

Nuclear Reaction on the Basis of statistical distribution Based on Nuclear Potential

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Abstract: when the nuclear or neutron stars are assembled by bringing their constituent from infinity, gravitational and electrostatic fields are generated. Using liquid drop model, statistical laws with kinetic beside potential term can describe the generation of both macroscopic fields by the nucleus or any astronomical objects. When special relativity is taken into account ,the rest mass energy can also be included beside macroscopic fields.

Key words: Statistical distribution, Nuclear potential, Gravity field , Electric field , Neutron stars, Special relativity.

I. Introduction

The nucleus is held by the forces which protect them from the enormous repulsion forces of the positive protons. It is a force with short range and not similar to the electromagnetic force. It is well know that the nucleus is consist of protons and neutrons. These are formed from quarks which are held together with strong force. This strong force is residual color force. The basic exchange particle is called gluon which works as mediator forces between quarks. Both the particles; gluons and quarks are present in protons and neutrons. [1, 2]The range of force between particles is not determined by the mass of particles. Thus, the force which balanced the repulsion force between the positively charged particles protons is a nuclear attraction which overcomes the electric repulsion force. [3]

Nuclear Force is defined as the force exerted between numbers of nucleons. This force is attractive in nature and binds protons and neutrons in the nucleus together. Since the protons are of same positive charge they exert a repulsive force among them.

Because of this attractive Nuclear Force, the total mass of the nucleus is less than the summation of masses of nucleons that is protons and neutrons. This force is highly attractive between nucleons at a distance of 10–15 m or 1 femtometer (fm) approximately from their centers. There are two types of nuclear forces, strong and weak nuclear force. [4]

Nuclear forces are independent of the charge of protons and neutrons. This property of nuclear force is called charge independence. It depends on the spins of the nucleons that is whether they are parallel or no and also on the non central or tensor component of nucleons.

The short range nuclear force field does not exist outside the nucleus. However the gravity and electric beside magnetic fields can distribute themselves around the nucleus affecting the surrounding electrons. In general the effect of gravity on electrons can be neglected compared to the electrostatic effects. But the gravitational field becomes important for some astronomical objects like neutrons stars. The gravity and electromagnetic fields manifests themselves as macroscopic potential, while nuclear short range field is a microscopic field [5].

In the statistical physics the role of macroscopic fields and their generation are not widely studied. Some attempts were made to accounts for the effect of potential on statistical distribution [6, 7, 8]. But no detailed studies were made to use statistical laws to explain generation of macroscopic fields by the nucleus and neutron stars.

This is done in section 2.section 3 and 4 are devoted for discussion and conclusion.

II. Newtonian statistical distribution laws for particles in a field

According to Newton laws , the total energy E is given by

$$E = \int E_n dPdV$$

for one particle the total energy takes the form

$$E = \frac{p^2}{2m} + V \quad (1-1)$$

Where

P is the momentum and V is the potential
Thus for n particles the total energy is given by

$$E = \int \left(\frac{p^2}{2m} + V \right) e^{-\beta \left[\left(\frac{p^2}{2m} + V \right) \right]} dP dV \quad (1-2)$$

Thus the average energy is given by

$$\langle E \rangle = \text{average energy} = \frac{\iint_0^\infty \left(\frac{p^2}{2m} + V \right) e^{-\beta \left(\frac{p^2}{2m} + V \right)} dP dV}{\iint_0^\infty e^{-\beta \left(\frac{p^2}{2m} + V \right)} dP dV} \quad (1-3)$$

$$\langle E \rangle = \frac{\int_0^\infty V e^{-\beta V} dV}{\int_0^\infty e^{-\beta V} dV} + \frac{\int_0^\infty \frac{p^2}{2m} e^{-\beta \frac{p^2}{2m}} dP}{\int_0^\infty e^{-\beta \frac{p^2}{2m}} dP} = \frac{I_1}{I_2} + \frac{I_3}{I_4}$$

Taking the integral

$$\int_0^\infty V e^{-\beta V} dV \quad (1-4)$$

By integrating by parts

$$u_1 = V \rightarrow du_1 = dV$$

$$dV_1 = e^{-\beta V} dV \rightarrow V_1 = -\frac{1}{\beta} e^{-\beta V}$$

$$\int_0^\infty u_1 dV = u_1 V_1 - \int_0^\infty v_1 du_1$$

Then

$$I_1 = \int_0^\infty V e^{-\beta V} dV = - \left[\frac{V}{\beta} e^{-\beta V} \right]_0^\infty - \int_0^\infty -\frac{1}{\beta} e^{-\beta V} dV$$

$$I_1 = 0 + \frac{1}{\beta} \int e^{-\beta V} dV$$

$$I_1 = \frac{1}{\beta} \int_0^\infty e^{-\beta V} dV = \frac{1}{\beta} \left[-\frac{1}{\beta} e^{-\beta V} \right]_0^\infty$$

$$I_1 = \frac{-1}{\beta^2} [0 - 1] = \frac{1}{\beta^2} \quad (1-5)$$

$$I_1 = \frac{1}{\beta^2} \quad (1-6)$$

The second integral is given by

$$I_2 = \int_0^\infty e^{-\beta V} dV = -\frac{1}{\beta} e^{-\beta V} \Big|_0^\infty$$

$$= \frac{-1}{\beta} [0 - 1] = \frac{1}{\beta} \quad (1-7)$$

Thus from (1-7) and (1-6) one gets

$$\frac{I_1}{I_2} = \frac{\int_0^\infty V e^{-\beta V} dV}{\int_0^\infty e^{-\beta V} dV} = \frac{\frac{1}{\beta^2}}{\frac{1}{\beta}} = \frac{\beta}{\beta^2} = \frac{1}{\beta}$$

$$\frac{I_1}{I_2} = \frac{\int_0^\infty V e^{-\beta V} dV}{\int_0^\infty e^{-\beta V} dV} = \frac{1}{\beta} \quad (1-8)$$

The third integral is given by

$$I_3 = \int_0^\infty \frac{p^2}{2m} e^{-\beta \frac{p^2}{2m}} dP \quad (1-9)$$

$$\text{Let } x = \beta \frac{p^2}{2m} \rightarrow \left(\frac{2m}{\beta} x \right)^{1/2} = P \quad (1-10)$$

$$\rightarrow dP = \frac{1}{2} \left(\frac{2m}{\beta} x \right)^{-1/2} \left(\frac{2m}{\beta} \right) dx = \frac{m}{\beta} (2m/P)^{-1/2} x^{-1/2} dx$$

At

$$P = 0 \rightarrow x = 0, P = \infty \rightarrow x = \infty$$

$$I_3 = \int_0^\infty \frac{x}{\beta} e^{-x} \frac{m}{\beta} \left(\frac{2m}{\beta} \right)^{-1/2} x^{-1/2} dx$$

$$= \frac{m}{\beta^2} \left(\frac{2m}{\beta} \right)^{-1/2} \int_0^\infty x^{1/2} e^{-x} dx \quad (1-11)$$

By using Gamma function integrations

$$[\Gamma(n)] = \int_0^\infty x^{n-1} e^{-x} dx \quad (1-12)$$

$$n - 1 = 1/2 \rightarrow n = 3/2$$

$$\int_0^\infty x^{1/2} e^{-x} dx = \left[\frac{3}{2} \right] = \frac{1}{2} \left[\frac{1}{2} \right] = \sqrt{\pi}/2 \quad (1-13)$$

Then

$$I_3 = \frac{m}{\beta^2} \left(\frac{2m}{\beta}\right)^{-1/2} \sqrt{\pi/2} \quad (1-14)$$

The fourth integral is also given by

$$I_4 = \int_0^\infty e^{-\beta \frac{p^2}{2m}} dP = \frac{m}{\beta} \left(\frac{2m}{\beta}\right)^{-1/2} \left[\frac{1}{2}\right]$$

$$I_4 = \frac{m}{\beta^2} \left(\frac{2m}{\beta}\right)^{-1/2} \sqrt{\pi} \quad (1-15)$$

Then from (1-14) and (1-15)

$$\frac{I_3}{I_4} = \frac{\frac{m}{\beta^2} \left(\frac{2m}{\beta}\right)^{-1/2} \frac{\sqrt{\pi}}{2}}{\frac{m}{\beta^2} \left(\frac{2m}{\beta}\right)^{-1/2} \sqrt{\pi}} = \frac{1}{2\beta} \quad (1-16)$$

Thus inserting equations (8) and (16) in equation (3) yields

$$\langle E \rangle = \frac{1}{\beta} + \frac{1}{2\beta} = \frac{2+1}{2\beta} = \frac{3}{2\beta} \quad (1-17)$$

According to liquid drop model, the nucleus can be treated as consisting of a large number of small tiny particles like massive photons. Thus the use of statistical in describing its behavior is justifiable.

Thus if these particles that constitute the nucleus are re-distributed to be at infinity, then no field is observed. But when a work is done to assemble and collect these tiny particles by bringing them from infinity the nucleus produces a macroscopic gravity field, besides an electrostatic field, with field strengths equal to E_g and E_e respectively.

Thus the total macroscopic energy produced by the nucleus is

$$E = \left(\epsilon E_e^2 + \frac{1}{4\pi G} E_g^2 \right) \left(\frac{4\pi}{3} R_0^3 \right) \quad (1-18)$$

Where E_g and E_e are the gravity and electric field strengths just outside the nucleus. But according to equation (1-17) to be

$$E = \frac{3N}{2\beta} \quad (1-19)$$

Where N are the number of particles forming the nucleus.

Comparing equations (1-19) and (1-18)

$$\frac{3N}{2\beta} = \left(\epsilon E_e^2 + \frac{1}{4\pi G} E_g^2 \right) \left(\frac{4\pi}{3} R_0^3 \right)$$

$$\beta = \frac{3}{2} \frac{N}{\left(\epsilon E_e^2 + \frac{1}{4\pi G} E_g^2 \right) \left(\frac{4\pi}{3} R_0^3 \right)} \quad (1-20)$$

This parameter is related to the macroscopic energy produced by nucleus. While the potential appearing in equations (1-1) and (1-3) is the microscopic potential which may have a functional form and thus a nature different from the macroscopic nucleus.

III. Massive and super massive Astronomical objects

Consider a massive astronomical body like planets or stars or super massive neutron stars. For such objects the macroscopic field produced is the gravitational field.

When any object is formed by assembling far tiny particles located at infinity to form this astronomical object, the work done to move them from infinity requires giving them a kinetic energy. A work is also done against the field that existed. Thus the total energy given by equation (1-17) is transformed to gravitational energy given by equation (1-18), thus

$$E = \frac{3N}{2\beta} = \frac{E_g^2 R^3}{3G} \quad (2-1)$$

Where R is the radius of the star.

IV. Statistical Laws Based on Generalized Special Relativity

According to generalized special relativity

$$E = mc^2 \quad (3-1)$$

The average energy is given by

$$\langle E \rangle = \frac{\int_0^\infty E e^{-\beta E} dE}{\int_0^\infty e^{-\beta E} dE} \quad (3-2)$$

$$\langle E \rangle = \frac{\int_0^\infty mc^2 e^{-\beta mc^2} dmc^2}{\int_0^\infty e^{-\beta mc^2} dmc^2} \quad (3-3)$$

$$I_1 = \int mc^2 e^{-\beta mc^2} dmc^2 \quad (3-4)$$

Let

$$x = \beta mc^2 \rightarrow \frac{x}{\beta c^2} = m \rightarrow dm = \frac{dx}{\beta c^2}$$

$$\text{then } \int \frac{x}{\beta} e^{-x} \frac{dx}{\beta} = \int \frac{x}{\beta^2} e^{-x} dx = \frac{1}{\beta^2} \int x e^{-x} dx$$

Use integration by parts

$$\text{let } u = x \rightarrow du = dx, dv = e^{-x} dx \rightarrow v = -e^{-x}$$

$$\int x e^{-x} dx = -x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x}$$

$$\therefore I_1 = \frac{1}{\beta^2 c^2} [-x e^{-x} - e^{-x}] = \frac{1}{\beta^2 c^2} \quad (3-5)$$

Let

$$I_2 = \int e^{-\beta c^2 m} dm c^2$$

Let

$$x = \beta c^2 m \rightarrow m = \frac{x}{\beta c^2} \rightarrow dm = \frac{dx}{\beta c}$$

$$I_2 = \int e^{-x} \frac{dx}{\beta c^2} = \frac{1}{\beta c^2} \int e^{-x} dx = \left[-\frac{e^{-x}}{\beta c^2} \right]_0^\infty = \frac{1}{\beta c^2} \quad (3-6)$$

$$\langle E \rangle = \frac{I_1}{I_2} = \frac{1}{\beta} \quad (3-7)$$

$$\langle E \rangle = \frac{\frac{1}{\beta^2} [-x e^{-x} - e^{-x}]_0^\infty}{\frac{1}{\beta} [e^{-x}]_0^\infty} = \frac{1}{\beta} \frac{[-(0+0) + (0+1)]}{1}$$

$$\langle E \rangle = \frac{1}{\beta} \quad (3-8)$$

Where $\frac{1}{\beta}$ is the energy per particle constant

$$m_0 c^2 + kT + \frac{\varepsilon E^2}{n} \quad (3-9)$$

For relativistic particles producing nucleus

$$E = N \langle E \rangle = \frac{N}{\beta} = N \left(m_0 c^2 + kT + \varepsilon E^2_e + \frac{E_g}{4\pi\theta} \right) \left(\frac{4\pi}{3} R_0^2 \right) \quad (3-10)$$

V. Discussion

The total energy and average energy of statistical systems consisting of particles having both kinetic as well as potential energy was derived as shown by equation (1-17) and (1-19).

The parameter β is related to the macroscopic energy as in the conventional statistical laws. In the case of nucleus the macroscopic energy includes gravity and electric fields produced by the nucleus as shown by equation (1-18). Thus β is given by (1-20). For Astronomical objects the β is related to the gravity field as shown by equation (2-1). when relativistic effects are taken in to account β includes also rest mass energy.

VI. Conclusion

[The new statistical law which incorporates potential energy beside kinetic one can be used to find new statistical laws that can describe the generation of macroscopic fields.

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