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Determination Of Threshold Density Of Some Astrophysical Bodies To Reduce To Schwarzschild`S Black Hole.

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Abstract
Gravitation, electromagnetism, and nuclear interactions constitute the fundamental forces governing physical phenomena in nature. Among these forces, gravitation exhibits a unique and universal property which is the function of density. In this work threshold density of some astrophysical bodies to reduce to black hole was determined .it was found that Jupiter have a lower threshold density of $3.294 \times 10^{16} Kgm^{-3}$ *while Mercury has higher threshold density of* $2.707 \times 10^{19} Kgm^{-3}$ *which means Jupiter has less mass to size ratio and gravity is less making it harder for thing to escape when it reduce to Schwarzschild's black hole compared to other planets. in the case of moons Luna have a lower threshold density of* $5.330 \times 10^{19} Kgm^{-3}$ *while Phobos has higher threshold density of* $5.227 \times 10^{24} Kgm^{-3}$ *.which means lunar has less mass to size ratio and gravity is less, making it harder for thing to escape when it reduces to Schwarzschild's black hole compared to other moons. In the case of Asteroids Psyche has higher threshold density of* $1.863 \times 10^{22}\ Kgm^{-3}$ *while ceres lower threshold density of* $7.106 \times 10^{20}\ Kgm^{-3}$*.Which means Ceres has less mass to size ratio and gravity is less, making it harder for thing to escape when it reduces to Schwarz child's black hole compared to asteroid.*
Keywords: *Astrophysical bodies, electromagnetism, Gravitation, nuclear interactions*

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I. Introduction

The nature of gravitation and its role in shaping the structure and evolution of the universe has remained a central theme in physics for centuries (Adams, J. C.1886). The gravitational force brings together any two objects that have mass. This force is called 'attractive' because it consistently seeks to draw masses closer together rather than pushing them apart. Every object, including ourselves, exerts a pull on all other objects throughout the universe (Tippler & Llewellyn, R..2012), (Kerr, 1963).

From the classical formulation of gravitational attraction by Isaac Newton to the revolutionary geometric interpretation introduced by Albert Einstein, our understanding of gravity has undergone profound transformation (Einstein, 1916), (Carroll 2004). Newtonian gravity successfully explains a wide range of phenomena, from planetary motion to terrestrial mechanics, through the inverse-square law of attraction. However, it fails to accurately describe extreme gravitational environments where spacetime curvature becomes significant. This limitation led to the development of General Relativity, which provides a more complete framework for understanding gravitational phenomena, particularly in regimes involving very high mass densities and strong gravitational fields (Einstein, 1916; Misner *et al*., 1973).

Several studies have been made to understand astrophysical bodies' example, the work of Herbert and Aliyu (2024). That Study of Radiation emission by some Astrophysical bodies using the laws of black Body radiation. Aliyu and Sambo,.(2021) that Determine the Schwarzschild's Radius of some Planetary Bodies in the Solar System Using Newtonian Mechanics

The study of threshold density is therefore essential for bridging the gap between classical astrophysical objects and relativistic compact objects. Although planets, moons, and asteroids are far from the conditions required for natural black hole formation, calculating their threshold densities allows for a deeper understanding of how far these objects are from gravitational collapse and highlights the role of geometric constraints in determining their stability.

Despite its importance, the concept of threshold density is often underrepresented in standard treatments of black hole physics, which tend to emphasize dynamical collapse processes rather than static conditions for collapse. As a result, there is a need for systematic studies that explicitly quantify the relationship between size and density for a variety of astrophysical bodies. Such analyses not only enhance conceptual understanding but also provide a basis for further theoretical and computational investigations.

The present study here aims to address this gap by determining the threshold density required for selected astrophysical bodies including planets, moons, and asteroids to collapse into Schwarzschild black holes. By employing analytical methods grounded in both Newtonian mechanics and General Relativity, the study derives a general expression for critical density and applies it to a range of celestial objects using observed physical parameters. The results are used to establish trends and comparisons that highlight the dependence of gravitational collapse on size and geometry.

In summary, this work seeks to deepen our understanding of gravitational collapse by focusing on the fundamental role of density in black hole formation. By extending the analysis beyond stellar objects to include a broader class of astrophysical bodies, it provides a comprehensive perspective on the conditions required for the formation of Schwarzschild black holes and reinforces the predictive power of General Relativity in describing extreme physical phenomena.

II. Methodology

To determine the Critical Density (ρ_c) the density at which an object of radius R would become a black hole we assume the object is compressed until its actual radius equals its Schwarzschild radius $R = R_{SH}$

Determination of Volume and Density of Planetary Bodies

We can calculate a planet's volume from its radius using the formula

$$V = \frac{4}{3}\pi r^3. \quad (1)$$

Where r is radius of the body

By doing so we assume the planet is spherical.

The physical density of any object is simply its mass divided by its volume; density is measured in units such as pounds per cubic foot, grams per cubic centimeter or kilograms per cubic meter. When calculating the density of a planet, look up its mass and radius, the latter of which is the distance from the surface to the center. Because planets are roughly spherical, calculate the volume of a sphere using the radius. Then divide the mass by the volume of the sphere to get the density.

The density of a planet (or anything else) is simply calculated as

$$\rho = \frac{M}{V} \quad (2)$$

Density = (ρ), M= Mass of the planet, V= Volume of the planet.

Determination of Schwarzschild's Radius

The distance from the centre of a non-rotating black hole to the event horizon is known as the Schwarzschild radius. This is like taking a ruler and measuring the distance from the dark well's edge to its centre. The Schwarzschild radius depends only on the mass of the object that creates the black hole.

We can derived the same result by considering how the escape velocity from the surface of the star depend upon the size of the star.

Newtonian expression for escape velocity is given by

$$v = \sqrt{\frac{2GM}{R}} \quad (3)$$

Where M is the mass of star with radius R If light is retarded by gravity it will be just fail to escape from the surface. When the escape velocity has the same value as c.

There fore

$$c = \sqrt{\frac{2GM}{R}} \quad (4)$$

The above equation can be re-arrange to give the radius R_{sh} which for a given mass must be compressed to form a black hole.

$$R_{Sh} = \frac{2GM}{c^2} \quad (5)$$

Where R_{sh} is called a schwarzschild's radius. Substituting value of constant parameters

Then

$$R_{sh} = 1.48 \times 10^{-27} M \quad (6)$$

Determination of Schwarzschild's Volume

We can calculate a black hole volume from its radius using the formula

$$V = \frac{4}{3}\pi R_{sh}^3 \quad (7)$$

Where R_{sh} is the Schwarzschild's radius of the body
The density of a black hole is simply calculated as

$$\rho = \frac{M}{V} \quad (8)$$

Density = (ρ) M = Mass of black hole
V= Volume black hole

$$R_{SH} = \frac{2GM}{c^2}$$

And

$$M = \rho V = \frac{4\pi R^3}{3}\rho \quad (9)$$

$$\therefore R_{sh} = \frac{8\pi G R^3}{3C^2}\rho \quad (10)$$

Critical (Threshold Density (ρ_C)

Critical density can be defined as the maximum density required for the astrophysical bodies to compress to forms a black hole. To get a critical density the reduction ratio must be equals to one

Threshold Condition: Setting $\frac{R_{sh}}{R}$ = 1, we solve for ρ
From equation 3.1.4 we have

$$\frac{R_{sh}}{R} = \frac{8\pi G R^2}{3C^2}\rho \quad (11)$$

From equation above, for a black hole to be formed

$$R = R_{sh} \quad (12)$$

Which Implies

$$\rho = \rho_c \quad (13)$$

Therefore equation 3.11 becomes

$$\rho_C = \frac{3C^2}{8\pi G R^2} \quad (14)$$

By substituting the value of c, G, and π in equation 3.14 we have
The above equation it becomes

$$\rho_C = \frac{1.610\times 10^{26}}{R^2} \quad (15)$$

III. Result And Discussion

Results

The threshold density for any astrophysical body (Planets, Asteroids and Moons) can be computed using equation (15)
Table 1: show the critical density of planets in the solar system.

Table 4.1 Threshold density (ρ_c) for planets in the solar system.

Planet	Radius (km)	Threshold density (ρ_c) (kgm^{-3})
Mercury	2439	2.707×10^{19}
Venus	6052	4.397×10^{18}
Earth	6378	3.958×10^{18}
Mars	3396	1.396×10^{19}
Jupiter	69911	3.294×10^{16}
Saturn	60268	4.433×10^{16}
Uranus	25559	2.465×10^{17}
Neptune	2476	2.626×10^{19}
Pluto (dwarf planet)	1185	1.147×10^{20}

Table 2: Threshold density for Moons of Earth and Mars

Name of planet	Name of moon	Radius (km)	Threshold density (ρ_c) **(kgm^{-3})**
Earth	lunar	1738	5.330×10^{19}
Mars	Phobos	5.55	5.227×10^{24}
	Deimos	6.25	4.122×10^{24}

Table 3: Threshold density for Moons of Jupiter

Name of moon	Radius (km)	Threshold density (ρ_c)
Metis	20	4.025×10^{23}
Andrastea	10	1.610×10^{24}
Amalthe	86	2.177×10^{22}
Thebe	50	6.440×10^{22}
IO	1818	4.879×10^{19}
Eropah	174	5.318×10^{21}
Leda	8	2.516×10^{24}
Ganymede	2634	2.321×10^{19}
Sinope	14	8.214×10^{23}
Himalia	85	2.228×10^{22}
Lysithea	12	1.118×10^{24}
Elera	40	1.086×10^{23}
Ananka	10	1.610×10^{23}
Pasphae	18	4.969×10^{23}

Table 4: Threshold density for Moons of Saturn

Name of moon	Radius (km)	Threshold density (ρ_c) (kgm^{-3})
Epimetheus	60	4.444×10^{22}
Janus	90	1.988×10^{22}
Mimas	199	4.066×10^{21}
Encaladus	249	2.597×10^{21}

Table 5: Threshold density for Moons of Uranus

Name of moon	Radius (km)	Threshold density (ρ_c) (kgm^{-3})
Coradeli	13	9.527×10^{23}
Ophelia	16	6.289×10^{23}
Bianca	22	3.326×10^{23}
Cresida	33	1.478×10^{23}
Desdemona	29	1.914×10^{23}
Juliet	42	9.127×10^{22}
Portia	55	5.322×10^{22}
Rosalind	27	2.209×10^{23}
Belinda	29	1.914×10^{23}

Table 6: Threshold density for Moons of Neptune

Name o moon	Radius (km)	Threshold density (ρ_c) (kgm^{-3})
Thalassa	40	1.086×10^{23}
Naiad	29	1.914×10^{23}
Despina	74	2.940×10^{22}
Galatea	79	2.580×10^{22}
Lraissa	193	4.322×10^{21}
Proteus	105	1.460×10^{22}
Triton	1352	8.808×10^{19}

Table 4.7: Threshold density for Moons of Pluto

Name of moon	Radius (km)	Threshold density (ρ_c)
Charon	586	4.688×10^{20}
Nix	225	3.180×10^{21}
Hydra	80	2.516×10^{22}

ASTEROIDS

Table 8: Show the Threshold density for Asteroids in solar system.

Table 4.8: Threshold density for Asteroids in solar system

Name of asteroid	Radius (km)	Threshold density (ρ_c) (kgm^{-3})
Ceres	476	7.106×10^{20}
Vesta	264.5	2.301×10^{21}
Pallas	272	2.176×10^{21}
Hygiea	265.5	2.284×10^{21}
Remnia	163	6.060×10^{21}
Davida	143	7.873×10^{21}
Eunimia	134	8.966×10^{21}
Juna	129	9.675×10^{21}
Psyche	93	1.863×10^{22}

IV. Discussion

Planetary Variation: Larger planets like Jupiter have a lower critical density ($3.294 \times 10^{16} Kgm^{-3}$) compared to smaller planets like Mercury (2.707×10^{19}kg/m^3). This is because the larger initial radius of Jupiter makes the "gap" to its Schwarzschild state relatively easier to bridge in terms of density, compared to a small, compact body. Earth and Venus has value with 10^{18} while Uranus has 10^{17}.

The Lunar Case: The Earth's moon (Luna) requires a lower density to become a black hole ($5.330 \times 10^{19} Kgkm^{-3}$) than the moons of Mars, such as Phobos ($5.227 \times 10^{24} Kgm^{-3}$) Kgm^{-3}confirms that smaller bodies like Phobos are much further from becoming black holes than larger ones like the Moon.

Metis, andrastea, amalthe, thebe, leda, sinope, himalia, lysithea, elera, ananka, pasphae are moons of Jupiter with the power of 10^{22} and above while Eropah is having the average value with 10^{21}. IO and Ganymede have10^{17}. The least threshold density with the power of 10^{19}.

Moons of Saturn (Epimetheus, Janus) are in same range of threshold density, while Mimas and Encaladus are also same range. This shows that the moons may attain equal threshold density to form Schwarschilds' black hole.

Neptune with largest moon among the planet and even bigger than Pluto (Triton) and all rest have the critical density of 10^{21} and above

In asteroids Ceres has less critical density followed by Pellas , others have almost the same range of critical density.

It was found that among all astrophysical bodies in our solar system Jupiter has a lower threshold density while Phobos has higher threshold density. Which means Jupiter has less mass to ratio and gravity is less making it harder for things to escape when it reduce to schwarzschild`s black hole compare to other astrophysical bodies.

V. Conclusion

The threshold density for some astrophysical bodies in the solar system were calculated where we avoid complex mathematical difficulty of General relativity. Results shows that not only the star can collapse to form a black hole rather anybody can form a black hole by attaining its threshold density it also shows that transformation of any astrophysical body in the solar system is strongly depend only on its radius.

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