

## Evaluation of Empirical Formulae for Estimating Global Radiation at Jos

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**Abstract:** This paper compares the prediction of three empirical models for estimating the global radiation at Jos with values obtained by measurement using 5 years data collected from the Nigerian Meteorological Agency (NIMET). Two of these models, Angstrom's and Reitveld's, related the global radiation to the sunshine duration. The third model, due to Monteith, related the global radiation to the maximum irradiance at solar noon. Mean daily values of insolation,  $\overline{H}$ , were computed and averaged over the 5-year period. Mean bias errors (MBE), root-mean-square errors (RMSE) and correlation coefficients (R) were also computed. Comparison of the results obtained showed that Monteith's model provided the best overall performance with  $\overline{H}_M = 22.1 \pm 3.4 \text{ MJm}^{-2}$ ,  $\text{MBE} = 0.81\%$ ,  $\text{RMSE} = 1.9\%$  and  $R^2 = 0.910$ . Angstrom's model with  $\overline{H}_A = 21.2 \pm 3.3 \text{ MJm}^{-2}$ ,  $\text{MBE} = -2.8\%$ ,  $\text{RMSE} = 6.3\%$  and  $R^2 = 0.863$  was rated next best, whereas Reitveld's model with  $\overline{H}_R = 20.5 \pm 3.3 \text{ MJm}^{-2}$ ,  $\text{MBE} = -6.2\%$ ,  $\text{RMSE} = 8.4\%$  and  $R^2 = 0.853$  was rated least accurate. The measured insolation had a mean of  $\overline{H}_G = 21.9 \pm 3.3 \text{ MJm}^{-2}$ . The values of insolation obtained in this work strongly suggest that solar energy could be utilized in Jos and its environs to meet some energy needs, at least at the domestic level.

**Key words:** empirical models, global radiation, maximum irradiance, solar noon, insolation.

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### I. Introduction

The sun can serve as a practical bountiful source of energy that will sustain civilization as the conventional fuels become expensive and scarce. Detailed information about solar radiation availability at any location is essential for the design and economic evaluation of a solar energy system. For example, in order to design a solar energy system, one must calculate the amount of solar energy that will fall on the collector each month. A simple way of doing this is to use the average daily solar radiation on a horizontal surface. Long-term measured data of solar radiation are required for this. Where long-term measured data are not available, various models based on available climatic data can be used to estimate the solar energy availability.

This paper uses three empirical models, namely Monteith's, Angstrom's and Reitveld's, to calculate the mean daily global radiation or insolation on a horizontal surface at Jos. The results obtained are then compared with the measured values.

### II. Method of Data Collection

The data set used in this paper was obtained from the Nigerian Meteorological Agency (NIMET), national headquarters, Lagos. These data contain records of monthly mean daily values of global radiation and sunshine hours for the years 1995 and 1998-2001, recorded from Haipang airport, near Jos, and are shown in appendix A.

### III. Method of Data Analysis

The above data was used in this paper to estimate the mean daily insolation for Jos using the three empirical models earlier mentioned. These empirical models are as follows:

#### Monteith's Model

Monteith's model for estimating the average insolation  $\overline{H}$  is (Monteith, 1975):

$$\overline{H} = (2N / \pi) S_{im} \quad (1)$$

where  $N$  = daylength of the average day of the month, calculated from (Halouani, Nguyen & Vo-ngoc, 1993) :

$$N = \frac{2}{15} \cos^{-1} [\cos 85^\circ - \tan(\phi)\tan(\delta)] \quad (2)$$

$\phi = \text{latitude of location (north positive)} = 9.95^\circ$  for Jos (Gelshik, 1997) and  $\delta$  is the solar declination given by Cooper (1969):

$$\delta = 23.45 \sin\{360(n + 284)/365\} \quad (3)$$

$n$  is the day of the year:  $1 \leq n \leq 365$ .

Values of  $S_{im}$  for Jos were adopted from Musa (2010) and used in equation (1) to obtain estimates of  $\overline{H}$ .

#### Angstrom Model

The Angstrom equation for  $\overline{H}$  is (Angstrom, 1924):

$$\overline{H} = \overline{H}_o \left( a + b \frac{\overline{n}}{N} \right) \quad (4)$$

where  $\overline{H}_o$  is the monthly average of the radiation outside of the atmosphere for the same location and  $a$  and  $b$  are constants dependent on site. Their values are determined by regression. Ojusu (1990) obtained these values as  $a = 0.192$  and  $b = 0.634$  for Jos. The Angstrom equation for Jos can therefore be written as:

$$\frac{H}{H_o} = 0.192 + 0.634 \sigma \quad (5)$$

where  $\sigma$  is the monthly average daily value of the fraction of possible sunshine hours given by:

$$\sigma \equiv \overline{n} / N \quad (6)$$

$\overline{n}$  is the actual sunshine duration for the day.

The extraterrestrial radiation  $\overline{H}_o$  is calculated from (Duffie & Beckman, 1991):

$$H_o = \left[ \frac{24 \times 3600}{\pi} \right] I_{sc} [1 + 0.033 \cos(360n/365)] \left[ \cos(\phi)\cos(\delta)\sin(\omega_s) + \frac{\pi\omega_s}{180} \sin(\phi)\sin(\delta) \right] \quad (7)$$

$I_{sc}$  is the solar constant and equals  $1367 \text{ Wm}^2$  (Tiwari, 2006).

#### Reitveld's Model

Reitveld's correlation for calculating the average insolation  $\overline{H}$  is (Reitveld, 1978):

$$H / H_o = 0.18 + 0.62\sigma \quad (8)$$

Reitveld's equation is believed to have universal validity (Halouani, Nguyen & Vo-ngoc, 1993).

### IV. Results and Discussion

Estimates of the average daily insolation made using the three empirical models, viz. Monteith's, Angstrom's and Rietveld's, are recorded in appendix B.

Fig.1 shows the prediction of these models along with the measured values. Monteith's model has its maximum and minimum values simultaneously with the measured data (in February and July respectively) while the maximum and minimum values of Angstrom's model coincides with that of Rietveld's (in November and August respectively). It is also noteworthy that peaks of insolation always occurred in the months of February and November.

Figures 2, 3 and 4 show the performance of the models in comparison with the observed values. Note the degree of sloping of these lines and the clustering of the points around the lines. A summary of this comparison is outlined below.

#### Measured Insolation: $\overline{H}_G$ in $\text{MJm}^{-2}\text{day}^{-1}$

- Averages – (a) Wet season:  $19.2 \pm 2.1$ , APE = 10.9%  
 (b) Dry season:  $24.7 \pm 1.1$ , APE = 4.5%  
 (c) 5 year:  $21.9 \pm 3.3$ , APE = 15.0%

**Monteith's Model:**  $\overline{H}_M$  in  $MJm^{-2}day^{-1}$

- (i) Averages – (a) Wet:  $19.4 \pm 2.1$ , 10.6%  
(b) Dry:  $24.82 \pm 1.7$ , 6.9%  
(c) 5 year:  $22.1 \pm 3.4$ , 15.2%
- (ii) Correlation with  $\overline{H}_G$ :  $R^2 = 0.910$
- (iii) Errors: MBE = 0.177(0.81%), RMSE = 0.420(1.9%).

**Angstrom's Model:**  $\overline{H}_A$  in  $MJm^{-2}day^{-1}$

- (i) Averages – (a) Wet:  $18.6 \pm 2.5$ , 13.5%  
(b) Dry:  $23.9 \pm 1.3$ , 5.2%  
(c) 5 year:  $21.2 \pm 3.3$ , 15.7%
- (ii) Correlation with  $\overline{H}_G$ :  $R^2 = 0.863$
- (iii) Errors: MBE = - 0.612(-2.8%), RMSE = 1.38(6.3%)

**Rietveld's Model:**  $\overline{H}_R$  in  $MJm^{-2}day^{-1}$

- (i) Averages – (a) Wet:  $17.9 \pm 2.5$ , 13.8%  
(b) Dry:  $23.1 \pm 1.2$ , 5.3%  
(c) 5 year:  $20.5 \pm 3.3$ , 16.0%
- (ii) Correlation with  $\overline{H}_G$ :  $R^2 = 0.853$
- (iii) Errors: MBE = - 1.36(-6.2%), RMSE = 1.38(8.4%)

We note that Monteith's model has the highest value of the coefficient of determination,  $R^2$ , while Rietveld's model has the least. In terms of errors, the Rietveld's model is the least accurate whereas Monteith's model is the most accurate. In all these, Angstrom's model comes between the other two models. But while Monteith's model overestimates the average insolation, the other two models underestimate it, each in proportion to its MBE.

A drawback of Monteith's model is that it is partly dependent on measurement of radiation: values of the irradiance at solar noon are needed as input before the model can be applied, and these may not be readily available. In general, values of insolation during the wet season are lower than in the dry season. The wet season values, therefore, contribute largely to errors in annual insolation. This can also be inferred from the values of the absolute percentage errors (APE) above.

## V. Conclusion

Out of the three empirical models of insolation that were tested in this work, it has been found that Monteith's model was the most accurate while Angstrom's model was rated second best. Rietveld's model was found to be the least accurate. The values of insolation obtained in this work strongly suggest that solar energy could be utilized in Jos and its environs to meet some energy needs, at least at the domestic level.

## VI. Recommendations

- In view of the significance of radiation to our biotic environment, we should equip our observatories with the latest and most reliable instruments for measurement of solar radiation and other related quantities.
- Further research should be conducted on solar radiation on tilted surfaces at Jos.

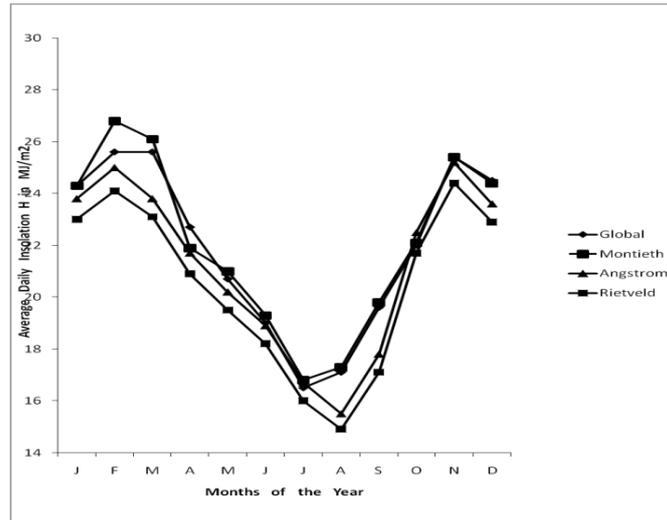


Fig.1: Variation of mean daily insolation with months at Jos: models and measured values

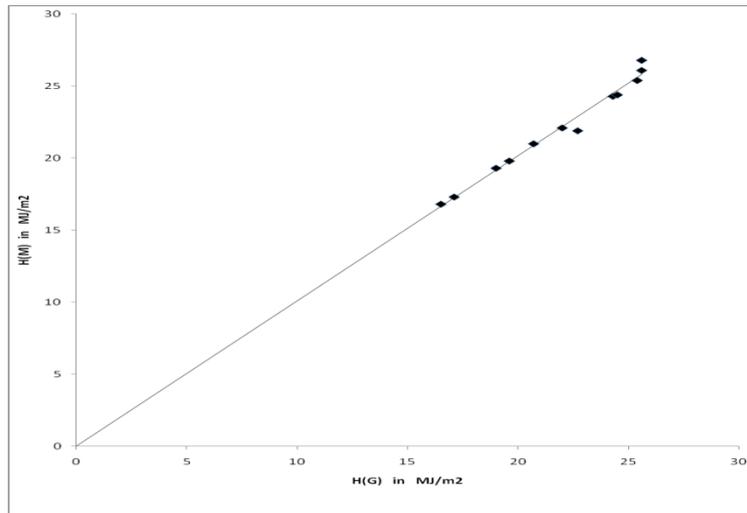


Fig. 2: Performance of Montieth's model for Jos

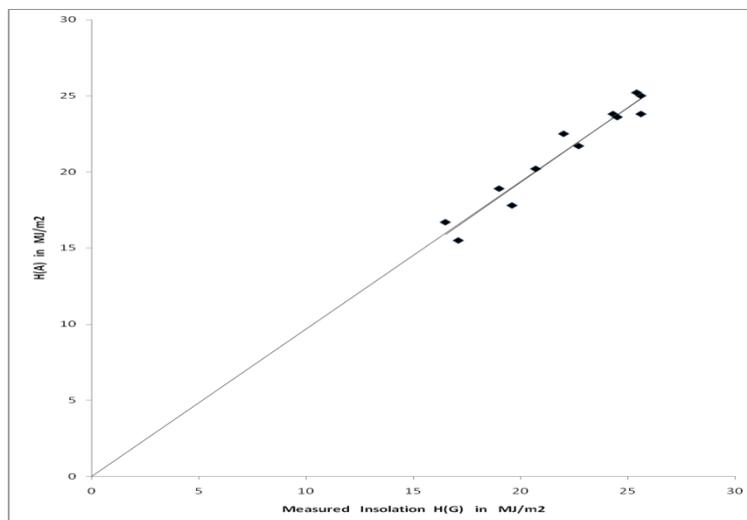


Fig. 3: Performance of Angstrom's model for Jos

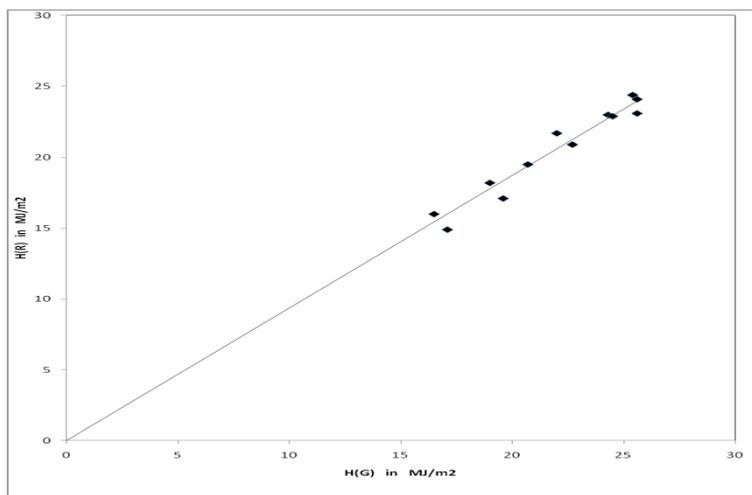


Fig. 4: Performance of Rietveld’s model for Jos

References

[1]. J.L. Monteith, Principles of Environmental Physics Edward Arnold (Publishers) Limited, London.1975.  
 [2]. N. Halouani, C.T. Nguyen and D. Vo-ngoc, Calculation of monthly average global solar radiation on horizontal surfaces using hours of bright sunshine, Solar Energy, 50 (3), 1993, 247-258.  
 [3]. M.J. Gelshik, Estimation of solar radiation using metrological parameters: sunshine hours, relative humidity and maximum air temperature, thesis, University of Jos, 1970.  
 [4]. P.T. Cooper, Absorption of solar radiation in solar stills, Solar Energy, 12 (3), 1969.  
 [5]. A. Musa, Estimation of average insolation and the radiation balance of the earth-atmosphere system at Jos, thesis, University of Jos, Nigeria, 2010.  
 [6]. A. Angstrom, Solar and terrestrial radiation, Quarterly Journal of Royal Meteorological Society, 50, 1924, 121-125.  
 [7]. J.O. Ojosu, On the bounds for global solar radiation estimates from sunshine hours for Nigerian cities. Nigerian Journal of Solar Energy, 9, 1990, 123-132.  
 [8]. J.A. Duffie and W.A. Beckman, Solar Engineering of Thermal Processes 2<sup>nd</sup> Edition. John Wiley and Sons, Chichester. 1991.  
 [9]. G.N. Tiwari, Solar Energy: Fundamentals, Design, Modeling and Applications Narosa Publishing House, New Delhi, India 2006.  
 [10]. [M.R. Rietveld, A new method for estimating the regression coefficients in the formula relating solar radiation to sunshine hours, Agricultural Meteorology, 19, 1978, 243-252.  
 [11]. G.N. Tiwari, Solar Energy: Fundamentals, Design, Modeling and Applications Narosa Publishing House, New Delhi, India.2006.

Appendix A

Raw Data for Jos Used for Analysis

Appendix A1: Global radiation measured in ml of Gunn-Bellani radiation integrator

Months	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1995	18.6	20.6	19.9	18.3	15.5	15.2	12.0	13.9	15.2	17.2	19.2	18.9
1998	19.5	21.6	20.0	18.8	15.7	14.8	12.8	12.4	14.2	16.0	19.3	18.9
1999	18.6	19.6	20.1	17.9	15.6	14.8	14.0	12.2	15.8	17.4	19.4	19.4
2000	19.3	19.3	20.6	17.2	17.4	14.2	13.1	15.5	16.2	17.2	20.3	19.1
2001	20.5	20.6	20.8	17.9	17.9	16.2	13.7	13.8	16.3	19.5	22.5	21.3

Appendix A2: Records of bright sunshine hours measured in hours

Months	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1995	9.2	9.3	7.7	6.7	5.8	6.1	4.9	5.1	5.6	7.7	9.7	9.8
1998	9.3	8.9	7.1	7.2	6.1	5.6	3.9	3.2	4.8	7.6	10.0	9.4
1999	9.1	9.2	8.3	7.0	6.3	5.8	5.3	3.6	4.8	7.3	9.7	9.6
2000	9.6	9.2	8.4	7.1	7.3	5.8	5.0	4.2	5.3	7.8	10.2	9.3
2001	9.9	9.4	8.9	7.0	6.7	6.8	4.9	4.3	5.3	8.8	10.5	9.9

**Appendix B: Average Daily Insolation at Jos**

**Appendix B1: Global radiation  $\bar{H}_G$  in  $MJm^{-2}day^{-1}$  converted from *ml* of Gunbellani**

Months	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	Year
1995	23.4	26.0	25.1	23.1	19.5	19.2	15.1	17.5	19.2	21.7	24.2	23.2	21.4
1998	24.6	27.2	25.2	23.7	19.8	18.7	16.1	15.6	18.1	20.2	24.3	23.8	21.4
1999	23.4	24.7	25.3	22.6	19.7	18.6	17.6	15.4	19.9	21.9	24.4	24.4	21.5
2000	24.3	24.3	26.0	21.7	21.9	17.9	16.5	19.5	20.4	21.7	25.6	24.1	22.0
2001	25.8	26.0	26.2	22.6	22.6	20.4	17.3	17.4	20.5	24.6	28.4	26.8	23.2
Mean	24.3	25.6	25.5	22.7	20.7	19.0	16.5	17.1	19.6	22.0	25.4	24.5	21.9

5year mean =  $21.9 \pm 3.3 MJm^{-2}day^{-1}$ , APE = 15.1%.

**Appendix B2: Monthly average daily insolation  $\bar{H}_M$  in  $MJm^{-2}day^{-1}$ , as predicted by Monteith's model**

Months	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	Year
1995	23.4	27.1	25.6	22.3	19.8	19.5	15.3	17.7	19.3	21.8	24.2	23.1	21.6
1998	24.6	28.4	25.7	22.9	20.1	19.0	16.4	15.8	18.2	20.3	24.3	23.7	21.6
1999	23.4	25.8	25.8	21.8	20.0	18.9	17.9	15.6	20.1	22.0	24.4	24.3	21.7
2000	24.3	25.4	26.5	20.9	22.3	18.2	16.8	19.8	20.6	21.8	25.6	24.0	22.3
2001	25.8	27.1	26.7	21.8	23.0	20.7	17.6	17.6	20.7	24.7	28.4	26.7	22.7
Mean	24.3	26.8	26.1	21.9	21.0	19.3	16.8	17.3	19.8	22.1	25.4	24.4	22.1

5year mean =  $22.1 \pm 3.4 MJm^{-2}day^{-1}$ , APE = 15.5%, MBE = 0.177, 0.81%, RMSE = 0.420, 1.9%,  $R^2 = 0.910$ .

**Appendix B3: Monthly average daily insolation  $\bar{H}_A$  in  $MJm^{-2}day^{-1}$ , as predicted by Angstrom's model**

Months	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	Year
1995	23.4	25.2	23.1	21.0	18.9	19.1	16.9	17.6	18.7	22.2	24.6	24.0	21.2
1998	23.6	24.4	21.8	22.1	19.5	18.1	14.9	13.7	17.0	22.0	25.2	23.3	20.5
1999	23.2	25.0	24.3	21.7	19.9	18.5	17.6	14.5	17.0	21.4	24.6	23.6	20.9
2000	24.1	25.0	24.5	21.9	21.9	18.5	17.0	15.8	18.1	22.4	25.6	23.1	21.5
2001	24.7	25.4	25.5	21.7	20.7	20.5	16.9	16.0	18.1	24.4	26.1	24.2	22.0
Mean	23.8	25.0	23.8	21.7	20.2	18.9	16.7	15.5	17.8	22.5	25.2	23.6	21.2

5year mean =  $21.2 \pm 3.3 MJm^{-2}day^{-1}$ , APE = 15.7%, MBE = -0.612, -2.8%, RMSE = 1.38, 6.3%,  $R^2 = 0.863$ .

**Appendix B4: Monthly average daily insolation  $\bar{H}_R$  in  $MJm^{-2}day^{-1}$ , as predicted by Rietveld's model**

Months	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	Year
1995	22.6	24.3	22.3	20.3	18.2	18.4	16.2	16.9	18.0	21.4	23.8	23.2	20.5
1998	22.8	23.6	21.0	21.3	18.8	17.4	14.2	13.1	16.4	21.2	24.4	22.5	19.7
1999	22.4	24.1	23.5	20.5	19.2	17.8	17.0	13.1	16.4	20.6	23.8	22.9	20.2
2000	23.4	24.1	23.7	21.1	21.1	17.8	16.4	15.1	17.4	21.6	24.8	22.3	20.7
2001	23.9	24.5	24.7	20.9	20.0	19.8	16.2	15.3	17.4	23.6	26.3	23.4	21.3
Mean	23.	20.1	23.1	20.9	19.5	18.2	16.0	14.9	17.1	21.7	24.4	22.9	20.5

5year mean =  $20.5 \pm 3.3 MJm^{-2}day^{-1}$ , APE = 16.0%, MBE = -1.38, -6.2%, RMSE = 1.83, 8.4%,  $R^2 = 0.853$ .