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Effect of Gravity Modulation on the Onset of Ferroconvection in a Densely Packed Porous Layer

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Abstract: *The stability of a horizontal porous layer of a ferromagnetic fluid heated from below is studied when the fluid layer is subject to a time-periodic body force. Modified Darcy law is used to describe the fluid motion. The effect of gravity modulation is treated by a perturbation expansion in powers of the amplitude of modulation. The stability of the system, characterized by a correction Rayleigh number, is determined as a function of the frequency of modulation, magnetic parameters, and Vadasz number. It is found that subcritical instability is possible for low frequency g-jitter and that the magnetic and g-jitter mechanisms work against each other for small and moderate values of frequency of modulation. The effect of Vadasz number is shown to be reinforcing the influence of gravity modulation for small and moderate values of frequency. The magnetic, porous and modulation effects disappear altogether for sufficiently large values of the frequency of modulation.*
Keywords*-Ferromagnetic fluid, Gravity modulation, Perturbation method, Porous layer, Stability.*

I. INTRODUCTION

Ferrofluids can be used to transfer heat as heat and mass transport in such magnetic fluids can be controlled by means of an external magnetic field. Numerous applications can be associated with these fluids including novel energy conversion devices, levitation devices and rotating seals [1]. Finlayson [2] first explained how an external magnetic field imposed on a horizontal layer of ferrofluid with varying magnetic susceptibility due to a temperature gradient results in a non-uniform magnetic body force, which leads to thermomagnetic convection. This form of heat transfer can be useful for cases where conventional convection fails to provide adequate heat transfer, for instance, in miniature microscale devices or under reduced gravity conditions.

Gupta and Gupta [3] investigated thermal instability in a layer of ferromagnetic fluid subject to coriolis force and permeated by a vertical magnetic field. It is substantiated that overstability cannot occur if the Prandtl number is greater than unity. Gotoh and Yamada [4] investigated the linear convective instability of a ferromagnetic fluid layer heated from below and confined between two horizontal ferromagnetic boundaries. The Galerkin technique is used and the Legendre polynomials are taken as the trial functions. It is shown that the magnetization of the boundaries and the nonlinearity of fluid magnetization reduce the critical Rayleigh number and the effects of magnetization and buoyancy forces are shown to compensate each other.

Blums [5] examined the possibility of having convection in ferromagnetic fluids as a result of magneto-diffusion of colloidal particles which give rise to non-uniform magnetization. Stiles and Kagan [6] examined the thermoconvective instability of a horizontal layer of ferrofluid in a strong vertical magnetic field. Their work also questioned the satisfactory agreement claimed to exist between the experiments and the theoretical model of Finlayson which has been generalized by them.

Odenbach [7] investigated the convective flow generated by the interaction of a magnetic field gradient with a gradient in magnetization in a magnetic fluid. This gradient was caused by the diffusion of the magnetic particles in the field gradient. Aniss *et al*. [8] investigated the effect of a time-sinusoidal magnetic field on the onset of convection in a horizontal magnetic fluid layer heated from above. The Floquet theory is used to determine the convective threshold for free-free and rigid-rigid cases. The possibility to produce a competition between the harmonic and subharmonic modes at the onset of convection is discussed.

Abraham [9] investigated the Rayleigh-Bénard problem in a micropolar ferromagnetic fluid layer in the presence of a vertical uniform magnetic field analytically. It is shown that the micropolar ferromagnetic fluid layer heated from below is more stable as compared with the classical Newtonian ferromagnetic fluid. The effect of radiative heat transfer on ferroconvection has been studied by Maruthamanikandan [10] using the linear stability analysis. Consideration is given to two asymptotic cases, *viz.*, transparent and opaque layers of fluid. The critical values marking the onset of convection are obtained using the Galerkin technique.

Bajaj [11] considered thermosolutal convection in magnetic fluids in the presence of a vertical magnetic field and bifrequency vertical vibrations. The regions of parametric instability have been obtained using the Floquet theory. Ramanathan and Muchikel [12] investigated the effect of temperature-dependent viscosity on ferroconvective instability in a porous medium. It is found that the stationary mode of instability is preferred to oscillatory mode and that the effect of temperature-dependent viscosity has a destabilizing effect on the onset of convection. Maruthamanikandan [13] investigated the problem of gravitational instability in ferromagnetic fluids in the presence of internal heat generation, surface tension, and viscoelasticity.

Saravanan [14] made a theoretical investigation to study the influence of magnetic field on the onset of convection induced by centrifugal acceleration in a magnetic fluid filled porous medium. The layer is assumed to exhibit anisotropy in mechanical as well as thermal sense. Numerical solutions are obtained using the Galerkin method. It is found that the magnetic field has a destabilizing effect and can be suitably adjusted depending on the anisotropy parameters to enhance convection. The effect of anisotropies of magnetic fluid filled porous media is shown to be qualitatively different from that of ordinary fluid filled porous media.

Singh and Bajaj [15] investigated numerically the effect of frequency of modulation, applied magnetic field, and Prandtl number on the onset of a periodic flow in the ferrofluid layer using the Floquet theory. Some theoretical results have also been obtained to discuss the limiting behavior of the underlying instability with the temperature modulation. Depending upon the parameters, the flow patterns at the onset of instability have been found to consist of time-periodically oscillating vertical magnetoconvective rolls. Singh and Bajaj [16] considered the effect of time-periodic modulation in temperatures on the onset of ferroconvection with rigid boundaries. It is found that, under modulation, subcritical instabilities are found to occur in the form of subharmonic response. Also, the onset of instability in the ferrofluid layer is found to heavily depend upon the frequency of modulation when it is driven solely by the magnetic forces alone, the effect being the greatest for the low frequency modulation and negligible for the case of high frequency modulation.

In view of the fact that heat transfer can be greatly enhanced due to thermomagnetic convection, the ferroconvection problems offer fascinating applications including cooling with motors, loudspeakers and transmission lines.

On the other hand, thermal convection induced by modulated gravitational forces has received much attention in recent time. When a system with density gradient is subject to vibrations, the resulting buoyancy force produced by the interaction of the density gradient and the gravitational field has a complex spatio-temporal structure. The time dependent gravitational field is of interest in space laboratory experiments, in areas of crystal growth and other related applications. It is reported by Wadih *et al*. [17, 18] that this fluctuating gravity, referred to as g-jitter, can either substantially enhance or retard heat transfer and thus drastically affect the convection.

Govender [19] made stability analysis to investigate the effect of low amplitude gravity modulation on convection in a porous layer heated from below. It was shown that increasing the frequency of vibration stabilizes the convection. Saravanan and Purusothaman [20] carried out an investigation on the influence of non-Darcian effects in an anisotropic porous medium and found that non-Darcian effects significantly affect the synchronous mode of instability. Govender [21] examined the influence of the Vadasz number on vibration in a rotating porous layer placed far away from the axis of rotation. It is shown that a frozen time approximation is appropriate for large Vadasz numbers provided the effect of vibration is modeled as small variations in the Rayleigh number.

Saravanan and Sivakumar [22] studied the effect of harmonic vibration on the onset of convection in a horizontal anisotropic porous layer. The influence of vibration parameters and heating condition on the anisotropy effects and the competition between the synchronous and sub-harmonic modes are discussed. Malashetty and Begum [23] analyzed the effect of thermal/gravity modulation on the onset of convection in a Maxwell fluid saturated porous layer. They found that low frequency gravity modulation has destabilizing effect on the stability of the system whereas moderate and high frequency modulation produces a stabilizing effect on the onset of convection.

More recently, Saravanan and Premalatha [24] have investigated thermovibrational convection in a porous layer permeated by a fluid exhibiting antisymmetric stress due to the presence of couple-stress. Low amplitude vibrations are considered. The critical values of the parameters are found with the help of the Mathieu functions. The instability limits for both synchronous and subharmonic responses and the transition between them are predicted.

The problem of control of convection is of relevance and interest in innumerable ferromagnetic fluid applications and is also mathematically quite challenging. The unmodulated Rayleigh-Bénard problem of convection in a ferromagnetic fluid has been extensively studied. However, attention has not been paid to the effect of gravity modulation on Rayleigh-Bénard convection in a horizontal porous layer of a ferromagnetic fluid. It is with this motivation that we study, in this paper, the problem of Rayleigh-Bénard convection in a ferromagnetic fluid saturated porous layer induced by gravity modulation with emphasis on how the stability criterion for the onset of ferroconvection would be modified in the presence of both porous matrix and gravity modulation.

II. MATHEMATICAL FORMULATION

We consider a ferromagnetic fluid saturated densely packed porous layer confined between two infinite horizontal surfaces $z=0$ and $z=h$ under the influence of a uniform, vertical magnetic field $\boldsymbol{H}_0$ and a time periodically varying gravity force $\boldsymbol{g} = (0, 0, -g(t))$ acting on it, where $g(t) = g_0\left(1 + \varepsilon \cos \bar{\omega} t\right)$ with g_0 being the mean gravity, ε the small amplitude, $\bar{\omega}$ the frequency and t the time. A uniform adverse temperature gradient ΔT is maintained between the lower and upper boundaries. The Boussinesq approximation is invoked to account for the effect of density variation. It is assumed that the fluid and solid matrix are in local thermal equilibrium. With these assumptions the governing equations are

$$\nabla \cdot \boldsymbol{q} = 0 \tag{1}$$

$$\rho_R \left[\frac{1}{\varepsilon_p} \frac{\partial \boldsymbol{q}}{\partial t} + \frac{1}{\varepsilon_p^2} \left(\boldsymbol{q} \cdot \nabla\right) \boldsymbol{q} \right] = -\nabla p - \rho g_0 \left(1 + \varepsilon \cos \bar{\omega} t\right) \hat{\boldsymbol{k}} - \frac{\mu_f}{k} \boldsymbol{q} + \nabla \cdot \left(\boldsymbol{H B}\right) \tag{2}$$

$$\begin{aligned} &\varepsilon_p C_1 \left[\frac{\partial T}{\partial t} + \left(\boldsymbol{q} \cdot \nabla\right) T \right] + \left(1 - \varepsilon_p\right) \left(\rho_0 C\right)_s \frac{\partial T}{\partial t} \\ &+ \mu_0 T \left(\frac{\partial \boldsymbol{M}}{\partial t} \right)_{V,H} \cdot \left[\frac{\partial \boldsymbol{H}}{\partial T} + \left(\boldsymbol{q} \cdot \nabla\right) \boldsymbol{H} \right] = K_1 \nabla^2 T \end{aligned} \tag{3}$$

$$\rho = \rho_R \left[1 - \beta \left(T - T_R\right) \right] \tag{4}$$

where $\boldsymbol{q} = \left(u, v, w\right)$ is the fluid velocity, ρ the density, ρ_R a reference density, ε_p the porosity of the porous medium, p the pressure, $\boldsymbol{H}$ the magnetic field, $\boldsymbol{B}$ the magnetic induction, μ_f the dynamic viscosity, k the permeability of the porous medium, μ_0 the magnetic permeability, T the temperature, $\boldsymbol{M}$ the magnetization, K_1 the thermal conductivity, β the coefficient of thermal expansion, T_R a reference temperature and C the specific heat. Here subscript s represents the solid and $C_1 = \rho_R C_{V,H} - \mu_0 \boldsymbol{H} \cdot \left(\frac{\partial \boldsymbol{M}}{\partial T} \right)_{V,H}$, where $C_{V,H}$ is the specific heat at constant volume and constant magnetic field.

Maxwell's equations simplified for a non-conducting fluid with no displacement current take the form

$$\nabla \cdot \boldsymbol{B} = 0, \qquad \nabla \times \boldsymbol{H} = \boldsymbol{0} \tag{5a, b}$$

$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{M} + \boldsymbol{H} \right) \tag{6}$$

Since the magnetization $\boldsymbol{M}$ is aligned with the magnetic field and is a function of temperature and magnetic field, we may write

$$\boldsymbol{M} = \frac{\boldsymbol{H}}{H} M \left(H, T \right) \tag{7}$$

The magnetic equation of state is linearized about the magnetic field $\boldsymbol{H}_0$ and the reference temperature T_R according to

$$M = M_0 + \chi \left(H - H_0 \right) - K_m \left(T - T_R \right) \tag{8}$$

where χ is the magnetic susceptibility and K_m is the pyromagnetic coefficient. The surface temperatures are $T_R + \frac{\Delta T}{2}$ at $z = 0$ and $T_R - \frac{\Delta T}{2}$ at $z = h$.

III. BASIC STATE

The basic state is quiescent and is described by

$$\left.\begin{aligned} &\boldsymbol{q} = \boldsymbol{q}_H = (0,0,0),\ \rho = \rho_H\left(z\right),\ p = p_H\left(z\right) \\ &T = T_H\left(z, t\right),\ \boldsymbol{H} = \boldsymbol{H}_H\left(z\right),\ \boldsymbol{M} = \boldsymbol{M}_H\left(z\right),\ \boldsymbol{B} = \boldsymbol{B}_H\left(z\right) \end{aligned}\right\}. \tag{9}$$

In the undisturbed state, the temperature T_H, the pressure p_H, the magnetic field $\boldsymbol{H}_H$, magnetic induction $\boldsymbol{B}_H$ and magnetization $\boldsymbol{M}_H$ satisfy the following equations

$$-\frac{\partial p_H}{\partial z} = \rho_H\, g_0\left(1+\varepsilon\cos\bar{\omega}t\right) - B_H\,\frac{\partial H_H}{\partial z} \tag{10}$$

$$T_H = T_R + \Delta T\left(\frac{1}{2} - \frac{z}{h}\right) \tag{11}$$

$$\rho_H = \rho_R\left[1 - \beta\Delta T\left(\frac{1}{2} - \frac{z}{h}\right)\right] \tag{12}$$

$$H_H = H_0 + \frac{K_m\,\Delta T}{\left(1+\chi\right)}\left(\frac{1}{2} - \frac{z}{h}\right) \tag{13}$$

$$M_H = M_0 - \frac{K_m\,\Delta T}{\left(1+\chi\right)}\left(\frac{1}{2} - \frac{z}{h}\right) \tag{14}$$

and

$$B_H = \mu_0\left(M_0 + H_0\right). \tag{15}$$

In what follows we examine the stability of the equilibrium state by means of the linear stability analysis.

IV. LINEAR STABILITY ANALYSIS

Let the basic state be perturbed by an infinitesimal thermal perturbation so that

$$\left.\begin{aligned} &\boldsymbol{q} = \boldsymbol{q}_H + \boldsymbol{q}',\ p = p_H + p',\ \rho = \rho_H + \rho',\ T = T_H + T' \\ &\boldsymbol{H} = \boldsymbol{H}_H + \boldsymbol{H}',\ \boldsymbol{B} = \boldsymbol{B}_H + \boldsymbol{B}',\ \boldsymbol{M} = \boldsymbol{M}_H + \boldsymbol{M}' \end{aligned}\right\} \tag{16}$$

where prime indicates that the quantities are infinitesimal perturbations. Substituting (16) into Eqs. (1) – (8) and using basic state solution, we obtain the following equations

$$\nabla .\, \boldsymbol{q}' = 0 \tag{17}$$

$$\rho' = -\beta\rho_R T' \tag{18}$$

$$\begin{aligned} \frac{\rho_R}{\varepsilon_p}\left[\frac{\partial \boldsymbol{q}'}{\partial t}\right] = &-\nabla p' - \rho' g_0\left(1+\varepsilon\cos\bar{\omega}t\right)\hat{\boldsymbol{k}} - \frac{\mu_f}{k}\boldsymbol{q}' + \mu_0\left(M_0+H_0\right)\frac{\partial \boldsymbol{H}'}{\partial z} \\ &-\frac{\mu_0 K_m}{\left(1+\chi\right)}\left(\frac{\Delta T}{h}\right)\left[\left(1+\chi\right)H_3' - K_m T'\right]\hat{\boldsymbol{k}} \end{aligned} \tag{19}$$

$$C_2\frac{\partial T'}{\partial t} - \varepsilon_p C_1\left(\frac{\Delta T}{h}\right)w' - \mu_0 K_m T_R\frac{\partial}{\partial t}\left(\frac{\partial \phi'}{\partial z}\right) + \frac{\mu_0 K_m^2 T_R}{\left(1+\chi\right)}\left(\frac{\Delta T}{h}\right)w' = K_1\nabla^2 T' \tag{20}$$

$$\left(1+\frac{M_0}{H_0}\right)\nabla_1^2\phi' + \left(1+\chi\right)\frac{\partial^2\phi'}{\partial z^2} - K_m\frac{\partial T'}{\partial z} = 0 \tag{21}$$

where $C_2 = \varepsilon_p C_1 + \left(1-\varepsilon_p\right)\left(\rho_0 C\right)_s$, $\boldsymbol{q}' = \left(u', v', w'\right)$, $\boldsymbol{H}' = \nabla\phi'$ with ϕ' being the magnetic potential. We eliminate the pressure term p' from Eq. (19) and then render the resulting equation and Eqs. (20) and (21) dimensionless through the following transformations $\left(x^*, y^*, z^*\right) = \left(\frac{x}{h}, \frac{y}{h}, \frac{z}{h}\right)$, $w^* = \frac{C_1 h w'}{K_1}$, $T^* = \frac{T'}{\Delta T}$, $t^* = \frac{K_1 t}{C_1 h^2}$ and $\phi^* = \frac{\left(1+\chi\right)\phi'}{K_m\,\Delta T\, h}$ to obtain (after dropping the asterisks for simplicity)

$$\left(\frac{1}{Va}\frac{\partial}{\partial t}+1\right)\nabla^2 w=\left[R\left(1+\varepsilon\cos\omega t\right)+RM_1\right]\nabla_1^2 T-RM_1\frac{\partial}{\partial z}\left(\nabla_1^2\phi\right) \tag{22}$$

$$\frac{\partial T}{\partial t}-w-M_2\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}\right)+M_2 w=\nabla^2 T \tag{23}$$

$$\left(\frac{\partial^2}{\partial z^2}+M_3\nabla_1^2\right)\phi=\frac{\partial T}{\partial z} \tag{24}$$

where ω is the dimensionless frequency of modulation given by $\varpi=\frac{C_1 h^2}{K_1}\omega$, $\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ and $\nabla^2=\nabla_1^2+\frac{\partial^2}{\partial z^2}$. The dimensionless parameters are $Va=\frac{\varepsilon_p\gamma h^2}{k\kappa}$, the Vadasz number, $R=\frac{\beta\Delta T g_0 hk}{\gamma\kappa}$, the Darcy-Rayleigh number, $M_1=\frac{\mu_0 K_m^2\Delta T}{(1+\chi)\beta g_0\rho_R h}$, the buoyancy-magnetization parameter, $M_2=\frac{\mu_0 K_m^2 T_R}{(1+\chi)C_1}$, the magnetization parameter, and $M_3=\frac{M_0+H_0}{H_0(1+\chi)}$, the non-buoyancy magnetization parameter, where $\kappa=\frac{K_1}{C_1}$ and $\gamma=\frac{\mu_f}{\rho_R}$.

To this end we note that the typical values of M_2 are of the order of 10^{-6} [2]. Hence we neglect M_2 and proceed further. The appropriate boundary conditions for the problem at hand are

$$w=T=\frac{\partial\phi}{\partial z}=0 \text{ at } z=0,1. \tag{25}$$

The magnetic boundary conditions in (25) are based on the assumption of infinite magnetic susceptibility. Finlayson [2] used this type of boundary condition in order to obtain exact solution to the ferroconvective instability problem with free-free, isothermal boundaries. The systematic derivation of the general boundary conditions for the magnetic potential is given in the work of Maruthamanikandan [13]. It is convenient to express the entire problem in terms of w. Upon combining Eqs. (22) – (24), we obtain an equation for the vertical component of the velocity w in the form

$$\begin{aligned}&\left(\frac{1}{Va}\frac{\partial}{\partial t}+1\right)\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial^2}{\partial z^2}+M_3\nabla_1^2\right)\nabla^2 w\\&=R\left(1+\varepsilon f\right)\left(\frac{\partial^2}{\partial z^2}+M_3\nabla_1^2\right)\nabla_1^2 w+RM_1M_3\nabla_1^4 w\end{aligned} \tag{26}$$

where $f(t)=\cos\omega t$. The boundary conditions in (25) can also be expressed in terms of w in the form [25]

$$w=\frac{\partial^2 w}{\partial z^2}=\frac{\partial^4 w}{\partial z^4}=0 \quad \text{at } z=0,1. \tag{27}$$

V. METHOD OF SOLUTION

We now seek the eigenfunction w and eigenvalue R of Eq. (26) for the basic temperature profile that departs from the linear profile by quantities of order ε. It follows that the eigenfunctions and eigenvalues of the problem differ from the classical Rayleigh-Bénard problem of ferroconvective instability in a Darcy porous layer by quantities of order ε. We therefore assume the solution of Eq. (26) in the form

$$(w,R)=(w_0,R_0)+\varepsilon(w_1,R_1)+\varepsilon^2(w_2,R_2)+\cdots\cdots \tag{28}$$

where R_0 is the Darcy-Rayleigh number for the unmodulated Rayleigh-Bénard convection in a ferromagnetic fluid saturated porous layer. Substituting (28) into Eq. (26) and equating the coefficients of like powers of ε, we obtain the following system of equations up to $O\left(\varepsilon^2\right)$

$$Lw_0=0 \tag{29}$$

$$Lw_1=R_1\left[\frac{\partial^2}{\partial z^2}+M_3\left(1+M_1\right)\nabla_1^2\right]\nabla_1^2 w_0+R_0\left(\frac{\partial^2}{\partial z^2}+M_3\nabla_1^2\right)f\,\nabla_1^2 w_0 \tag{30}$$

$$\begin{aligned}Lw_2=&R_1\left[\frac{\partial^2}{\partial z^2}+M_3\left(1+M_1\right)\nabla_1^2\right]\nabla_1^2 w_1+R_2\left[\frac{\partial^2}{\partial z^2}+M_3\left(1+M_1\right)\nabla_1^2\right]\nabla_1^2 w_0\\&+R_0\left(\frac{\partial^2}{\partial z^2}+M_3\nabla_1^2\right)f\,\nabla_1^2 w_1+R_1\left(\frac{\partial^2}{\partial z^2}+M_3\nabla_1^2\right)f\,\nabla_1^2 w_0\end{aligned} \tag{31}$$

where

$$L=\left(\frac{1}{Va}\frac{\partial}{\partial t}+1\right)\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial^2}{\partial z^2}+M_3\nabla_1^2\right)\nabla^2-R_0\left(\frac{\partial^2}{\partial z^2}+M_3\left(1+M_1\right)\nabla_1^2\right)\nabla_1^2.$$

The zeroth order problem is equivalent to the problem of Rayleigh-Bénard ferroconvection in a porous layer in the absence of thermal modulation. The marginally stable solution for the unmodulated problem is given by

$$w_0=\exp\left[i\left(l\,x+m\,y\right)\right]\sin\pi z \tag{32}$$

where l and m are wavenumbers in the x and y directions. Upon substituting (32) into Eq. (29), we obtain the following expression for the Rayleigh number

$$R_0=\frac{\left(\pi^2+M_3\alpha^2\right)\left(\pi^2+\alpha^2\right)^2}{\alpha^2\left[\pi^2+M_3\left(1+M_1\right)\alpha^2\right]} \tag{33}$$

where $\alpha^2=l^2+m^2$ is the overall horizontal wavenumber of the convective disturbance. Since changing the sign of ε amounts to a shift in the time origin and such a shift does not affect the stability of the problem, it follows that all the odd coefficients $R_1, R_3,$ ..... in Eq. (28) must vanish. Following the analysis of Malashetty and Padmavathi [26], we obtain the following expression for R_2

$$R_2=K_3\sum_{n=1}^{\infty}\left(n^2\pi^2+M_3\alpha^2\right)\frac{C_n}{D_n} \tag{34}$$

where

$$K_3=\frac{R_0^2\,\alpha^2\left(\pi^2+M_3\alpha^2\right)}{2\left[\pi^2+M_3\left(1+M_1\right)\alpha^2\right]}$$

$$\begin{aligned}C_n=&\frac{1}{Va}\omega^2\left(n^2\pi^2+\alpha^2\right)\left(n^2\pi^2+M_3\alpha^2\right)-\left(n^2\pi^2+\alpha^2\right)^2\left(n^2\pi^2+M_3\alpha^2\right)\\&+R_0\,\alpha^2\left[n^2\pi^2+M_3\left(1+M_1\right)\alpha^2\right]\end{aligned}$$

$$D_n = A_1^2 + A_2^2$$

with

$$A_1 = \frac{1}{Va}\omega^2\left(n^2\pi^2+\alpha^2\right)\left(n^2\pi^2+M_3\alpha^2\right)-\left(n^2\pi^2+\alpha^2\right)^2\left(n^2\pi^2+M_3\alpha^2\right)$$
$$+R_0\alpha^2\left[n^2\pi^2+M_3\left(1+M_1\right)\alpha^2\right]$$

and

$$A_2 = \frac{\omega}{Va}\left(n^2\pi^2+\alpha^2\right)^2\left(n^2\pi^2+M_3\alpha^2\right)+\omega\left(n^2\pi^2+\alpha^2\right)\left(n^2\pi^2+M_3\alpha^2\right).$$

The value of R obtained by this procedure is the eigenvalue corresponding to the eigenfunction w, which though oscillating, remains bounded in time. Since R is a function of the horizontal wavenumber α and the amplitude of modulation ε, we may write

$$R(\alpha,\varepsilon) = R_0(\alpha)+\varepsilon^2 R_2(\alpha)+\ ...... \tag{35}$$

$$\alpha = \alpha_0+\varepsilon^2\alpha_2+...... \tag{36}$$

The critical value of the Rayleigh number R is computed up to $O\left(\varepsilon^2\right)$ by evaluating R_0 and R_2 at $\alpha_0=\alpha_c$, where α_c is the value at which R_0 is minimum. It is only when one wishes to evaluate R_4, α_2 must be taken into account [26]. In view of this, we may write

$$R_c(\alpha,\varepsilon) = R_{0c}(\alpha)+\varepsilon^2 R_{2c}(\alpha)+..... \tag{37}$$

where R_{0c} and R_{2c} are respectively the value of R_0 and R_2 evaluated at $\alpha=\alpha_c$. If R_{2c} is positive, supercritical instability exists and R has the minimum at $\varepsilon=0$. On the other hand, when R_{2c} becomes negative, subcritical instability is possible.

VI. RESULTS AND DISCUSSION

In this paper we carried out an analytical study of the effect of time-periodically varying gravity field on the onset of convection in a ferromagnetic fluid saturated porous layer. The regular perturbation method based on small amplitude of modulation is employed to compute the value of Darcy-Rayleigh number and the corresponding wavenumber. The expression for critical correction Darcy-Rayleigh number R_{2c} is computed as a function of the frequency ω of modulation, the magnetic parameters M_1 and M_3, and the Vadasz number Va. The effect of these parameters on the stability of the system is elucidated. The sign of R_{2c} characterizes the stabilizing or destabilizing effect of modulation. A positive R_{2c} indicates that the modulation effect is stabilizing, while a negative R_{2c} is indicative of the destabilizing effect of modulation.

The variation of critical correction Darcy-Rayleigh number R_{2c} with frequency ω for different values of the parameters is exhibited in Figs. 1 through 6. We observe that for small values of ω, R_{2c} is negative implying that the effect of gravity modulation is to destabilize the system with convection occurring at an earlier point when compared with the unmodulated system. However, for moderate and large values of ω, R_{2c} is positive, meaning the effect of gravity modulation is to stabilize the system with convection occurring at a later point in comparison with the unmodulated system. As a result, for small values of ω, subcritical instability is possible and supercritical instability exists otherwise. It should also be noted that the effect of gravity modulation disappears altogether when ω is sufficiently large.

The effect of buoyancy-magnetization parameter M_1 on the stability of the system is displayed in Figs. 1 and 2. The parameter M_1 is the ratio of magnetic force to gravitational force. It is found that R_{2c} increases with an increase in M_1 provided that ω is small. However, the trend reverses for moderate and large values of the frequency of gravity modulation ω. Also, the magnetic mechanism reduces the destabilizing effect of gravity modulation when ω is small and again reduces the stabilizing effect of gravity modulation when ω is moderate and large. Further, we find that R_{2c} increases with increasing values of ω, attains a peak value and then decreases with further increase of ω. The frequency at which the peak value is attained depends on the strength of magnetic forces.

We present in Figs. 3 and 4 the effect of M_3 on the critical correction Darcy-Rayleigh number R_{2c} when other parameters are fixed. The parameter M_3 measures the departure of linearity in the magnetic equation of state. The results concerning Figs. 3 and 4 are qualitatively similar to that of Figs. 1 and 2. Therefore the effect of M_3 is to reduce the destabilizing effect of gravity modulation for small values of ω and the stabilizing effect of gravity modulation for moderate and large values of ω.

In Figs. 5 and 6, we display the effect of Vadasz number Va on the critical correction Darcy-Rayleigh number R_{2c} when other parameters are fixed. We observe from these figures that the critical correction Rayleigh number R_{2c} decreases with increasing Va for small values of ω indicating the destabilizing effect of Vadasz number on the convection in a gravity modulated ferrofluid saturated porous medium. However, for moderate and large values of ω, the trend reverses. Therefore the effect of increasing Va is to enhance the influence of gravity modulation on the threshold of ferroconvection in a porous layer.

The analysis presented in this paper is based on the assumption that the amplitude of the modulation is very small and the convective currents are weak so that nonlinear effects may be neglected.

VII. FIGURES

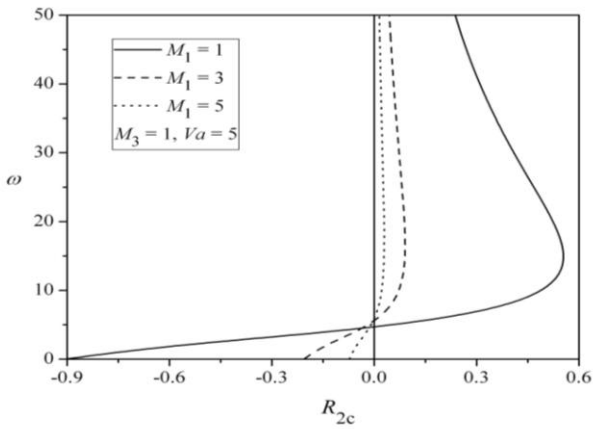

Fig. 1 Variation of R_{2c} with small and moderate values of ω for different values of M_1.

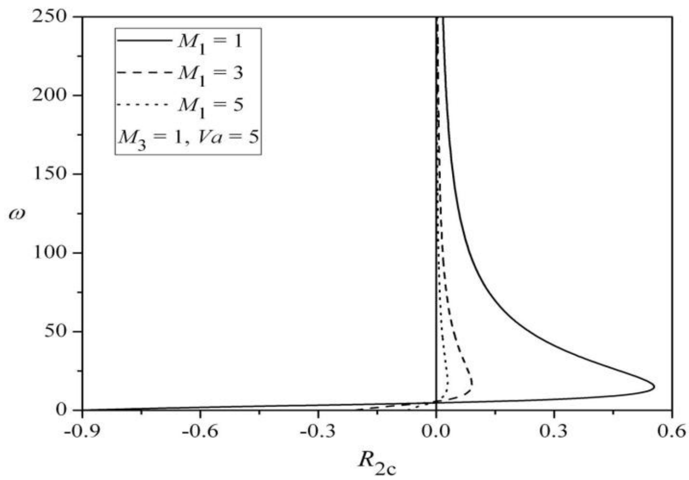

Fig. 2 Variation of R_{2c} with ω for different values of M_1.

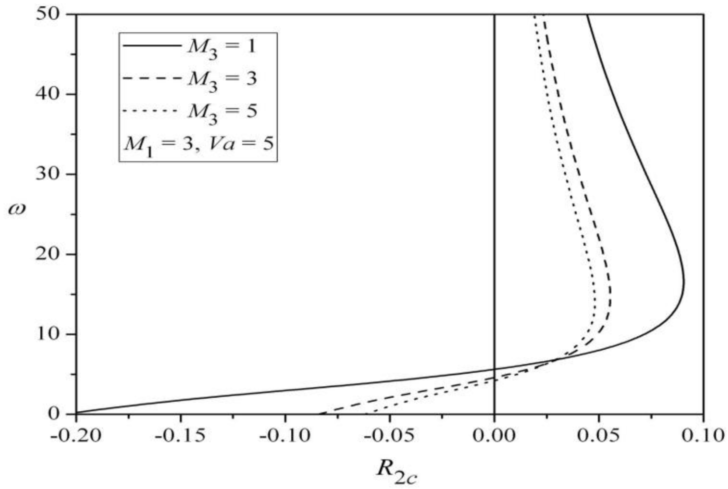

Fig. 3 Variation of R_{2c} with small and moderate values of frequencies ω for different values of M_3.

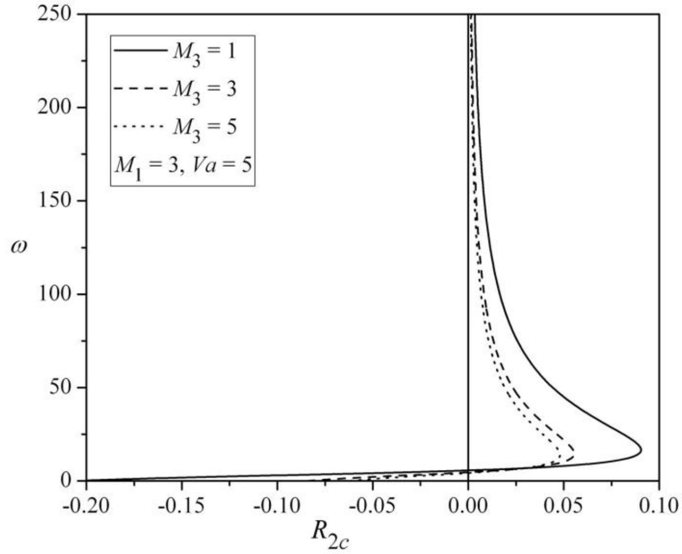

Fig. 4 Variation of R_{2c} with ω for different values of M_3.

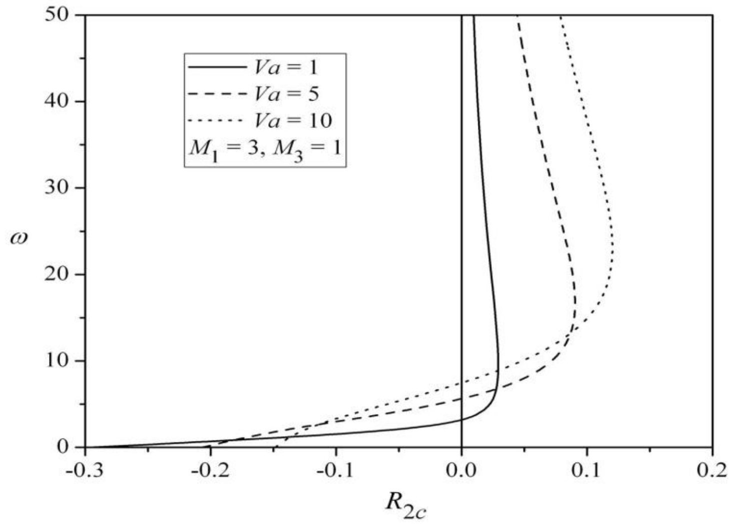

Fig. 5 Variation of R_{2c} with small and moderate values of ω for different values of Va.

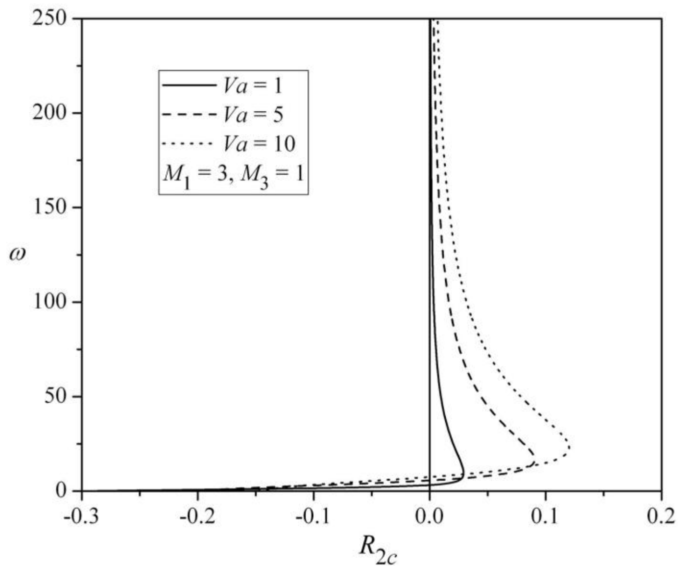

Fig. 6 Variation of R_{2c} with ω for different values of Va .

VIII. CONCLUSIONS

The effect of gravity modulation on the onset of convection in a ferromagnetic fluid saturated porous layer is studied by means of the regular perturbation method. The following conclusions are drawn:

(i) Subcritical instability manifests on account of gravity modulation for low frequency.

(ii) The effects of gravity modulation and the magnetic mechanism on the system are mutually antagonistic for small and moderate values of the frequency of modulation.

(iii) Vadasz number enhances the destabilizing effect of gravity modulation for small frequency, while for moderate and large frequency, its effect is to augment the stabilizing effect of gravity modulation.

(iv) The effects of magnetic forces, porous medium and gravity modulation disappear for sufficiently large values of the frequency of gravity modulation.

In conclusion, gravity modulation in the presence of porous medium can advance or delay the onset of ferroconvection depending on the frequency of gravitational modulation. The effect of gravity modulation could be exploited to control convective instability in a ferromagnetic fluid saturating a porous medium.

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