

## Quantum Explanation of Conductivity at Resonance

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**Abstract:** Mineral Exploration is very important for industry. There are many spectral techniques used for identification of elements. Unfortunately these techniques are complex and expensive. There is a need for simple technique for exploration. This work utilizes simple technique based on electrical conductivity. The experimental work shows variation of conductivity with frequency, with line shape similar to absorption line. There is a minimum frequency for each element, which can be used as a finger print characterizing it. Fortunately this conductivity –frequency relation can be explained on the basis of quantum and statistical physics.

**Key Words:** conductivity, resonance, quantum, statistical physics, frequency.

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### I. Introduction

Spectrometers are very important in civilization [1]. They are widely used in mineral exploration [2, 3]. They are also utilized in soil tests [4], beside their applications in detecting trace elements in plants and organisms [5, 6]. Recently spectrometers are used in medicine in diagnosis [7].

Spectrometers are devices to account for the concentration of elements in any sample [8]. They consist of detectors which detect electromagnetic waves by converting them to corresponding electrical pulses.

A frequency or wavelength splitter unit is important for the spectrometer to split the spectrum of the sample [9]. The spectrum of sample is displayed on the display unit screen as a wavelength (or frequency) versus the photons intensity. The wavelengths are related to elemental content of samples [9]. The photon intensity is related to the concentration of elements in the sample [9].

Most of spectrometers utilize electromagnetic spectrum to identify elements existing in sample [Al, Cu, Au, Ag, Sn, Fe]. However some attempts were made to identify some materials by determining their energy gap, using simple electrical methods. Unfortunately such attempts can identify insulators and semiconductors but they cannot identify conductors or minerals. This requires searching for simple alternative to do this. This paper is concerned with performing this task by using simple electric method based on conductivity.

This work is done experimentally in section 2 and 3, and verified theoretically in section 4, sections 5 and 6 are devoted for results and discussion.

### II. Experimental Change of conductivity with frequency

In this experiment a transmitter coil emits electromagnetic waves.

These electromagnetic waves are allowed to incident on certain materials. The re emitted electromagnetic waves are received by a receiver.

#### (2-1) Apparatus:

- 10 Resistors (10k $\Omega$ , 2.2G $\Omega$ , 39k $\Omega$ ), 12 Capacitors (0.1 $\mu$ F, 0.01 $\mu$ F, 220 $\mu$ F), 6 Transistors (NPN), 2 transmitter and receptor Coils (400, 500, 600, 700, 1000 turns), Wire connection, Speakers, Cathode Ray Oscillator, Board connection, Battery (9V), Signal generator.

#### (2-2) Samples:

A pieces of metal (Cu, Al, Fe, Au, Ag, Sn).

#### (2-3) Method:

The transmitter coil current is varied by using signal generator. The emitted photons are allowed to incident on the sample. The sample absorbs photons and re emit them. The metal detector design is the circuit which connected as shown in fig (3.1). The signals appearing at oscilloscope were taken before mounting the sample, and after photon emission. The frequency and the corresponding conductivity of sample are recorded and determined from signal generator, current, voltage, the length and cross sectional area of samples. The current and voltage gives resistance, which allows conductivity determination from the dimensions of the sample.

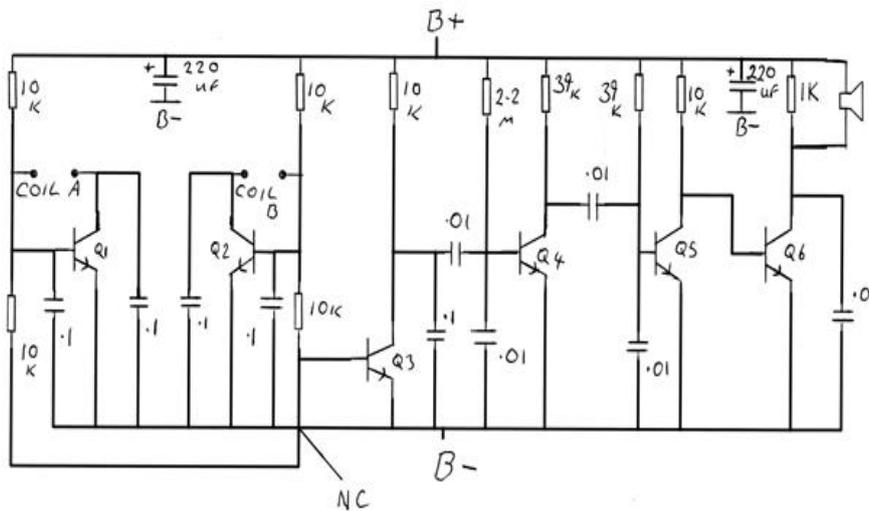


Fig (3-1)

(2-4): Tables and Results:

Table (2-4-1) Relation between frequency (f) and Conductivity ( $\sigma$ ) without applied magnetic field for Cu, Al, Fe, Au, Ag, Sn

Frequency ( Hz )	Conductivity ( $10^6$ cm. $\Omega$ )
24	0.452
27	0.596
29	0.0993
34	0.0917
50	0.143
56	0.337

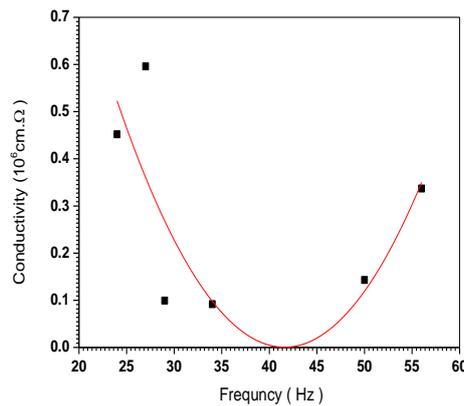
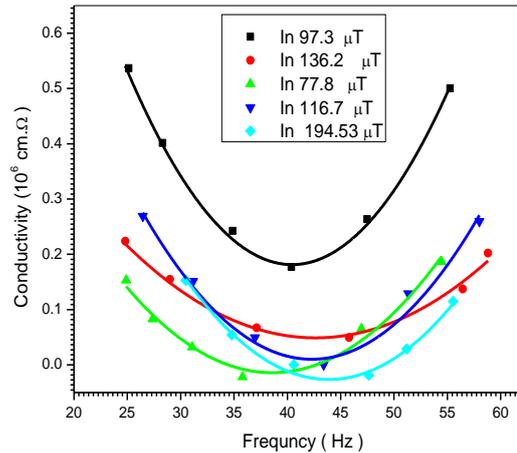


Fig (2-4-1-1) Relation between resonance frequency and Conductivity for Cu, Al, Fe, Au, Ag, Sn

Table (2-4-2) Relation between frequency (f) and Conductivity ( $\sigma$ ) for different magnetic flux densities for gold

Frequency ( Hz )	Conductivity ( $10^6$ cm. $\Omega$ ) In 97.3 $\mu$ T	Conductivity ( $10^6$ cm. $\Omega$ ) In 77. $\mu$ T	Conductivity ( $10^6$ cm. $\Omega$ ) In 116.7 $\mu$ T	Conductivity ( $10^6$ cm. $\Omega$ ) In 136.2 $\mu$ T	Conductivity ( $10^6$ cm. $\Omega$ ) In 194.53 $\mu$ T
55.25746	0.50026	0.12637	0.04739	0.15165	0.06824
47.48594	0.26365	0.03724	0.01396	0.04468	0.02011
40.49329	0.16368	0.03439	0.0129	0.04126	0.01857
35.4219	0.24726	0.05362	0.02011	0.06435	0.02896
26.8109	0.39107	0.1695	0.06356	0.2034	0.09153
24.81177	0.58753	0.2235	0.08381	0.2682	0.12069



**Fig (2-4-2-2) Relation between frequency (f) and Conductivity (σ) for different magnetic flux densities for gold**

### III. Theoretical Interpretation

#### (3-1) Quantum Theoretical Model:

$$\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = -c^2 \cdot \hbar^2 \nabla^2 \psi + m_0^2 c^4 \psi \quad (3-1-1)$$

Sub

$\psi = u(r) f(t)$  in (1) yields

$$-u \hbar^2 \frac{\partial^2 f}{\partial t^2} = -f c^2 \cdot \hbar^2 \nabla^2 u + m_0^2 c^4 f u \quad (3-1-2)$$

$$-\frac{1}{f} \hbar^2 \frac{\partial^2 f}{\partial t^2} = -\frac{1}{u} c^2 \cdot \hbar^2 \nabla^2 u + m_0^2 c^4 = E^2 \quad (3-1-3)$$

Where

$$-\frac{1}{f} \hbar^2 \frac{\partial^2 f}{\partial t^2} = E^2 \quad (3-1-4)$$

$$\therefore -\hbar^2 \frac{\partial^2 f}{\partial t^2} = E^2 f \quad (3-1-5)$$

Where

$\hbar \omega_0$  = electron energy in bounded state.

$\hbar \omega$  = energy given to the electron.

$E = \hbar \omega - \hbar \omega_0$  = excitation energy.

Consider solution

$$f = \sin \alpha t \quad (3-1-6)$$

$$\hbar^2 \alpha^2 f = E^2 f \quad (3-1-7)$$

$$\therefore \hbar \alpha = E$$

$$\alpha = \frac{E}{\hbar} = \omega - \omega_0 \quad (3-1-8)$$

$$\therefore \sigma = \frac{ne^2 \tau}{m} = \frac{|\psi|^2 e^2 \tau}{m} = \frac{e^2 \tau}{m} |\sin(\omega - \omega_0) t|^2 \quad (3-1-9)$$

At near resonance  $\omega - \omega_0 \ll 1$

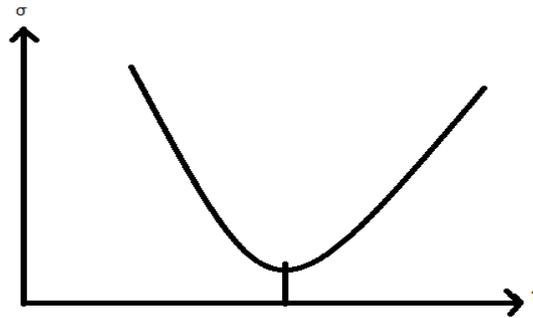


Fig (3-1-1) Theoretical relation between frequency (f) and Conductivity (σ)

$$\sin(\omega - \omega_0)t \approx (\omega - \omega_0)t \quad (3-1-10)$$

Inserting (3-1-10) in (3-1-9) yields

$$\therefore \sigma = \frac{e^2 \tau}{m} |(\omega - \omega_0)t|^2 \quad (3-1-11)$$

**(3-2) Classical Absorption Conductivity Resonance Curve:**

Consider an electron of mass m oscillate with natural frequency  $\omega_0$ . If an electric field of strength

$$E = E_0 e^{i\omega t} \quad (3-2-1)$$

Was applied, Then the equation of motion of the electron, in a frictional medium of friction coefficient  $\gamma$ , is given by

$$m\ddot{x} = eE - m\omega_0^2 x - \gamma\dot{x} \quad (3-2-2)$$

Consider the solution

$$x = x_0 e^{i\omega t} \quad (3-2-3)$$

Thus

$$v = \dot{x} = i\omega x \quad \ddot{x} = -\omega^2 x \quad (3-2-4)$$

Inserting (14) and (15) and (12) in (13) yields

$$-m\omega^2 x = e \frac{E_0}{x_0} x - m\omega_0^2 x - \gamma v$$

Thus

$$v = -m \frac{(\omega^2 - \omega_0^2)}{\gamma} x + e \frac{E_0}{\gamma x_0} x \quad (3-2-5)$$

For simplicity consider large displacement amplitude  $x_0$  compared to the electrical one  $E_0$ . Thus the last term in (4-5) can be neglected to get

$$v = -m \frac{(\omega^2 - \omega_0^2)}{\gamma} x \quad (3-2-6)$$

But the conductivity is given by

$$\sigma = \frac{e\tau}{m} n = \frac{e\tau}{m} n_0 e^{-\beta m v_e^2} \quad (3-2-7)$$

Where the effective value  $v_e$  is related to the maximum value through the relation

$$v_e = \frac{v_0}{\sqrt{2}} \quad (2-2-8)$$

For small value of the power of e , one can expand exponential term to be

$$e^{-x} = 1 - x \quad (3-2-9)$$

Therefore equation (3-2-7) becomes

$$\sigma = \frac{e\tau}{m} n_0 \left[ 1 + \frac{\beta m v_0^2}{4} \right] \quad (3-2-10)$$

Inserting (3-2-5) in (3-2-9) yields

$$\sigma = \frac{e\tau}{m} n_0 \left[ 1 + \frac{\beta m^2 x_0^2 (\omega - \omega_0)^2 (\omega + \omega_0)^2}{4\gamma} \right]$$

$$\sigma = \frac{e\tau}{m} n_0 \left[ 1 + \frac{\beta m^2 \omega_0^2 x_0^2 (\omega - \omega_0)^2}{\gamma} \right] \quad (3-2-11)$$

Where near resonance

$$\omega \approx \omega_0 \quad \omega + \omega_0 \approx 2\omega_0 \quad (3-2-12)$$

The relation between conductivity and frequency resembles that of (2-2-1) in its dependence on  $\omega$  . This relation is displayed graphically in Fig (2-4-2-2).

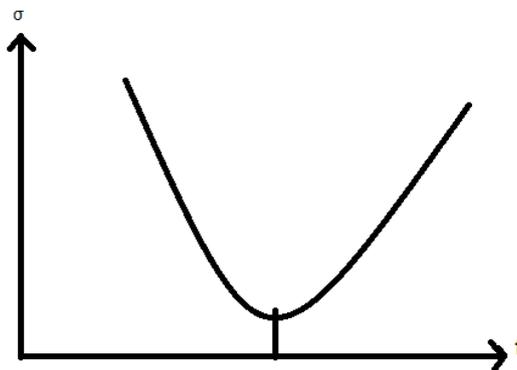


Fig (3-2-1) Theoretical relation between frequency (f) and Conductivity ( $\sigma$ )

#### IV. Discussion

The experimental work which was done shows variation of conductivity for gold according to Figs (2-4-2-1) and (2-4-2-2). The conductivity decreases then attains a minimum value in the range of (40-50 Hz), then increases a gain.

The theoretical expression (3-1-11) which is displayed graphically in Fig (3-1-1) is based on the ordinary expression for the conductivity. The electrons density n is found by solving Klein-Gordon equation for free particle. This is obvious as far as conduction electrons are free. The electron density is found from the square of the wave function, which is a sin function. Since at resonance  $\omega$  is very near to  $\omega_0$ , thus one can replace  $\sin x$  by  $x$  . The theoretical relation for f and  $\sigma$  obtained by this model resembles the experimental one in Fig(3-1-1).

Another classical approach based on Maxwell –Boltzmann distribution in section (3) shows a relation between  $\sigma$  and f in Fig (3-2-1) similar to experimental relation. The relations between  $\sigma$  and f resembles that of resonance, with minimum conductivity.

It is very interesting to note that each element has its own resonance conductivity at which conductivity is minimum.

In this model the ordinary expression for  $\sigma$  in eq\_n (3-1-9) is used. But n here is found from Maxwell statistical distribution.

#### V. Conclusion

The experimental work done here shows that conductivity changes with the frequency and have a minimum at a certain frequency. This frequency can be used to identify elements. Fortunately this experimental

relation can be explained theoretically on the bases of Klein –Gordon eq\_n or on the bases of Maxwell – Boltzmann distribution.

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