IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 17, Issue 4 Ser. 1 (Jul. – Aug. 2025), PP 23-29
www.iosrjournals.org

Edge Ringing Suppression in Highly Aberrated Coherent Optical Systems

[1]P.Shailaja, [2] D. Karuna Sagar and [3] S. Venkateshwara Rao
[1, 3] Department of Physics, Jawaharlal Nehru Technological University, Hyderabad, India
[2]Optics Research Group, Department of Physics, Osmania University, Hyderabad, India

Abstract.
The formation of straight-edge images by an optical system with a circular aperture apodized using Bartlett filters has been studied. One of the most common problems in coherent imaging systems is edge ringing. As the apodization parameter increases from 0 to 1, the severity of edge ringing reduces, and the edge gradient becomes smoother, but this comes at the cost of increased edge shift. Notably, when the Bartlett amplitude filter is applied with an apodization value of $\beta = 1$, and the system is affected by primary spherical aberration and defocus, edge ringing is almost completely eliminated.

Date of Submission: 02-07-2025 Date of Acceptance: 12-07-2025

I. Introduction

Apodisation involves intentionally altering the transmittance of an optical system, thereby influencing its imaging performance. This technique also proves effective in managing the impact of aberrations in coherent imaging systems. In coherent imaging of edge objects, three primary factors contribute to image degradation: edge ringing, edge gradient, and edge shifting. Instead of displaying a sharp geometric edge, the image appears as a series of fringes. Edge ringing arises mainly due to the abrupt cutoff in the transfer function of optical systems (Considine, 1966), and can also be interpreted through the amplitude impulse response (or point spread function, PSF), which contains negative amplitude regions responsible for this ringing effect (Mills and Thompson, 1986). Physically, this phenomenon is analogous to the mathematical "Gibbs phenomenon" (Papoulis, 1962).

Edge shifting occurs because of the non-linear nature of the coherent imaging process, resulting in the apparent displacement of the edge toward the region of higher transmittance (Gaskill, 1978). This effect can be reduced by increasing the transmission of the optical pupil (Asakura and Araki, 1976). Edge gradient refers to the steepness of the intensity transition across the edge. For the same object and optical system, the edge gradient tends to be higher in coherent images compared to incoherent ones. Apodization is a technique used to alter the imaging characteristics of an optical system by shaping its entrance pupil, thereby suppressing ringing in the system's impulse response (1–3).

Since the introduction of lasers, coherent edge imaging has become increasingly important in the field of image science. One of the key issues in such imaging is edge ringing, which is quantified by the height of the first intensity peak above the normalized average irradiance level of one [4]. While the term "ringing" is typically associated with ripples in the time domain, it is also used to describe similar effects in the frequency domain. Ringing introduces visual artifacts and must be minimized for clearer imaging. Edge gradient refers to the rate of change in image intensity per unit variation in the Z-direction near the geometric edge—specifically at $Z = 0$ [6].

Most studies on image formation by aberration-free optical systems, as well as efforts to improve image quality, have been extensively carried out under the two extreme cases of coherent and incoherent illumination [5–12]. Apodization can be implemented through various methods, such as modifying the geometry or transmission properties of the aperture [13]. One approach, known as aperture shaping, involves altering the shape or size of the aperture itself. Another method, called aperture shading, uses an apodizing filter placed over the pupil to adjust light transmission. In essence, apodization is the intentional modification of the pupil function to control the light distribution in the point spread function (PSF), thereby enhancing image quality [14]. The formation of straight-edge images by an optical system with a circular aperture apodized using Bartlett filters has been studied.

Theory and Formulation

The Mathematical expression of Amplitude transmittance of an opaque straight edge object (15) is given by

$$A(u,v) = 1 \quad \text{for} \quad u \geq 0$$

$$=0 \quad \text{for} \quad u \geq 0<0 \qquad (1)$$

It is evident that $A(u,v)$ has a discontinuity at $u = 0$. It cannot be, therefore, Fourier transformed directly. However, its Fourier transform is obtained by expressing it in terms of the Signum function.

$$A(u,v) = \frac{1}{2}[1 + Sgn(u)] \qquad (2)$$

Where $Sgn(u)$ is defined as

$$Sgn(u) = 1 \quad \text{for} \quad u > 0$$

$$=-1 \quad \text{for} \quad u < 0 \qquad (3)$$

A sequence of transformed functions which approach $Sgn(u)$ as a limit should considered, as this function also has a discontinuity at u =0.

As an example, the function

$$f(u)=[\exp(-\gamma|u|)Sgn(u)] \to Sgn(u) \quad as \quad \gamma \to 0 \qquad (4)$$

The transform of (4) is given by

$$F.T.[f(u)]=\int_{-\infty}^{\infty} \exp(-\gamma|u|)Sgn(u)\exp(-2i\pi u x)dx$$

$$= \int_{-\infty}^{0} -\exp[(\gamma - i2\pi x)u]dx + \int_{0}^{\infty} \exp[(-\gamma + i2\pi x)u]dx$$

$$= \frac{1}{(\gamma - i2\pi u)} + \frac{1}{(\gamma + i2\pi u)} \qquad (5)$$

As $\gamma \to 0$, the above expression equals $\left(\frac{1}{i\pi u}\right)$ ie., the Fourier transform of

$$f(u)=\exp[(-\gamma|u|)Sgn(u)] = \frac{1}{i\pi u} \qquad (6)$$

Thus, expressing the straight edge in terms of as given in (2) its Fourier transform can be obtained as

$$F.T.[A(u,v)] = F.T.\left[\frac{1}{2}\{1+Sgn(u)\}\right]$$

$$= \int_{-\infty}^{\infty} [1+(-\gamma|u|Sgn(u))]\exp(-i2\pi u x)dx$$

$$= \frac{1}{2}\left[\delta(x)+\frac{1}{i\pi x}\right] \qquad (7)$$

Where $\delta(x)$, the Fourier transform of unity, is the well –known Dirac-delta function. The expression (7) gives the Fourier spectrum of the object amplitude distribution defined by (1).In this spectrum, the presence of a large zero frequency impulse at is observed, in addition to the other non-zero frequency components. Looking at the object function, it appears at the first sight that is purely zero frequency input to the optical system and therefore, the presence of those non-zero frequencies in the spectrum of such an object may appear rather strange. It should be, however, observed that the object function has zero transmission over one-half in its own plane and a transmission equal to unity over the other half. In other words, $A(u,v)$ is zero for $u < 0$ and then there is an abrupt discontinuity at $u = 0$. Thus, $A(u,v)$ is not a true d.c. signal as it is not constant over the entire interval ranging from $-\infty$ to ∞, and this explains the presence of the other frequency components in the spectrum.

The imaging situations, encountered in optics are generally concerned with objects where amplitude or intensity variations are to be considered in two dimensions. The complex object amplitude distribution as defined in (1) implies that there is no variation in amplitude transmission of the object along the entire v-direction. This will give rise to an infinite impulse at $y=0$ in the spectrum plane and can be represented by the Dirac-delta function $\delta(y)$. Finally, therefore, the two-dimensional F.T. of the object function is obtained as

$$a(x,y)=\frac{1}{2}\left[\delta(x)+\frac{1}{i\pi x}\right]\delta(y) \qquad (8)$$

The above expression gives the spectrum of the object amplitude distribution $A(u,v)$ at the entrance pupil of the optical system. The modified object amplitude spectrum at the exit pupil of the optical system will be given by

$$a^{'}(x,y)=a(x,y).T(x,y) \qquad (9)$$

Where the pupil function of the given optical system containing aberrations is can be expressed (ARAKI and SAAKURA, 1978) by

$$T\,(x,y)=f(x,y)\exp[i\phi(x,y)] \qquad (10)$$

Where f(x, y) denote the amplitude transmittance over the pupil and Φ(x, y) indicated the wave aberration function of the optical system. In the absence of apodisation, f(x, y) is taken to be equal to unity i.e., for the Airy pupils, f(x, y) = 1.

For defocusing, primary spherical aberration, the aberration function can be expressed as

$$\phi\,(x,y)=\left[-i\left(\frac{1}{2}\phi_d r^2+\frac{1}{4}\varphi_s r^4\right)\right] \qquad (11)$$

ϕ_d is the defocusing parameter, φ_s is the primary spherical aberration parameter. Hence, expression (10) becomes on substitution of expressions (11)

$$T\,(x,y)\;=\;f(x,y)\exp[i\left[-\left(\frac{1}{2}\phi_d r^2+\frac{1}{4}\varphi_s r^4\right)\right]] \qquad (12)$$

From expressions (8), (9) and (10) the modified amplitude spectrum at the exit pupil is given by

$$a^{'}(x,y)=\frac{1}{2}\left[\delta(x)+\frac{1}{i\pi x}\right]\delta(y)\,f(x,y)\exp[i\phi(x,y)] \qquad (13)$$

The above equation (13) gives the modified spectrum of the object at the exit pupil of the system. The complex amplitude distribution in the image plane will be given by the inverse F.T. of (13). Therefore,

$$A^{'}(u',v')\;=\;\frac{1}{2}\int_{-\infty}^{+\infty}\!\!\int\left[\delta(x)+\frac{1}{i\pi\;x}\right]\delta(y)\,f\,(x,\;y)\,\exp[\,i\,\phi(\,x,\;y\,)]$$
$$\exp[2\pi i\,(\,u'x+v'y\,)]\,dx\,dy \qquad (14)$$

The integration limits of equation (14) are only formal because the pupil function given by T(x, y) vanishes outside the pupil and can be assumed to be unity inside. Thus, after some manipulation in the integration of Eq. (14) by employing the shifting property of Dirac-delta function the expression (14) can be simplified as

$$A^{'}(u',v')=\frac{f(0)}{2}+\frac{1}{2\pi}\int_{-1}^{1}f\,(x,y)\cos[\phi(x,y)]\frac{\sin(u'x)}{x}dx$$
$$+\frac{i}{2\pi}\int_{-1}^{1}f\,(x,y)\sin\,[\,\phi\,(x,y)]\,\frac{\sin(u'x)}{x}\,dx \qquad (15)$$

The shift property of Dirac-delta function is represented by

$$\int_{-\infty}^{+\infty}\delta(x)\;f(x)\,dx=\,f(0) \qquad (16)$$

The aberration function ϕ(x, y) and the amplitude transmittance f(x, y) have been considered to be even for deriving expression (15). For the central transmittance of the pupil function f(0)=1 the expression (15) can be expressed as

$$A^{'}(u',v') = \frac{1}{2} + \frac{1}{2\pi}\int_{-1}^{1} f(x,y)\cos[\phi(x,y)]\frac{\sin(u'x)}{x}dx$$

$$+\frac{i}{2\pi}\int_{-1}^{1} f(x,y)\sin[\phi(x,y)]\frac{\sin(u'x)}{x}dx \qquad (17)$$

When optical system is free from aberrations, Φ(x,y)=0. For the rotationally symmetric pupil function $f(x,y)= f(-x,-y)$

Setting $2\pi u^{'} = Z$ in equation (17), then it reduces to the more explicit formula for the image of an edge object.

$$A^{'}(Z)=\frac{1}{2}+\frac{1}{2\pi}\int_{-1}^{1} f(x,0)\exp[i\phi(x,0)]\frac{Sin(Z x)}{x}dx \qquad (18)$$

On further simplification equation (18) reduces to

$$A^{'}(Z)=\frac{1}{2}+\frac{1}{\pi}\int_{0}^{1} f(x,y)\exp[i\phi(x,0)]\frac{\{Sin(Z x)\}}{x}dx \qquad (19)$$

The present work deals with the 1-D straight edge object and hence the general form of amplitude distribution is given by For Airy expression 19 becomes

$$A^{'}(Z)=\frac{1}{2}+\frac{1}{\pi}\int_{0}^{1} f(x,0)\exp[i\phi(x,0)]\frac{\{Sin(Z x)\}}{x}dx \qquad (20)$$

The intensity distribution of an edge image formed by an apodized aberrated optical system is given by the squared modulus of expression (21)

$$A^{'}(Z) = \frac{1}{2}+\frac{1}{\pi}\int_{0}^{1} f(x)\exp[-i(\phi_d\frac{x^2}{2}+\frac{1}{3}\phi_c Cos(\theta)r^3+\phi_s\frac{x^4}{4})]\frac{\sin(Zx)}{x}dx \qquad (21)$$

$$B^{'}(Z) = |A'(Z)|^2 \qquad (22)$$

$$A^{'}(Z) = \{\frac{1}{2}+\frac{1}{\pi}\int_{0}^{1} f(x)\exp[-i(\phi_d\frac{x^2}{2}+\frac{1}{3}\phi_c Cos(\theta)r^3+\phi_s\frac{x^4}{4})]\frac{\sin(Zx)}{x}dx\}^{2} \qquad (23)$$

Where f(x) is the amplitude Filter. In the present study barlett amplitude filter is employed for circular aperture For Barlett Amplitude filter f(x) = (1-βr)

II. Results and Discussion

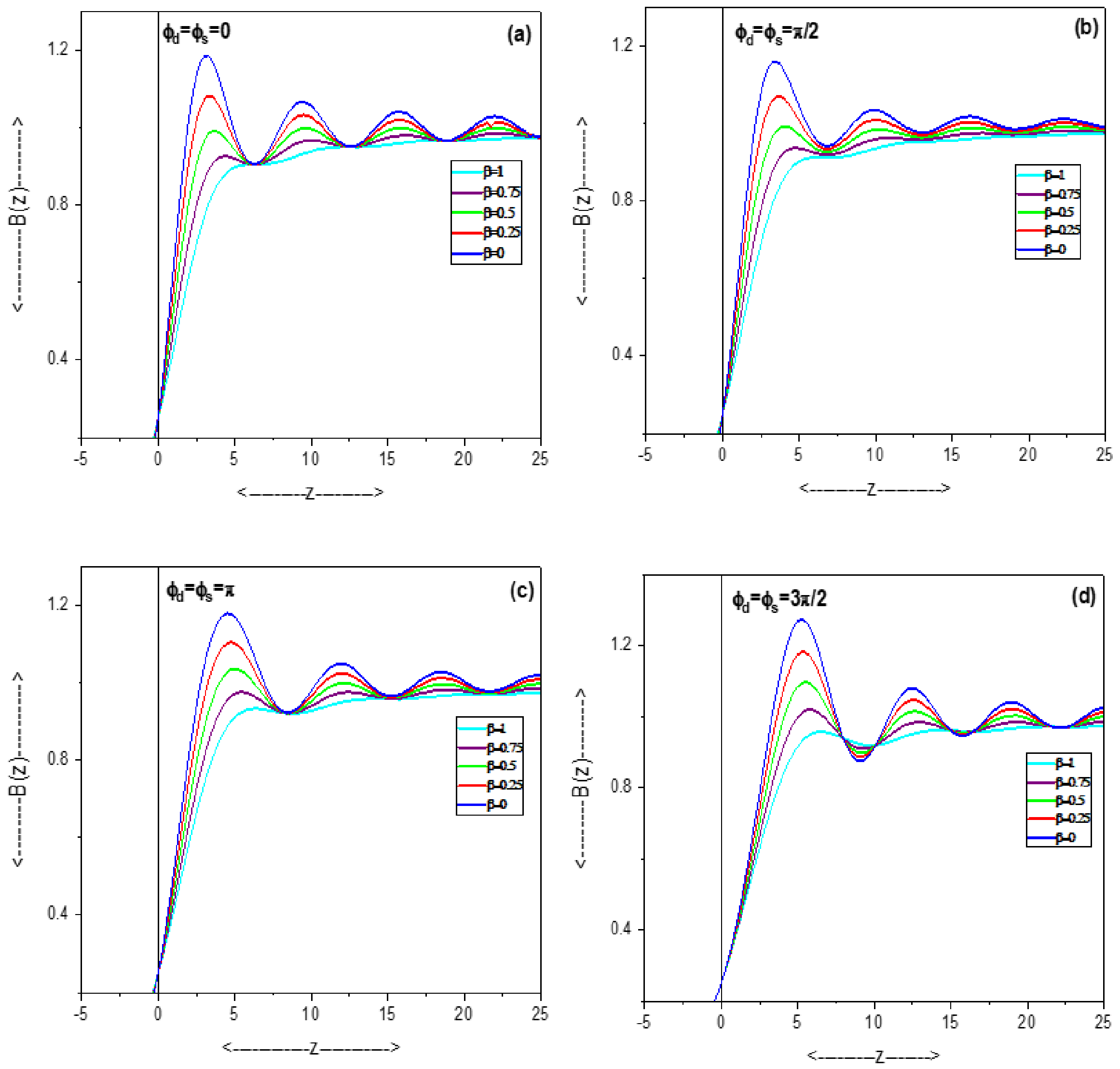

Fig: 1 INTENSITY DISTRIBUTION CURVES

Figure 1 (a) shows the intensity profile curves of straight-edge objects imaged by a coherent optical system with a circular, aberration-free aperture, apodized using a Bartlett amplitude filter. It is clear that edge ringing is most severe when the aperture is clear (β = 0), indicating that un-apodized systems exhibit the highest ringing levels. As the apodization parameter β increases from 0 to 1, edge ringing and edge gradient both decrease, while edge shift increases.

Figures 1 (b) to 1(d) illustrate the system's response under varying levels of defocus and primary spherical aberration. For apodization values increasing from β = 0 to 1 in increments of 0.25, the best suppression of edge ringing occurs at β = 1, particularly for aberration combinations of Φd = Φs = π/2, π, and 3π/2.

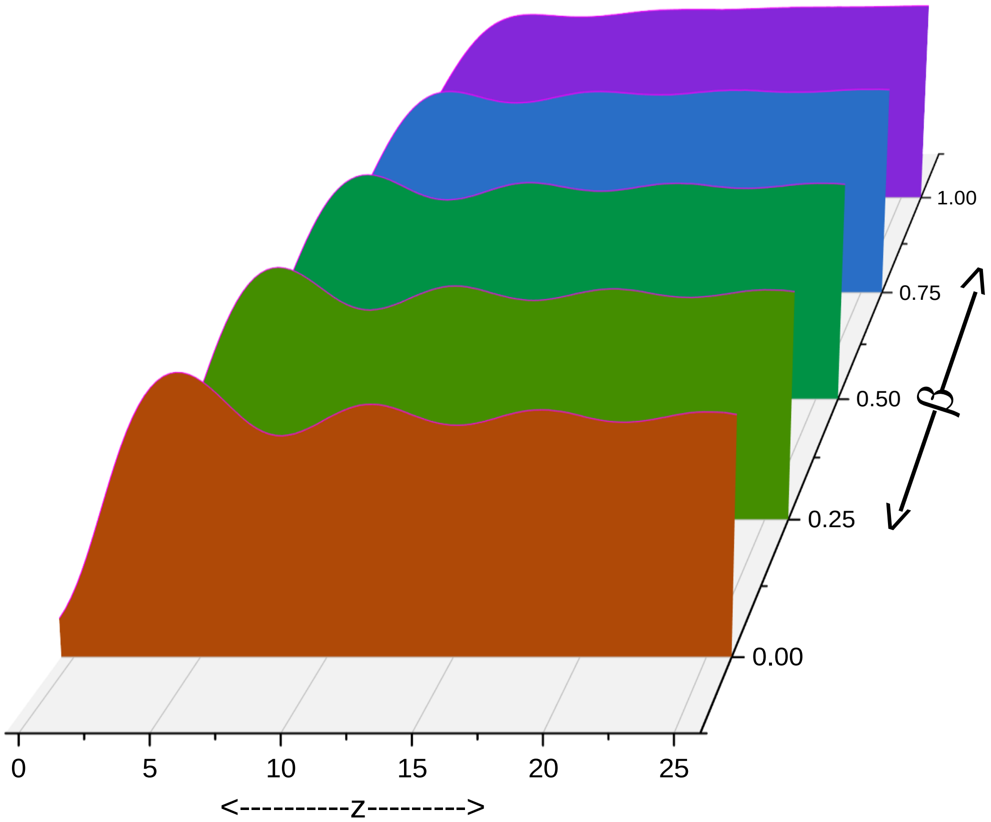

Fig: 2: 3D Graph: Straight Edge objects with aberrations $\varphi_d = \varphi_s = \pi$ for and Barlett Amplitude filter

III. Conclusions

Apodizing the optical system with a selected filter effectively reduces the unwanted effects of edge ringing. In such cases, the image intensity profile at a straight edge transitions smoothly and monotonically from the dark side to the bright side. However, this reduction in ringing comes at the cost of increased edge shift and a decrease in edge gradient. Notably, when the system is apodized using a Bartlett amplitude filter with $\beta = 1$, and influenced by primary spherical aberration and defocus, edge ringing is almost completely eliminated.

References

[1]. T. Asakura And T. Araki, "Apodization In Coherent Image Formation," Optik, Vol. 46, No. 4, Pp. 365–375, 1976
[2]. P. K. Mondal, K. Prabhakar Rao, And S. Subramayam, "A Unified Review Of Phase Contrast, Dark-Ground And Annular Imaging Techniques," Atti Della Fondazione Giorgio Ronchi, Vol. 35, No.4, Pp. 527–540, 1980
[3]. R. Barakat, "Solution Of The Luneberg Apodization Problems,"The Journal Of The Optical Society Of America, Vol. 52, No. 3, Pp.264–272, 1964. Papoulis, A., "Systems And Transforms With Applications In Optics", (Mcgraw-Hill, Usa) P.442, (1968).
[4]. Asakura And Araki, Optika, 46, 365 (1976)
[5]. S. Ramanathan, Ph.D.Thesis, Osmania University (1986).
[6]. G. B. Parrent, Jr. And T. J. Skinner, Opt. Acta 8, 93 (1961)
[7]. R. A. Shore, Proc. Of The Symposium On Eta, Copenhagen, Denmark, P. 787 (Pergamon, London, 1963).
[8]. W. T. Cathey, J. Opt. Soc. Am. 55, 1035 (1965).
[9]. R. A. Shore, B. J. Thompson, And R. E. Whitney, J. Opt. Soc. Am. 56, 733 (1966).
[10]. J. G. Meadors, Opt. Acta 12, 379 (1965).
[11]. R. A. Shore, J. Opt. Soc. Am. 58, 1484 (1968).
[12]. K. Singh And H. S. Dhillon, J. Opt. Soc. Am. 59, 397 (1969).
[13]. K. Singh And B. N. Gupta, Opt. Acta 17, 609 (1970).
[14]. Hecht, E., Zajac, A., "Optics", (Addison – Wesley, Usa), 2nd Ed., P.422, 496, 497 & 512, (1987).
[15]. Wetherell, W.B. "Applied Optics And Optical Engineering" (Academic Press, N.Y., Eds. R. R. Shannon & J.C. Wyant) Vol.8, Ch.6, P.173-283, (1980).
[16]. T. Araki And T Asakura, Apodized Images Of Coherently Illuminated Edges In The Presence Of Defocusing And Spherical Aberration, Optika Aplicata, 1978 Viii/4, 159-170