

Computations on Gabor lens having two different field distributions

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Abstract : The focusing capability of a Gabor lens look very promising, relatively little experimental research has been performed to test and develop this device for practical use. Since there is little published work on the properties of Gabor lens, the present computational work has been concentrated on studying such lens. The magnetic and the electrostatic fields constituting Gabor lens have been represented by the rectangular field model and the Glaser field model. The focal length of the lens has been computed with the aid of the electron beam trajectory under zero magnification conditions. The present work has paved the path for new investigation on Gabor lens.

Keywords: Gabor lens, trajectory equation, electron beam, field models, focal length.

I. INTRODUCTION

When a particle beam propagates through background gas, a plasma is formed by collisional ionization resulting in (Partial) neutralization of the beam's space charge and decrease of the beam radius. This "gas focusing" effect occurs naturally and often utilized to improve high-current beam transport. Gabor, in 1947, proposed a non-neutral electron plasma confined in a magnetron – type trap as an effective “space – charge lens” for positive ion beams [1]. This “Gabor lens” which offers better control and focusing strength than both gas focusing and applied fields, has been investigated by several research groups since its invention.

A Gabor lens is a charged particle lens which may be useful for matching an ion beam into a radio frequency quadrupole (RFQ)[2]. It is also interesting from a purely physical perspective since it contains a non – neutral plasma. Focusing an ion beam with a given sign of charge in a Gabor lens requires a non- neutral plasma with the opposite sign of charge as the beam. It is the electric field in the Gabor lens which provides the external forces to focus the beam ions [3].

II. GABOR LENS CONSTRUCTION

The design concept of a Gabor lens for an electron beam is illustrated somewhat simplistically in Fig.1. There is a small opposition coil at one end of the lens to create a cusp in the magnetic field. This gives the plasma hydromagnetic stability. It has been found that it is necessary to place an iron shell around the lens to shield the source from the magnetic field of the lens. The advantage of this configuration is that the electrons emitted by the cathode are born in a region with $B=0$, i.e., their canonical angular momentum is zero [4].

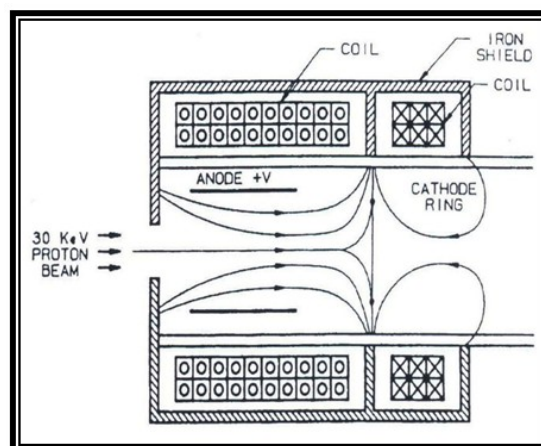


Figure1 Gabor lens with magnetic field cusp. The cusp adds stability to the plasma.

In the design shown in Fig. 1, a magnetic field is produced by two coils of uneven length, in opposition. A hot cathode in the form of circular loop is arranged at or near the magnetic field line which crosses the axis. This arrangement is necessary, as electrons which have to cross magnetic flux lines are prevented by their angular momentum from reaching the axis.

In the region where the magnetic field is approximately axial and homogeneous, a cylindrical electrode, the 'anode', is arranged, with high positive potential with respect to the cathode. So long as the anode potential is below a certain critical value, the magnetic field prevents all but a small fraction of the electrons from reaching the anode. At the two ends, guard electrodes with potential somewhat below the cathode potential prevent the electrons from escaping. Thus the space accessible to electrons is limited from all sides. Into this space the cathode will pour electrons until the potential in the axis is depressed to very nearly cathode potential, and equilibrium is established [5].

III. Trajectory Equations

Electron and ion optics is the theory and practice of production, control, and utilization of charged particle beams. Charged particle beams can only be controlled (accelerated, focused, and deflected) by external electromagnetic fields. Electric field are produced by set of electrodes held at suitably chosen voltage.

Coils surrounded by ferromagnetic materials provide magnetic field. Different symmetries may be utilized for electron and ion optical element such as lenses, deflectors, etc. [6].

In a typical device, the beam originates with some energy from the electron source and enters the focusing field of an objective lens at a certain acceptance angle, to be focused to a point image at a reasonable working distance behind the lens. However, due to the geometrical and chromatic aberrations of the lens, a point image can never be achieved. There will be a crossover of different trajectories instead. This crossover can be imaged by a projector lens and deflected by a suitable deflector element. The deflector itself is a source of additional aberrations.

When a charged particle travels through a region containing a magnetic field or an electrostatic field, or both, forces will be exerted on it that deflect it and, in the case of electrostatic field, alter its speed.

The trajectory of electrostatic symmetrical electron or ion optical system is given by:

$$\frac{d^2r}{dz^2} + \frac{dr}{dz} \frac{V'}{2V} + \frac{V''}{4V} r = 0 \text{ --- (1)}$$

Where r is the radial displacement of the beam from the optical axis z , and the primes denote a derivative with respect to z . $V=V(z)$ is the electrostatic potential distribution along the optical axis z . Equation (1) is a linear homogeneous second order differential equation, known as the paraxial – ray equation which describes the paths of charged particles moving through a rotationally symmetrical electrostatic field characterized by the potential function V . The paraxial ray equation was first derived by Busch in 1926. Many important deductions can be made from this equation [7]:

(a) The quotient of charge-to-mass (Q/m) does not appear in the equation. Therefore, the trajectory is the same for any charged particle entering the field with the same initial kinetic energy, but arrive to the same focus at different times.

(b) The equation is homogeneous in V . Therefore, an equal increase (or decrease) in the potential V at all the points of field (multiplying the potential by any constant) does not change the trajectory.

(c) The equation is homogeneous in r and z which indicates that any increase in the dimensions of the whole system produce a corresponding increase in the dimension of the trajectory, since the equipotential, though of the same form, are enlarged. If the object is doubled in size, the image will be doubled in size, the ratio between the two remaining constant.

The motion of a charged – particle (an electron in this case) in an axially symmetrical field may be derived starting from many departure points. One can start from a more familiar method of elementary mechanics [2]. The paraxial – ray equation of an electron in a magnetic field of axial symmetry is given by:

$$\frac{d^2r}{dz^2} + \left(\frac{e}{8mV_r} \right) (B_z)^2 r = 0 \text{ --- (2)}$$

Where e and m are the charge and mass of the electron respectively, B_z is the axial flux density, and V_r is the relativistically corrected accelerating voltage which is given by:

$$V_r = V_a (1 + 0.978 * 10^{-6} V_a) \text{ --- (3)}$$

Where V_a is the accelerating voltage. It can be easily realized from equation (2) that the force driving the electrons towards the axis is directly proportional to the radial distance r . This is the principle of a focusing field. Furthermore, this force is proportional to the square of the magnetic flux density which means that if the direction of the magnetic field is reversed by reversing the current, the direction of the force towards the axis should not change, i.e. there will be no change in the focus.

For combined electrostatic and magnetic field in the non – relativistic case the trajectory equation is given by [7]:

$$\frac{d^2r}{dz^2} + \frac{1}{2V} \frac{dV}{dz} \frac{dr}{dz} + \frac{1}{4V} \left[\frac{e}{2m_0} (B_z)^2 + \frac{d^2V}{dz^2} \right] r = 0 \text{ ----- (4)}$$

Equation (4) is applied in the present work since Gabor lens consists of combined electrostatic and magnetic fields.

IV. Field Models

It is often desirable to perform a rapid approximate evaluation of lens properties without actually carrying out a detailed analysis. This can be accomplished if we have a simple mathematical model for the lens, i.e. an approximation for the axial field distribution that is reasonably close to the real one and allows a solution in closed form or an approximation in simple terms [6].

A- The rectangular model

The axial flux density distribution in rectangular model is given by:

$$B(z) = B_m \text{ ----- (5)}$$

Where B_m is the maximum value of the magnetic field. The general shape of axial flux density distribution $B(z)$ is shown in fig. 2.

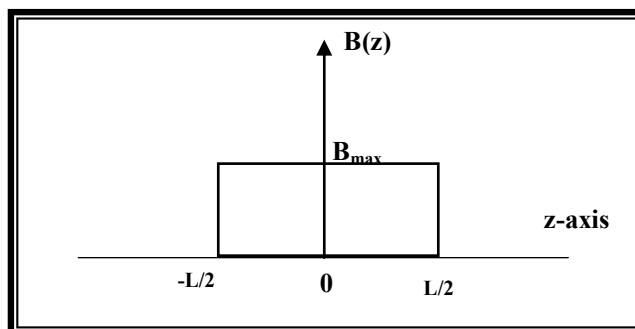


figure 2 the axial flux density distribution in rectangular field model.

B- The Glaser field model

The axial flux density distribution in Glaser model is given by:

$$B(z) = \frac{B_m}{1 + \left(\frac{z}{a}\right)^2} \text{ ----- (6)}$$

Where B_m is the maximum value of magnetic field, and a : is the width at half maximum ($B_m/2$); B_m mostly depends on the excitation. The general shape of axial flux density distribution $B(z)$ is shown in fig.3 [7].

In the present work the rectangular and Glaser field models have been taken into account to represent the electrostatic and magnetic fields constituting Gabor lens.

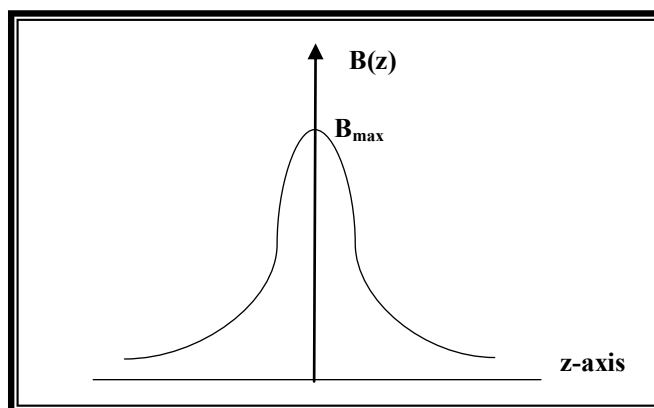


figure 3 Glaser's model

V. Definitions and Operating Conditions

Some definitions and operating conditions of charged – particle optical system are given in this section.

Object side: The side of lens or at which the charged particles enter.

Image side: The of lens or deflector at which the charged particles leave.

The object plane (z_o): The plane at which the physical object is placed, or a real image is formed from a previous lens, on the object side.

The image plane (z_i): The plane at which the real image of the object plane z_o is formed on the image side.

Magnification (M): In any optical system the ratio between the transverse dimension of the final image and the corresponding dimension of the original object is called the lateral magnification:

$$M = \frac{(\text{imageheight})}{(\text{objectheight})} \text{-----}(7)$$

There are three magnification conditions in which a lens or deflector can operate, namely:

- (i) Zero magnification condition: In this case of operating condition $Z_0 = -4$ as shown in fig. 4. The final probe – forming lens in a scanning electron microscope (SEM) is usually operated in this condition.

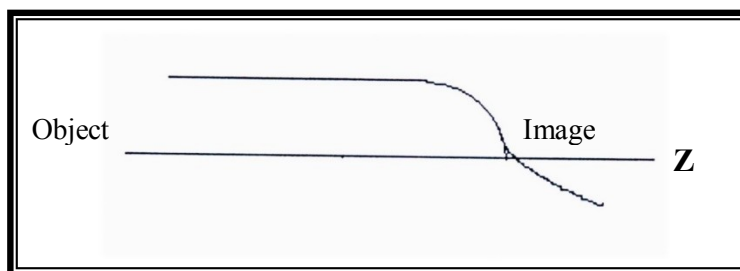


figure 4 Zero magnification condition.

- (ii) Infinite magnification condition: The operating condition is such that $Z_i = +4$ as shown in fig.5. The objective lens in a transmission electron microscope (TEM) is usually operated in this condition.

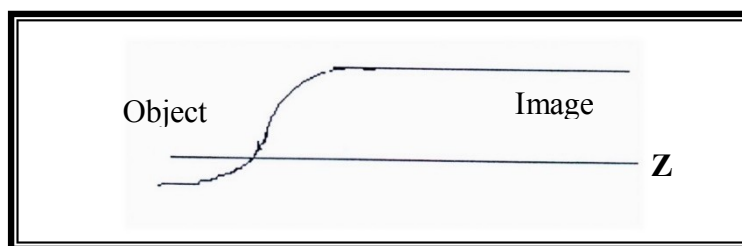


figure 5 Infinite magnification condition.

(iii) Finite magnification condition: The operating condition in this case is that Z_0 and Z_i are at finite distance, as shown in fig.6. The electrostatic lens in field –emission gun is usually operated in this condition [7].

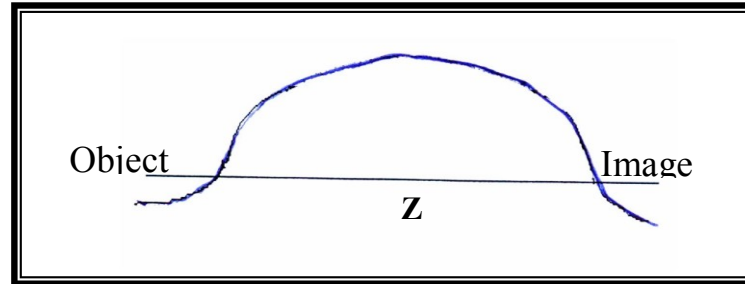


figure 6 Finite magnification condition.

In the present work the zero magnification condition has been taken into consideration.

VI. Results and Discussion

This research aims to study the trajectories of an electron beam in magnetic and electrostatic field constituting a Gabor lens. One field is assumed to be that of Glaser and the other is of rectangular shape. This research is preformed computationally.

A computer program with MathCAD Professional, has been used for determining the trajectory of the electron with the aid of the second – order Runge – Kutta method. The Program has shown high ability of drawing the general shapes of axial flux density distributions and high facility and proficiency of drawing the trajectory of the electron beam traversing rotationally symmetric field.

Two different field distributions have been considered in Gabor lens. The Glaser field distribution given by equation (6) firstly represented the magnetic field and the rectangular field distribution given by equation (5) represented the electrostatic field. Then the situation was reversed where the Glaser distribution represented the electrostatic field and the rectangular field distribution represented the magnetic field. The trajectory and the focal length were computed for each case.

The electron beam path along the axis of Gabor lens under zero magnification condition is shown in fig. 7.

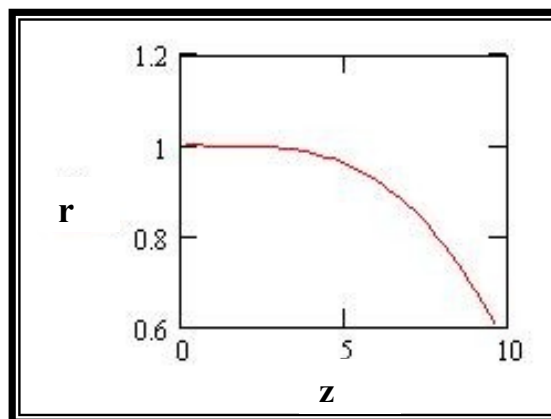


figure 7 the radial displacement r of the electron beam trajectory as a function of the axial distance z .

Under zero magnification condition the image side focal length f of the lens depends on the voltage V as shown in fig. 8. It is seen that f increases with increasing voltage, this is due to the decrease of the lens refractive power.

Gabor's space – charge lens shows that this lens is capable of providing much stronger focusing than other lenses.

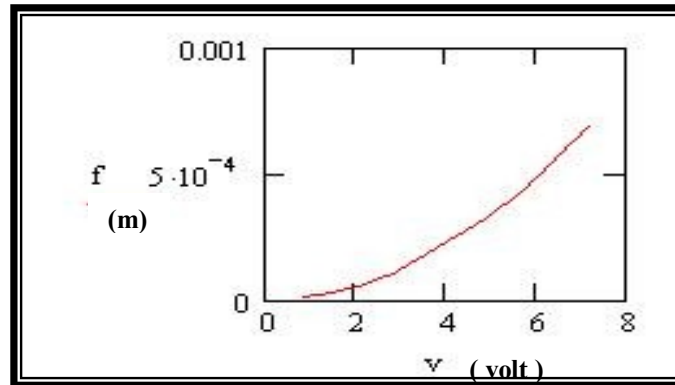


figure 8 The focal length as a function of the potential V of a Gabor lens having Glaser and rectangular field distributions.

We need reduced magnetic and electrostatic field strength or a reduced installation length to provide a given focal length compared with conventional systems like quadrupoles and magnetic solenoids. The density distribution of the enclosed space charge is given by the enclosure conditions in transverse and longitudinal direction. For homogeneous charge density distribution the resulting electrostatic field and the focusing forces inside the space charge cloud are linear. Additionally in case of a positive ion beam the space charge of the confined electrons causes compensation of the ion beam space charge forces. Hence all resolving forces on the beam ions are linear and thus the transformation is linear. Therefore space charge lenses are a serious alternative to inject space charge dominated low energy heavy ion beam into a RFQ.

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