On The Origin of Electromagnetic Waves from Lightning Discharges

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Abstract: Interaction of up going ion beam forming current flow in the pre-ionized stepped leader plasma and the way, how the kinetic energy of the beam particles is converted into electromagnetic energy have been discussed. The ion beam interaction with the plasma wave modes in the stepped leader channel produces perturbations in the return stroke current flow and changes its uniformity and becomes non-uniform. In the present study, the return current is taken to be deeply modulated at a given modulation frequency, and considered that it behaves like an antenna for electromagnetic radiation. In this paper the total amount of energy associated with return stroke is given to electromagnetic waves is estimated.

Keywords: Anomalous Doppler Shift, Brillouin radius, Cernkov resonance, Cyclotron frequency, Cyclotron resonance, Equipartition of energy, Fourier – Laplace Transform, Gyro frequency, Modulation frequency.

I. Introduction

Lightning return stroke is an up going ion beam within a pre-ionized channel called stepped leader in which the excessive electrons deposited during the leader process are drained to the earth. The partially ionized stepped leader channel is supposed to bring the negative charge from the cloud base and distributes it homogeneously along the channel. The front of the return stroke moves upward with the velocity near to the velocity of light in the partially ionized channel [1 - 6].

The electromagnetic energy radiated by lightning return stroke covers a wide frequency range, extending from a few hertz in the ELF band to the visible light [7 - 11]. The exact physical processes, due to which radiation of electromagnetic waves from the ionized channel, taking place is not known clearly. The current in the channel is the main cause of electromagnetic radiation. The approximate assumption of the simple harmonic form of return current flow in the pre-ionized channel of stepped leader largely governs the radiated electromagnetic power and its spectral characteristics [12 - 13]. The double exponential form of return current flow does explain certain features of atmospherics but it is not based on the plasma processes taking place in the return stroke current flow in the lightning channel. Singh and Singh [13] have reported in their presentation that the upwelling ion beam interacts with the pre-ionized channel and gives rise to resulting return current. The resulting current in the lightning channel may become an oscillatory as a result of plasma wave interaction. The energy extraction from upwelling ion beam requires, for emission of the electromagnetic waves, that the phase matching between waves and particles should be maintained for as long as possible. This desirable property is maintained by Cernkov and Cyclotron resonances.

In this presentation, the idea of modulated current which flows into the pre-ionized plasma channel is taken [14 - 16] and is applied to the electromagnetic emission from the ion beam forming the return stroke. For this study it is considered that the modulated beam behaves like an antenna for the electromagnetic radiation. The value of modulation frequency, \( \Omega \) is chosen such that force resonance condition (i.e. \( \omega = \pm \Omega \)) may be sustained (\( \omega = \pm \omega \) for Cernkov resonance, and \( \omega = \pm \Omega \) for cyclotron resonance). In this presentation it has been reported that the electromagnetic power as a form of ion whistler Anomalous Doppler Shifted (ADS) mode is radiated from lightning return stroke channel. The energy in an ion whistler ADS mode is mainly due to the oscillatory damping of lower and upper wave modes in the stepped leader channel [10]. The electromagnetic radiation from an oscillatory current source becomes variable. Such variations in recorded whistler are often observed in whistler sonograms.

This paper is organized as follows: In sec. (II) return stroke ion beam interaction with pre-ionized stepped leader is discussed. Sec. (III) is devoted to describe the beam model on the basis of subsequent considerations. In sec. (IV) expression of electromagnetic power for ion whistler ADS mode is carried out. In sec. (V) applications of our proposal and its validity are argued. Results are presented and discussed in this section of the study. Finally, sec. (VI) states the conclusions.

II. Interaction Of Return Stroke Ion Beam To Pre-Ionized Stepped Leader

The dissociation, excitations and ionization processes of lower atmosphere result into partial ionized channel known as stepped leader. As the stepped leader descends downward, the length of the ionized column steadily increases. The overall degree of ionization in the stepped leader channel increases and attains peak
ionization. The ion beam originates from the ground and moves upward with a velocity close to the velocity of light. On its upward journey the ion beam meets the stepped leader at a height of about 100 – 500 m above the ground generating thundering sound and light. A part of the return stroke charges are neutralized by the down coming stepped leader. The partially neutralized return stroke charges move further upward with slower velocity and interact with the ionized plasma in the stepped leader. As a result of beam particle interaction, the return stroke ion beam is retarded by the plasma wave turbulence in the stepped leader plasma. The current thus generated does not vary linearly. The major part of the electromagnetic radiation takes place from the meeting point of upward going ions and downward coming ions.

In the steady state system a constant return current is set up in the stepped leader channel. The upward going positive ion beam interacts with the plasma wave modes in the pre-ionized stepped leader channel [13]. The beam interaction with plasma wave modes changes the uniform passage of ion beam resulting into a non-uniform flow of return stroke current.

### III. Beam Geometry Of Return Stroke

The realistic beam model for upwelling ion beam forming return stroke is shown in Fig. (1). The return stroke is considered as a circular cross-section helical beam model with a Brillouin radius $r_0$. The upwelling beam velocity is taken parallel to the magnetic field (i.e. pitch angle, $\theta = 0^\circ$). With this consideration, $V_0$ is taken as the average velocity of upwelling ion beam into stepped leader channel.

The flow of ion beam into the pre-ionized stepped leader is stable and continuous with velocity near to the velocity of light. Stable and continuous flow of ion beam into the stepped leader is maintained when Lorenz and electrostatic forces are balanced. In this way Brillouin radius arises and describes the equilibrium of upwelling ion beam. In the background plasma (i.e. stepped leader) the beam behavior is governed by the ratio $\omega_{pe} / \omega_{ce}$ where $\omega_{pe}$ and $\omega_{ce}$ are background plasma frequency and cyclotron frequency of electron respectively; when this ratio exceeds unity (in this case) neutralization effects play an important role. In term of the gyro frequency of the beam particles, Brillouin radius which is taken as the radius of the return stroke is given as [14]

$$r_0 = \left( \frac{I_0 q_i^2 \pi \varepsilon_0 V_0 m_i \omega_{ci}^2}{2} \right)^{\frac{1}{2}}$$  \hspace{1cm} (1)

Where, $I_0$ is the return current, $q_i$ and $m_i$ are the charge and mass of the beam particle, respectively, and $\omega_{ci}$ is the gyro frequency of the beam particle. The value of $r_0$ is found from Eq. (1), as 0.5 cm which is equal to the experimental value [17].

![Fig. (1) Schematic diagram shows helical model of the upwelling ion beam](image)

On the basis of this discussion, we consider the beam geometry as the sum of individual particle trajectories, and attention is focused on fluid description for the modulated current density. The current density is then taken as [14, 18]

$$j = N_0 e V_0 = \begin{cases} 0 & \text{if } z \leq 0 \\ 0 & \text{if } z > 0 \end{cases}$$

$$j_z = I_0 (\sin \Omega \tau + 1) Y(\tau) P(r_z) Y(z)$$  \hspace{1cm} (2)

Where, $\Omega$ is the modulation frequency, $\tau = t - \frac{z}{V_0}$ is the retarded time, $Y$ is the unit step function whose significance is related to the spatial and temporal variations of return stroke current and $P(r_z)$ is the transverse dimension of the beam constituting the return current. Beam is considered in the plane perpendicular to the magnetic field which is taken as the z axis. The beam dimension is defined as
\[ P (r_+) = 0 \text{ for } r_+ > r_0 \quad \text{and} \quad P (r_-) = \frac{1}{\pi r_0^2} \text{ for } r_- < r_0. \] (3)

### IV. Power Lost By The Return Stroke

The modulated current into the plasma is the desired source term for the generation of electromagnetic waves and is highly dependent on the transverse dimension of the beam. Using Maxwell’s field equations and the harmonic source current \( \mathbf{J} \), the electromagnetic wave equation in terms of the electric field vector is written as \([14, 15]\)

\[
(\mu_0 \mathbf{B} - \mu_0^2 + K) \mathbf{E} = \frac{1}{\iota \omega_\mathbf{E} \Delta} \mathbf{J}
\] (4)

Where, \( \mathbf{B} = -\mathbf{k} c / \omega \) is the refractive index of the background plasma, \( \mathbf{k} \) is wave number, \( c \) is the velocity of light; \( \mathbf{I} \) and \( \Delta \) are the unitary and dielectric susceptibility tensors, respectively. The electric field which is governed by the dispersion property of the medium, corresponding to the return current can be obtained from Eq. (4) as

\[ \mathbf{E} = \frac{\Lambda(\omega, \mathbf{E})}{\iota \omega_\mathbf{E} \Delta} \mathbf{J} \] (7)

Where, \( \Lambda \) is cofactor matrix and \( \Delta \) is its determinant, respectively. Expression for \( \mathbf{E} (r, t) \) is obtained as

\[ \mathbf{E}(r, t) = \int \frac{\Lambda(\omega, \mathbf{E})}{\iota \omega_\mathbf{E} \Delta} \mathbf{J}_1 \exp[-i(\mathbf{k} \cdot \mathbf{r} - \omega t)] d^3k d\omega. \] (8)

The current vector \( \mathbf{J} \) is easily deduced by taking the Fourier – Laplace transform of Eq. (2) as

\[ \mathbf{J} = \frac{1}{(2\pi)^2} \int j \exp[-i(\mathbf{k} \cdot \mathbf{r} - \omega t)] \Delta^2 \] (9)

The current is taken to be deeply (i.e. 100 %) modulated. Now if we neglect a dc contribution (\( \omega = 0 \)), involving current neutralization effects, the integral over \( \omega \) yeilds the term \( \Omega^2 / (\Omega^2 - \omega^2) \). The Cerenkov and Cyclotron resonances appear clearly in denominator of the current transform and there is the force resonant frequency condition (\( \omega = \pm \Omega \)). In this presentation computation is made for the radiated power in different resonance modes. Solution of Eq. (9) is found as

\[ \mathbf{J} = \int _{r_0}^r \frac{\Lambda(\omega, \mathbf{E})}{\iota \omega_\mathbf{E} \Delta} \mathbf{J}_1 \frac{\Omega}{(2\pi)^2 r_0 k_1 \Delta^2 (k_1 - \omega)^2 (\omega^2 - \omega^2)} \] (10)

Where, \( \mathbf{J}_1 \) is the Bessel function of first order, \( k_1 = k \sin \theta \), \( k_\perp = k \cos \theta \) and \( \theta \) is the angle between the direction of propagation of oscillations and magnetic field. The power lost in form of electromagnetic wave is written as

\[ P_1 = -\int \mathbf{E}(r, t) \cdot d^3 r. \] (11)

Eqs. (2) and (8) are used in (11), we obtain \( P_1 \) as

\[ P_1 = -\int \frac{\iota^2}{(2\pi)^2 r_0^2 \Delta \omega_\mathbf{E} \Delta^2} \mathbf{J}_1 \exp[-i(\mathbf{k} \cdot \mathbf{r} - \omega t)] \frac{\Delta}{(k_1 - \omega)^2} \] (12)

Integration of Eq. (12) has been carried out by means of the residue theorem \([14]\).

Neutralization takes place due to the interaction of the fully ionized beam with the partially ionized channel. The results proceed from the fact that only the zeros of \( \Diamond_m \) gives a contribution to the power lost per unit length by the beam. Within the simple pole condition, power \( P_1 \) reads as

\[ P_1 = -\frac{\iota L^2}{2\pi^2 r_0^2 \omega_\mathbf{E} \Delta^2 \omega_\mathbf{E}} \frac{g_{zz}}{2} \left( k_1, r_0 \right) \] (13)

Where,

\[ \Delta_m = \mu^2 \left( \mu_0^2 - \frac{\mu_0^2}{\Delta_0} \right) \frac{\partial}{\partial k_1} \left( \mu^2 - \mu_0^2 \right), \]

\[ \mu_1 = \kappa_1 \sin^2 \theta + \kappa_3 \cos^2 \theta, \]

\[ \mu_2 = \left( \frac{\omega_0^2}{\omega_0^2 - \omega^2} - \frac{\omega_0^2}{\omega_0^2 - \omega^2} \right), \]

\[ \mu_2 = \left( \frac{\omega_0^2}{\omega_0^2 - \omega^2} - \frac{\omega_0^2}{\omega_0^2 - \omega^2} \right), \]

\[ \frac{\partial}{\partial k_1} \left( \mu^2 - \mu_0^2 \right) = \frac{\mu^2}{\omega^2} \left[ 1 - \left( \frac{\omega_0^2}{\omega} \right) \cos \theta \right] \]

\[ \kappa_1 = 1 - \frac{\omega_0^2}{\omega^2}, \]

\[ \kappa_3 = 1 - \frac{\omega_0^2}{\omega^2}, \]

\[ g_{zz} \approx \mu^2 \cos \theta. \]

Dispersion relation in low frequency ion mode is given as
\[ \mu^2 - \mu_{0k}^2 = \frac{k^2 \varepsilon^2}{(1 - \frac{\omega_{ce}}{\omega}) \omega k} \left( 1 + \frac{\omega_{pe} \omega}{\omega k} - \frac{\omega k}{\omega} \right) \]  

(14)

The solution of Eq. (14) for return stroke beam is obtained as

\[ k = \frac{\omega_{ci} k_i}{\omega} - \frac{\omega_{pe} \omega}{\omega k} \text{ and } k_\parallel = \frac{(\omega - m \omega_{ci})}{V_0} \]

Where, \( \omega_{ci} = \left( \frac{\omega_{pe} \omega_{ci} k_i}{2 \pi n e V_0 m_i r_0^2} \right)^{\frac{1}{2}} \)

The length of the continuously increasing upwelling ion beam which finally attains to its maximum length 5 km is given by [5, 12]

\[ L = \int_0^L \nu \, dt \]  

(15)

In which \( \nu = v_0 (\exp(-at) - \exp(-bt)) \) where, \( v_0 = 3 \times 10^8 \text{ ms}^{-1}, a = 6 \times 10^4 \text{ s}^{-1} \) and \( b = 7 \times 10^5 \text{ s}^{-1} \) is the velocity of return stroke [19 - 23]. From this consideration, Eq. (13) is written as

\[ P_1 = P_0 V_0 \left[ \frac{\exp(-bt)}{b} - \frac{\exp(-at)}{a} \right] \]  

(16)

Where, \( P_0 = -\frac{I_0^2}{2\pi r_0^2 \varepsilon_0} \frac{a}{\Delta m k_\perp} \left| J_1^2 (k_\perp, r_0) \right| \)

V. Applications And Discussions

Eq. (16) gives the power lost by the return stroke. Most of the electric energy which is dissipated by a lightning return stroke is thought to be stored initially in the stepped leader [24]. During the primary stages of the return stroke, much of this energy goes into the dissociation, ionization and heating of the channel, some fraction of it is radiated away in different modes. The radiated electromagnetic power is obtained by classical equipartition of energy between the wave energy and the kinetic energy of the plasma particles. Only one quarter of energy is given to waves; remaining part is distributed between plasma particles [15, 18]. Thus, radiated power is written as

\[ P_R = \frac{P_1}{4}. \]  

(17)

Fig. (2) Graph shows variation of the radiated power

For the sake of calculations of plasma parameters, following values have been taken for the typical lightning return stroke channel [6, 7, 25, 26].

\[ r_0 = 5 \times 10^{-3} \text{ m} \]  

[i.e. equal to the value calculated from Eq. (1)], \( I_0 = 20 \times 10^3 \text{ A} \) and \( V_0 = 1.3 \times 10^8 \text{ m s}^{-1} \). Values of plasma parameters for the lightning are found as

(i) \( \omega_{ce} = 1.3 \times 10^8 \text{ s}^{-1} \),  
(ii) \( \omega_{pe} = 5.6 \times 10^9 \text{ s}^{-1} \),  
(iii) \( \omega_{ci} = 8.7 \times 10^8 \text{ s}^{-1} \),  
(iv) \( \omega_{pi} = 1.1 \times 10^{12} \text{ s}^{-1} \), and  
(v) \( \Omega = \omega = 2.0 \times 10^4 \text{ s}^{-1} \).

Using these values, variation of the radiated power in ion ADS mode (i.e. for \( m = -1 \)) with time is calculated and presented in Fig. (2). Radiated power is found to increase due to the fact that the length of the ion beam increases continuously with its rise time. Increase of the length of the return stroke provides more space for the development of the instabilities caused due to the interaction of the upwelling ion beam within the partially ionized plasma. In the present study it has been assumed that the Brillouin radius arises during the passes of the return ion beam. As a result of the balance of Lorentz force and the electrostatic repulsive force, stable and continuous flow of return current is maintained. As a result of huge amount of beam current, magnetic field arises in the channel due to which ions are gyrated, and hence ion cyclotron frequency depends.
on the return current. Computation of Brillouin radius from Eq. (1) can be manifested as the beam radius. Interaction of upwelling ion beam to the pre-ionized stepped leader is resulted into the oscillatory nature of return beam current. The oscillatory nature of the current flow in the plasma system is found to result into modulation of the beam and self consignment of the beam current. The harmonically varying current radiates large amount of electromagnetic power which may be available for ion whistler ADS mode propagation. These microscopic processes are important for variations in the radiating characteristics of the source. The observed whistler wave intensity recorded at the ground based or satellite measurements can not be entirely attributed propagation effects through whistler ducts. The oscillatory variation should be carefully analyzed before using these data for any diagnostic study.

From this study it is argued that: For normal Doppler shifted mode (i.e. \( m = 1 \)) wave damping has been found to occur with the propagation of return stroke ion beam. For ion whistler mode (i.e. \( m = 0 \)) the contribution of return stroke beam is found to drop out.

VI. Conclusions

In the present study it is argued that the process of wave – particle interaction in the stepped leader and return stroke channels enhances the oscillations at times result into modulation and filamentation of the beam forming return stroke current. The radiated electromagnetic power in form of ion whistler ADS mode is mainly due to the oscillatory damping of waves in the pre-ionized channel. The oscillatory features in the recorded whistlers should be carefully analyzed before sing the data for any diagnostic studies. The recorded whistlers’ characteristics are combination of source variations and changes arising from propagation effect in the field aligned whistler ducts.

References