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On the Second Order Temporal Coherence: Two-Photon Interference

Vandana Arora[1], Jagneet Kaur Anand[2]
[1](*Department of Physics, Keshav Mahavidyalaya, University Of Delhi, India*)
[2](*Department of Electronics, Keshav Mahavidyalaya, University Of Delhi, India*)

Abstract:
The classical interpretation of 2nd order coherence is studied by considering the four-point correlation function in case of two-photon interference. Mathematical equations governing these phenomenon are established. The parametric down-conversion for the generation of two entangled photons is discussed.

***Key Word**: Correlation; Temporal Coherence; vacuum mode; Polarization; Two-photon interference; entanglement; idler frequency*

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I. Introduction

Starting with the concept of four-point correlation, the mathematics for various degrees of moment is established. The first and second order degrees of moment (mean and the standard deviation) are discussed in detail. Therefrom a 2nd order non-linear medium is considered and non-linear polarization in terms of the propagating electric field is discussed. The generation of two entangled photons by utilizing the parametric optical amplification/oscillation is highlighted. The vacuum modes which can only be explained using only the quantum theory are responsible for the down-conversion of a photon of high frequency into two entangled photons of lower frequencies. These two entangled photons interfere with themselves to capitulate the interference pattern.

II. Background: Coherence and Correlation

A pulsed laser source has perfect correlation among the frequency components emitted by it. So it has ideal degree of temporal coherence (unity) and ideal degree of spectral-coherence (unity). The pulsed structure in its intensity profile is due to fact that all its frequency components add up coherently. Common laser sources such as He-Ne and laser pointers have continuous wave (CW) fields due to zero correlation among its frequency components, implying that these are stationary sources w.r.t. time (Shimoda 2013; Thyagarajan and Ghatak 2010).

There are certain sources which have partial correlation among the frequency components emitted by them and their characteristics lie in between the perfect correlation and zero correlation. Typically, sun radiations have a frequency bandwidth of $\Delta\omega = 100\,THz$. It can be calculated that the sunlight has a coherence time $\tau_c \approx 10^{-14}\,s$ and the coherence length is $c\tau_c \approx 3\,\mu m$. For diode lasers and very well-stabilized lasers, the typical values for frequency bandwidth are $1\,GHz$ and $1\,MHz$ respectively; the coherence times are $1\,ns$ and $1\,\mu s$ respectively; and the coherence lengths are $0.3\,m$ and $300\,m$ respectively.

A coherence function (Luo and Sun 2017; Schlosshauer 2007; Schlosshauer 2019) describes the degree and nature of correlation that exists in any source of electromagnetic (E. M.) field. In classical physics, coherence yields the degree of order in a random field. Whereas, in quantum physics, the coherence explains the entangled fields.

Mathematically, if an experiment (engaging a random process) is performed repeatedly then the ensemble average or the expectation value of its outcome x can be evaluated in terms of its probability distribution $p(x)$ as:

$$\langle x \rangle = \int x\, p(x)\, dx \qquad (1)$$

In a random process, it is not just the expectation value which carries relevant information, but instead one can evaluate a hierarchy of expectation values from the experiment outcome (x), called the r^{th} order moment of the random process. Mathematically, it can be computed as below:

$$\langle x^r \rangle = \int x^r\, p(x)\, dx \qquad (2)$$

Eqn. (2) provides a broad over view and varying depths of information about the random process as discussed in the following paragraph.

The first moment, *i.e.*, for $r = 1$ is called the 'mean' value. One can calculate the higher moments around the 'mean' value and these are called the central moments of the order 'r':

$$\langle (x - \langle x \rangle)^r \rangle = \int (x - \langle x \rangle^r)\, p(x)\, dx \qquad (3)$$

It can be seen from Eqn. (3) that the first central moment, *i.e.*, for $r = 1$, is always zero. We define the variance (σ^2) and standard deviation (σ) as the 2nd central moment and its square root respectively.

$$\sigma = \sqrt{\langle (x - \langle x \rangle)^2 \rangle} = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \qquad (4)$$

In order to define the correlation function between two random processes x_1 and x_2, we define

$$\langle x_1^m \rangle \langle x_2^n \rangle = \int x_1^m x_2^n\, p(x_1 x_2)\, dx \qquad (5)$$

Eqn. (5) gives the correlation between m^{th} moment of x_1 and n^{th} moment of x_2.
Extending the above equations for correlation to real physical event/process/E. M. source with spatial and temporal dependence $V(\vec{\boldsymbol{r}}, t)$, we can define the Intensity $I(\vec{\boldsymbol{r}}, t)$, two-point correlation function $\Gamma(\vec{\boldsymbol{r}}_1, t_1; \vec{\boldsymbol{r}}_2, t_2)$, and four point correlation function $\Gamma(\vec{\boldsymbol{r}}_1, t_1; \vec{\boldsymbol{r}}_2, t_2; \vec{\boldsymbol{r}}_3, t_3; \vec{\boldsymbol{r}}_4, t_4)$ as:

$$I(\vec{\boldsymbol{r}}, t) = \langle V^*(\vec{\boldsymbol{r}}, t) V(\vec{\boldsymbol{r}}, t) \rangle \qquad (6)$$

Where $V^*(\vec{\boldsymbol{r}}, t)$ is the complex conjugate of $V(\vec{\boldsymbol{r}}, t)$.

$$\Gamma(\vec{\boldsymbol{r}}_1, t_1; \vec{\boldsymbol{r}}_2, t_2) = \langle V^*(\vec{\boldsymbol{r}}_1, t_1) V(\vec{\boldsymbol{r}}_2, t_2) \rangle \qquad (7)$$

And $\Gamma(\vec{\boldsymbol{r}}_1, t_1; \vec{\boldsymbol{r}}_2, t_2; \vec{\boldsymbol{r}}_3, t_3; \vec{\boldsymbol{r}}_4, t_4) = \langle V^*(\vec{\boldsymbol{r}}_1, t_1) V^*(\vec{\boldsymbol{r}}_2, t_2) V(\vec{\boldsymbol{r}}_3, t_3) V(\vec{\boldsymbol{r}}_4, t_4) \rangle$ (8)

Two-point correlation is also called cross-correlation function, and it describes the amount of correlation or the coherence between the values of the physical event/process at spatial points $\vec{\boldsymbol{r}}_1$ and $\vec{\boldsymbol{r}}_2$ at times t_1 and t_2 respectively. An example to study two-point correlation is the interference pattern in the Young's double slit experiment.

Four point correlation (and other higher order correlation) is used to study quantum coherence. It can be shown that the cross-correlation (spatial/temporal) makes a perfect Fourier Transform pair with the respective spectral density (time frequency/spatial frequency). For the case when there is no frequency correlation that means the physical event/process/source is stationary w.r.t. time, *e.g.*, He-Ne laser having CW fields. The example of perfect frequency correlation is a pulsed laser as discussed in the earlier in this section. Apart from temporal coherence, we have spatial and angular coherence where the Fourier Transform pairs are w.r.t. the linear momentum and orbital angular momentum respectively. In the next section, we will be elaborating on four-point correlation for the case of vector fields in the time domain where the direction of vibration is related with their polarization. We emphasize that we are not discussing the spatial and angular coherence in this paper.

III. Two-photon interference and 2nd Order Coherence: Non Linear Effects

In order to study 2nd order coherence, we will quantify four-point correlation in polarization basis (Glauber 1963; Luo and Sun 2017). To observe two-photon interference, we choose a two-photon Mach-Zehnder Interferometer (Rarity et al. 1990). Consider an electromagnetic field consisting of two-photons. The second order coherence (four-point correlation) is calculated as the expectation of detecting one photon at space-time point $(\vec{\boldsymbol{r}}_1, t_1)$ and the 2nd photon being detected at space-time point $(\vec{\boldsymbol{r}}_2, t_2)$. Upon absorption of a photon, the E. M. field moves from the initial quantum state $|i\rangle$ to some final quantum state $|f\rangle$. Defining the four space-time point correlation function for the two-photon case as:

$G^{(2)}(\vec{\boldsymbol{r}}_1, t_1; \vec{\boldsymbol{r}}_2, t_2; \vec{\boldsymbol{r}}_3, t_3; \vec{\boldsymbol{r}}_4, t_4) = \langle\langle f|\hat{E}^{(+)}(\vec{\boldsymbol{r}}_1, t_1)\hat{E}^{(+)}(\vec{\boldsymbol{r}}_2, t_2)\hat{E}^{(+)}(\vec{\boldsymbol{r}}_3, t_3)\hat{E}^{(+)}(\vec{\boldsymbol{r}}_4, t_4)|i\rangle\rangle_{en}$ (9)

Where the subscript 'en' stands for ensemble average. $\hat{E}^{(+)}(\vec{\boldsymbol{r}}_1, t_1)$ is the photon intensity operator at space-time point $(\vec{\boldsymbol{r}}_1, t_1)$ where one of the detectors of Mach-Zehnder interferometer is placed. The $(+)$ superscript on $\hat{E}$ signifies positive frequency components in its Fourier Transform. Eqn. (9) is significantly used to study quantum entanglement.

In the presence of an external field, the electrons in an atom acquire a net dipole moment from which we can compute the induced polarization as the dipole moment per unit volume. In a non-linear optical system when the applied electric field strength is strong, it can be written as (Ghatak and Thyagarajan 1989):

$\boldsymbol{P}(\vec{\boldsymbol{r}}, t) = \sum_{n=1}^{\infty} \epsilon_0 \chi^{(n)} \boldsymbol{E}^n$ (10)

Where $\chi^{(n)}$ is the n^{th} order optical susceptibility and ϵ_0 is the permittivity constant. Usually $\chi^{(1)} \approx unity$ which is the linear optical susceptibility. The 2nd and 3rd order susceptibilities are of the order of $\chi^{(2)} \approx 10^{-12} m/V$ and $\chi^{(3)} \approx 10^{-24} m^2/V^2$. The Displacement $\boldsymbol{D}$ can be computed as:

$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P} = \epsilon_0 \boldsymbol{E} + \sum_{n=1}^{\infty} \epsilon_0 \chi^{(n)} \boldsymbol{E}^n$ (11)

The refractive index of the medium is defined as $= \sqrt{1 + \chi^{(1)}}$.

In order to study two-photon phenomenon, let us consider the 2nd order non-linear optical effects. We neglect $\chi^{(3)}$ and other higher order susceptibility terms. The most common non-linear optical phenomenon due to finite $\chi^{(2)}$ are i) second harmonic generation (SHG), ii) sum and difference frequencies generation, and iii) optical parametric amplification/oscillation. Parametric term signifies the conservation of energy. The second order polarization term is given by:

$\boldsymbol{P}^{(2)} = \epsilon_0 \chi^{(2)} \boldsymbol{E}^2$ (12)

Considering the scalar fields, a monochromatic E. M. field $E(t) = E_0 e^{-i\omega t} + E_0^* e^{i\omega t}$, when passed through such a medium, would result in SHG due to the presence of E^2 term in Eqn. (12).

Another field (not monochromatic) consisting of two frequencies ω_1 and ω_2, when passed through such a 2nd order non-linear medium, would result in the generation of $\omega_2 \pm \omega_1, 2\omega_1, 2\omega_2$ terms. The sum/difference frequency generation is highly efficient process due to the presence of fields at ω_1 and ω_2.

However in order to produce the difference frequency generation $\omega_3 = \omega_2 - \omega_1$, the field at frequency ω_2 need not be present. This is possible due to the ever-presence of the vacuum mode or the idler frequency at ω_2. Now this process of generating difference frequency has a very poor efficiency because this process is stimulated by the idler frequency/vacuum mode. The presence of vacuum mode or the idler frequency can only be explained by the quantum theory, with the condition that the medium should be non-centrosymmetric. The input field at frequency ω_1 is split into ω_2 and ω_3 such that $\omega_1 = \omega_2 + \omega_3$ preserving the conservation of energy. With proper phase-matching with the input field, the two photons released at ω_2 and ω_3 are either collinear direction with ω_1 (pump) or non-collinear. The phase matching can also be adjusted to yield the two photons at ω_2 and ω_3 to be in same polarization state (type-I down conversion) or orthogonal (type-II down conversion) to each other. These two released photons obtained by the process of parametric down-conversion are indeed entangled with each other. It can be shown that the quantum state of these two released photons cannot be written as the product of their individual quantum states due to their entanglement.

In order to observe a two-photon interference pattern, we need two entangled photons and then we say that two entangled photon system is interfering with itself. The corresponding coherence is termed as 2nd order coherence.

IV. Discussion

A two-photon interference is studied from the quantum theory along with elaborate supporting mathematics. The process of generation of two-entangled photons is discussed employing a 2nd order non-linear medium. The concept of four-point correlation arising due to 2nd order coherence is elaborated.

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