ensures that the spatial and temporal relationships between the frames are well-defined prior to the commencement of their relative motion. Specifically, at the initial moment $t' = t = 0$, the origins of the two frames coincide, satisfying the relation $x' = x$. This establishes a clear initial correspondence between their space and time coordinates. Remarkably, this precondition can be obtained by substituting, $v = 0$, into the Lorentz transformation equations, which simplify to

$$x' = x$$
$$y' = y, z' = z,$$
$$t' = t$$

Clearly, inertial frames in standard configuration have a fundamental and intrinsic relationship with the Lorentz transformation equations, making this configuration a prerequisite for their applicability. It follows that if a pair of inertial frames satisfies the simplified form of the Lorentz transformation equations (2) before the onset of uniform translational motion relative to one another, i.e., when $v = 0$, then only the full Lorentz transformation equations (1) remain applicable at any subsequent time *t*. In other words, the two inertial frames must possess a well-defined initial space-time relationship, effectively "recognizing/knowing" each other in advance. This predefined relationship serves as the basis for determining unknown space-time relationships at any later time *t*. If this precondition is not met - if the frames are not in standard configuration at the start - the Lorentz transformations cannot be applied at any later time *t*.

(ii) While imparting a constant velocity to the origin of one of the two stationary systems, Einstein asserted[13], *"Let this velocity be communicated to the axes of the coordinates, the relevant measuring rods, and the clocks."* Despite its apparent simplicity, this important assertion has not received the attention it deserves, even though it encapsulates another critical precondition for the applicability of the Lorentz transformation equations. This assertion implies that the magnitude of the velocity must become known to both inertial frames, along with their associated measuring rods and clocks—that is, to both inertial observers at that very instant it is imparted. The significance of this condition lies in the fact that knowledge of the quantity, *v*, at the onset of relative motion is essential as it serves as a fundamental parameter for determining the precise spatial and temporal relationships between the two inertial frames at any later time *t* via the Lorentz transformation equations.

Thus, inertial frames in standard configuration not only "know" each other in terms of their initial spatial and temporal relationships but also "know" the magnitude of their relative velocity, ensuring the applicability of the Lorentz transformation.

(iii) The significance of the term, $x^{'} = x - vt$, which appears in both the Galilean and the Lorentz transformations, has already been emphasized[1]. This term is geometrically defined by the spatial location of the point event and the relative spatial positioning of the two inertial frames in relative translational motion and can be easily identified in Fig. 1. This diagram also reflects the fact that the relative motion began from the position of standard configuration. Once the relative motion begins from this configuration, the frame $S^{'}$ moves a distance *vt* along the positive *X* direction of *S* after time *t*. Therefore, the term, $x^{'} = x - vt$, not only establishes the relationship between the coordinates $x^{'}$ and *x*, but also encodes the information about the relative spatial positions of the two frames, through the displacement vector *vt,* from which these coordinates are measured.

IV. Inertial Frames In A Different Spatial Orientation And The Lorentz Transformation

Now, let us consider two other inertial frames *K* and $K^{'}$ in a different orientation as indicated in Fig. 2, relatively in motion towards one another, in contrast to the inertial frames *S* and $S^{'}$, which move away from each other. This '*towards'* motion may lead to a head-on collision. The same point event now has the coordinates (x, y, z, t) and $(-x^{'}, y^{'}, z^{'}, t^{'})$ relative to *K* and $K^{'}$ respectively, *if we choose to use the same notations as for S and* $S^{'}$. By comparing the orientations, directions of motion, and relative spatial positions of the inertial frames in Fig. 2 with those in Fig. 1, it becomes clear that,

(a) The $x^{'}$ and x no longer satisfy the relation, $x^{'} = x - vt$. The displacement vector *vt* is missing in Fig. 2.
(b) The inertial frames, *K* and $K^{'}$, do not satisfy the preconditions for the applicability of the Lorentz transformation equations outlined in the previous Section.
(c) Specifically, since this pair of inertial frames does not satisfy the simpler form of the Lorentz transformation equations (2) before the start of their relative motion, the Lorentz transformation (1) cannot be applied to them at any later time *t*.
(d) This limitation inherent in the Lorentz transformation is less surprising than it may initially seem, for we are already subtly aware of this restriction as explained in our previous work[1].

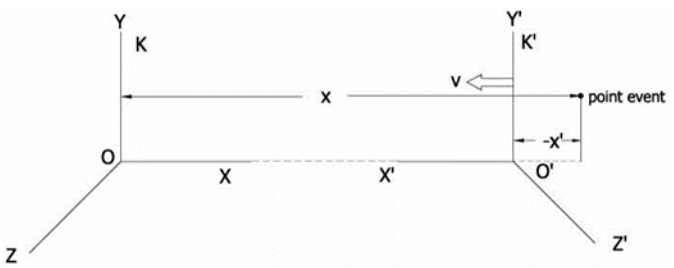

Fig. 2. In stark contrast to Fig. 1, the inertial frame $K^{'}$ is relatively in motion towards *K*. In this scenario, the coordinates, *x* and $x^{'}$, of the same point event do not satisfy the geometric relation $x^{'} = x - vt$.

Einstein was also aware that the Lorentz transformation is applicable only to a specific orientation of inertial frames, but he did not emphasize this fact. In his well-known book[15], *Relativity: The Special and General Theory (Chapter 11, "Lorentz Transformation")*, he poses the question: What are the values of $x^{'}, y^{'}, z^{'}, t^{'}$, of an event with respect to 'moving system', when the magnitudes x, y, z, t, of the same event with respect to 'stationary system' are given? He then states that, "*For the relative orientation in space of the* *coordinate systems indicated in the diagram* (This diagram is similar to our Fig. 1) *this problem is solved by means of the Lorentz transformation equations"*. This statement clearly affirms that the Lorentz transformation is valid only for a specific orientation of the inertial frames.

While the principle of relativity of the *physical* theory of relativity undoubtedly asserts the equivalence of all inertial frames, *mathematically* the Lorentz transformation derived from a particular orientation of inertial frames can not be universally applied to frames in a different orientation. A distinct mathematical framework is required for such cases.

Since the Lorentz transformation is not valid for the inertial frames K and $K^{'}$ in relative orientation in space as indicated in Fig. 2, it is theoretically invalid to Lorentz transform the collision, which may occur due to their motion relatively towards each other, from one frame to the other. Notably, the GZK cut-off was determined by *Lorentz transforming* the collision between the UHECR proton and the CMBR photon from the '*universal frame*' to the '*projectile rest frame*', where photopion production was expected to occur. However, these two frames also fail to satisfy the necessary preconditions for subjecting them to the Lorentz transformations, thereby rendering the operation of Lorentz transformation of the collision theoretically invalid. As a result, the anticipated photopion production did not occur. The same issue arises in the case of TeV-scale gamma-ray attenuation theories. Consequently, the cutoffs proposed by the GZK and TeV-γ photon theories are rendered theoretically flawed. Therefore, Einstein's Special Relativity does not need to be modified or replaced[16,17,18,19,20,21] to explain the predictions of these theories.

In the standard application of relativistic mechanics to particle collision problems, no distinction has ever been necessary between the S - S' and K - K' orientations of inertial frames. However, this observation underscores an important point: it is the mathematical framework of the Lorentz transformations, rather than relativistic mechanics itself, that differentiates between these orientations. This suggests that the successful application of relativistic mechanics in particle collision problems does not inherently depend on the use of Lorentz transformation. We demonstrated this conclusively, in our previous work[1] by re-examining well-studied cases of particle collisions.

It is common practice to denote the axes of any pair of inertial frames in motion relative to each other, as (X,Y,Z) and $(X^{'},Y^{'},Z^{'})$, *without considering their spatial orientation*. However, this leads to confusion, primarily because the same symbols originally assigned to the frames S and $S^{'}$ - from which the Lorentz transformation were derived, are reused even when referring to differently oriented inertial frames, such as, K and $K^{'}$. This reuse creates the misleading impression that the coordinates (x,y,z,t) and $(x^{'},y^{'},z^{'},t^{'})$ obtained from the K and $K^{'}$ are identical to those in the Lorentz transformation equations (1), thereby implicitly suggesting that the later remain applicable. However, we now recognize that this assumption is incorrect. To eliminate this ambiguity, it is preferable to use distinct symbols, such as (L,M,N) and $(L^{'},M^{'},N^{'})$ for the axes of differently oriented inertial frames K and $K^{'}$. With these notations, the coordinates of the given point event would be expressed as (l,m,n,t) and $(-l^{'},m^{'},n^{'},t^{'})$, requiring a clear relation with the coordinates (x,y,z,t), $(x^{'},y^{'},z^{'},t^{'})$ appearing in the Lorentz transformation to determine whether the transformation equations can be validly applied.

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