*IOSR Journal of Applied Physics (IOSR-JAP)*
*e-ISSN: 2278-4861.Volume 8, Issue 3 Ver. I (May. - Jun. 2016), PP 34-42*
*www.iosrjournals.org*

# Ion-Acoustic Double Layers in a Five Component Cometary Plasma with Kappa Described Electrons and Ions

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***Abstract:*** *We investigate the propagation characteristics of Ion-acoustic solitons and double layers in a five component cometary plasma consisting of positively and negatively charged oxygen ions, kappa described hydrogen ions, hot solar electrons, and slightly colder cometary electrons. The KdV and modified KdV equations are derived for the system and its solutios plotted for different kappa values and negatively charged oxygen ion densities. It is found that the strength of the double layer increases with increasing spectral indices. It, however, decreases with increasing negatively charged oxygen ion densities. The parameter for the transition from compressive to rarefactive soliton is also specified. The presence of negatively charged oxygen ions can significantly affect the nonlinearity coefficients (both quadratic and cubic) of a double layer.*
***Keywords:*** *Cometary multi-ion plasma, Ion-Acoustic solitary wave, Double layers, Modified KdV equation*

## I. Introduction

In recent decades, an intensively studied new field in plasma physics has been nonlinear wave propagation in different plasma environments. Among the nonlinear phenomena solitons, shock waves, double-layers, etc. have been studied with great interest in recent years.

The nonlinear dynamics of the ion-acoustic wave, which is an important wave in a plasma, has been studied extensively in recent years [1-5]. The formulation of Sagdeev in 1966 can be considered as the first nonlinear theory of ion-acoustic waves [6]. The experimental evidence for Sagdeev's formulation was by Ikezi et al. in 1970 [7].

In a soliton, nonlinearity is balanced by the dispersive effect of the medium. Theoretical formulations of nonlinear phenomena are usually carried out by the reductive perturbation technique. In the small amplitude approximation, one can derive partial differential equations like the Korteweg de-Vries (KdV) equation, the modified Korteweg de-Vries (mKdV) equation, the nonlinear Schrodinger equation, etc. The KdV and mKdV equations are used respectively in the study of solitons and double layers in strongly coupled plasmas [8-12].

The observation of high energetic particles in space plasmas gives rise to a long tailed velocity distribution which may significantly deviate from the well-known Maxwellian distribution [13]. A distribution function characterised by a real parameter kappa was proposed to interpret the power law dependence of velocity distributions observed in space plasmas. The kappa distribution has thus been extensively used in the study of many astrophysical and magnetospheric environments. The ion acoustic solitary wave in plasmas consisting of superthermal electrons and positrons has recently been studied by many authors [14-22].

A cometary plasma consists of heavier positively and negatively charged oxygen ions, hydrogen ions and electrons with relative densities depending on the distance from nucleus [23-25]. The positively charged oxygen ions and colder components of electrons are produced by the photo dissociation of water molecules in the cometary atmosphere; while the hotter components of electrons are of solar origin. Thus a cometary plasma is a multi-ion plasma consisting of both lighter and heavier ions and electrons with different temperatures.

We therefore study the ion acoustic solitary wave and double layer in this five component cometary plasma consisting of positively and negatively charged oxygen ions, kappa described hydrogen ions, hot solar electrons and slightly colder cometary electrons. The KdV and mKdV equations are derived using the reductive perturbation technique. For typical values observed at comet Halley, we find that the strength of double layer increases with increasing spectral indices kappa. It, however, decreases with increasing negatively charged oxygen ion densities. We have also studied the dependence of coefficients of quadratic and cubic nonlinearity in the mKdV equation on physical parameters relevant to comet Halley.

## II. Basic equations

We consider the existence of the Ion-Acoustic double layer in a five component plasma consisting of positively and negatively charged oxygen ions, kappa described hydrogen ions, hot electrons of solar origin and colder electrons of cometary origin. At equilibrium, the charge neutrality requires,

$$n_{ce0} + n_{se0} + Z_1 n_{10} = n_{H0} + Z_2 n_{20}$$

$n_{ce0}$, $n_{he0}$ represent the equilibrium densities of cometary electrons and solar electrons respectively, while $n_{10}$, $n_{20}$, $n_{H0}$ are the equilibrium densities of negatively charged oxygen ($O^-$) ions, positively charged oxygen ($O^+$) ions and hydrogen ions respectively. $Z_1$ and $Z_2$ represent the charge numbers of $O^-$ and $O^+$ ions respectively.
The kappa distribution of species 's' is given by,

$$n_s = n_{s0}\left[1 + \frac{e_s\varphi}{k_B T_s(\kappa_s - 3/2)}\right]^{-\kappa_s + 1/2} \quad \text{....................(1)}$$

In (1), s = H for hydrogen, $= se$ for solar electrons and $= ce$ for cometary photo-electrons. $n_s$ denotes the density (with the subscript '0' denoting the equilibrium value), $e_s$ the charge, $T_s$ the temperature and $\kappa_s$ the spectral index for species 's'. $k_B$ is the Boltzmann's constant and $\varphi$, the potential.
The dynamics of the heavier ions can be described by the following hydrodynamic equations:

$$\frac{\partial n_j}{\partial t} + \frac{\partial(n_j v_j)}{\partial x} = 0 \quad \text{...........(2)}$$

$$\left(\frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x}\right) v_j = \mp \frac{Z_j e}{m_j}\frac{\partial\varphi}{\partial x} \quad \text{.................(3)}$$

where $v_j$ and $m_j$, respectively, denote the fluid velocity and mass of the j-species of ions (j = $O^-$, $O^+$).

The Poisson's equation is given by

$$\frac{\partial^2\varphi}{\partial x^2} = -4\pi e\left(n_H + Z_2 n_2 - Z_1 n_1 - n_{ce} - n_{se}\right) \text{............(4)}$$

We normalize (2) to (4) using the parameters of the $O^-$ ions according to, $\phi = \frac{e\varphi}{k_B T_1}$, $V_j = \frac{v_j}{c_s}$, $T = \frac{t}{\omega_{p1}^{-1}}$ where $c_s = \left(\frac{Z_1 k_B T_1}{m_1}\right)^{1/2}$ and $\omega_{p1} = \left(\frac{4\pi Z_1^2 e^2 n_{10}}{m_1}\right)^{1/2}$. The variable x is normalized using $\lambda_{D1} = \left(\frac{Z_1 k_B T_1}{4\pi Z_1^2 e^2 n_{10}}\right)^{1/2}$ while $N_j = \frac{n_j}{n_{j0}}$.
Thus, equations (2) to (4) can be rewritten as

$$\frac{\partial N_1}{\partial T} + \frac{\partial(N_1 V_1)}{\partial x} = 0 \text{.................(5)}$$

$$\frac{\partial N_2}{\partial T} + \frac{\partial(N_2 V_2)}{\partial x} = 0 \text{.................(6)}$$

$$\frac{\partial V_1}{\partial T} + V_1\frac{\partial V_1}{\partial x} = \frac{\partial\phi}{\partial x} \text{...............(7)}$$

$$\frac{\partial V_2}{\partial T}+V_2\frac{\partial V_2}{\partial x}=\frac{-Z_2 m}{Z_1}\frac{\partial \phi}{\partial x}\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

where $m=\dfrac{m_1}{m_2}$ .

The normalized Poisson's equation after substitution of (1) is,

$$\frac{\partial^2\phi}{\partial x^2}=N_1-N_2\left(1+\mu_{ce}+\mu_{se}-\mu_H\right)+\mu_{ce}\left(1-\frac{\phi}{\sigma_{ce}\left(\kappa_{ce}-3/2\right)}\right)^{-(\kappa_{ce}-1/2)}$$

$$+\mu_{se}\left(1-\frac{\phi}{\sigma_{se}\left(\kappa_{se}-3/2\right)}\right)^{-(\kappa_{se}-1/2)}-\mu_H\left(1+\frac{\phi}{\sigma_H\left(\kappa_H-3/2\right)}\right)^{-(\kappa_H-1/2)}\ldots\ldots\ldots\ldots(9)$$

where $\mu_{ce}=\dfrac{n_{ce0}}{Z_1 n_{10}}$, $\mu_{se}=\dfrac{n_{se0}}{Z_1 n_{10}}$, $\mu_H=\dfrac{n_{H0}}{Z_1 n_{10}}$, $\sigma_{ce}=\dfrac{T_{ce}}{T_1}$, $\sigma_{se}=\dfrac{T_{se}}{T_1}$ and $\sigma_H=\dfrac{T_H}{T_1}$

## III. Dynamics of solitons

We use the reductive perturbation method to study the dynamics of solitons by introducing the transformations

$\xi=\varepsilon^{1/2}(x-\lambda T)$ , $\tau=\varepsilon^{3/2}T$

where ε is a smallness parameter and λ is the wave phase speed.

To apply the reductive perturbation technique the various parameters are expanded as

$$N_{1,2}=1+\varepsilon N_{1,2}^{(1)}+\varepsilon^2 N_{1,2}^{(2)}+\ldots\ldots\ldots\ldots(10)$$

$$V_{(1,2)}=\varepsilon V_{(1,2)}^{(1)}+\varepsilon^2 V_{(1,2)}^{(2)}+\ldots\ldots\ldots\ldots(11)$$

$$\phi=\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+\ldots\ldots\ldots\ldots\ldots(12)$$

We substitute (10) to (12) in (5) to (9) and equating the coefficients of different powers of ε yielding,

$$V_1^1=\lambda N_1^1\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

$$\phi^1=-\lambda^2 N_1^1\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

$$V_2^1=\frac{m}{\lambda}\frac{Z_2}{Z_1}\phi^1\ldots\ldots\ldots\ldots\ldots(15)$$

$$N_2^1=\frac{m}{\lambda^2}\frac{Z_2}{Z_1}\phi^1\ldots\ldots\ldots\ldots(16)$$

Equating the coefficients of $\varepsilon^{5/2}$ in (5) and (6), we get

$$\frac{\partial N_1^1}{\partial\tau}-\lambda\frac{\partial N_1^2}{\partial\xi}+\frac{\partial V_1^2}{\partial\xi}+\frac{\partial(N_1^1V_1^1)}{\partial\xi}=0\ldots\ldots\ldots\ldots(17)$$

$$\frac{\partial N_2^1}{\partial\tau}-\lambda\frac{\partial N_2^2}{\partial\xi}+\frac{\partial V_2^2}{\partial\xi}+\frac{\partial(N_2^1V_2^1)}{\partial\xi}=0\ldots\ldots\ldots\ldots\ldots(18)$$

And equating the coefficient of order $\varepsilon^{5/2}$ in (7) and (8) results in,

$$\frac{\partial V_1^1}{\partial\tau}-\lambda\frac{\partial V_1^2}{\partial\xi}+V_1^1\frac{\partial V_1^1}{\partial\xi}=\frac{\partial\phi^2}{\partial\xi}\ldots\ldots\ldots(19)$$

$$\frac{\partial V_2^1}{\partial \tau}-\lambda\frac{\partial V_2^2}{\partial \xi}+V_2^1\frac{\partial V_2^1}{\partial \xi}=-\frac{Z_2 m}{Z_1}\frac{\partial \phi^2}{\partial \xi}\text{........(20)}$$

Finally, equating the coefficients of terms of order $\varepsilon^2$ from Poisson's equation (9) gives,

$$\frac{\partial^2 \phi^1}{\partial \xi^2}=N_1^2-N_2^2\left(1+\mu_{ce}+\mu_{se}-\mu_H\right)+T\phi^2+L(\phi^1)^2\text{..........(21)}$$

Where

$$T=\frac{\mu_{ce}\left(\kappa_{ce}-1/2\right)}{\left(\kappa_{ce}-3/2\right)\sigma_{ce}}+\frac{\mu_{se}\left(\kappa_{se}-1/2\right)}{\left(\kappa_{se}-3/2\right)\sigma_{se}}+\frac{\mu_H\left(\kappa_H-1/2\right)}{\left(\kappa_H-3/2\right)\sigma_H}$$

and

$$L=\frac{\mu_{ce}\left(\kappa^2{}_{ce}-1/4\right)}{\left(\kappa_{ce}-3/2\right)^2\sigma_{ce}^2}+\frac{\mu_{se}\left(\kappa^2{}_{se}-1/4\right)}{\left(\kappa_{se}-3/2\right)^2\sigma_{se}^2}-\frac{\mu_H\left(\kappa^2{}_H-1/4\right)}{\left(\kappa_H-3/2\right)^2\sigma_H^2}$$

Substituting the values from (13) to (16) into (17) to (21) and eliminating the second order terms, we obtain the KdV equation as

$$\frac{\partial \phi^1}{\partial \tau}+\alpha\phi^1\frac{\partial \phi^1}{\partial \xi}+\beta\frac{\partial^3 \phi^1}{\partial \xi^3}=0\text{.............(22)}$$

where α is the quadratic nonlinearity and is given by

$$\alpha=-\beta\left[3-3m^2\left(\frac{Z_2}{Z_1}\right)^2\left(1+\mu_{ce}+\mu_{se}-\mu_H\right)+2L\right]$$

and

$$\beta=\frac{1}{2\left[1+m\frac{Z_2}{Z_1}\left(1+\mu_{ce}+\mu_{se}-\mu_H\right)\right]}$$

Transforming the KdV equation with $\phi^1=\psi$ to a frame $\eta=\xi-U\tau$ and integrating the transformed equation with respect to $\eta$ with help of boundary conditions $\psi$, $\frac{\partial \psi}{\partial \eta}$ and $\frac{\partial^2 \psi}{\partial \eta^2}\to 0$ as $\eta\to\infty$, the following equation is obtained

$$\frac{1}{2}\left(\frac{\partial \psi}{\partial \eta}\right)^2+V(\psi)=0\text{............(23)}$$

Here $V(\psi)$ is the Sagdeev potential and is given by

$$V(\psi)=\frac{\psi^2}{2\beta}\left(\frac{\alpha\psi}{3}-U\right)\text{.........(24)}$$

For the existence of solitary waves the following conditions have to be satisfied [6].

(i) $V(\psi,U)\big|_{\psi=0}=0=\frac{\partial V(\psi,U)}{\partial \psi}\bigg|_{\psi=0}$

(ii) $V(\psi,U)\big|_{\psi=\psi_s}=0$ ……………………(25)

(iii) $V(\psi,U)<0; \quad 0<|\psi|<|\psi_s|$

where $\psi_s$ is the amplitude of solitary wave. The soliton solution of KdV equation (22) is

$$\phi^1=\psi_s Sech^2\left(\frac{\eta}{\omega}\right);\ \psi_s=\frac{3U}{\alpha};\quad \omega=\sqrt{\frac{4\beta}{U}}\text{.................(26)}$$

where $\omega$ is the spatial width of the soliton. The nature of the soliton depends on the sign of the nonlinearity coefficient (α). Compressive solitons exist for α > 0 and rarefactive solitons exist for α < 0. The corresponding condition for their existence is obtained as

$$\alpha > (<) 0 \Rightarrow \quad m^2\left(\frac{Z_2}{Z_1}\right)^2\left(1+\mu_{ce}+\mu_{se}-\mu_H\right)+2L > (<) 1 \quad ...............(27)$$

## IV. Dynamics of weak double layers

In order to study the double layer propagation characteristic we must consider the higher order nonlinear effects. Hence, for the dynamics of weak double layers, the transformations used are

$\xi = \varepsilon(X - \lambda T)$, $\tau = \varepsilon^3 T$

Using the above transformations and reductive perturbation technique in (2) to (6) and equating the coefficients of order $\varepsilon^3$, the following second order relations are obtained

$$V_1^2 = \frac{(\phi^1)^2}{2} - \phi^2 ............(28)$$

$$N_1^2 = \frac{3}{2}(\phi^1)^2 - \phi^2 ...........(29)$$

$$V_2^2 = \frac{m^2\left(\frac{Z_2}{Z_1}\right)^2}{2}(\phi^1)^2 + \frac{Z_2}{Z_1}m\phi^2 ..........(30)$$

$$N_2^2 = \frac{3}{2}m^2\left(\frac{Z_2}{Z_1}\right)^2(\phi^1)^2 + \frac{Z_2}{Z_1}m\phi^2 ..........(31)$$

Equating the terms of order $\varepsilon^2$ and $\varepsilon^3$ in Poisson's equation gives,

$$N_1^2 - N_2^2\left(1+\mu_{ce}+\mu_{se}-\mu_H\right) + T\phi^2 + L\left(\phi^1\right)^2 = 0............(32)$$

$$\frac{\partial^2\phi^1}{\partial\xi^2} = N_1^3 - N_2^3\left(1+\mu_{ce}+\mu_{se}-\mu_H\right) + T\phi^3 + 2L\phi^1\phi^2 + \gamma(\phi^1)^2 ...........(33)$$

Substituting (29) and (31) in (32), we get

$$\gamma\left(\phi^1\right)^2 = 0.................(34)$$

where $\gamma = \frac{1}{2}\left[3 - 3m^2\left(\frac{Z_2}{Z_1}\right)^2\left(1+\mu_{ce}+\mu_{se}-\mu_H\right) + 2L\right]$

Since $\phi^1 \neq 0$, it follows that at least $\gamma \sim O(\varepsilon)$ and thus $\gamma(\phi^1)^2 \sim O(\varepsilon^3)$ [9]. Hence it should be included in the next higher order i.e. $O(\varepsilon^3)$ of the Poisson equation. Finally, eliminating all the third order variables i.e. the terms $O(\varepsilon^4)$ using the first and second order relations and neglecting a higher order term $\gamma\frac{\partial(\phi^1\phi^2)}{\partial\xi} \sim O(\varepsilon^5)$, the following modified form of KdV equation is derived.

$$\frac{\partial\phi^1}{\partial\tau} + \alpha\phi^1\frac{\partial\phi^1}{\partial\xi} + \rho(\phi^1)^2\frac{\partial\phi^1}{\partial\xi} + \beta\frac{\partial^3\phi^1}{\partial\xi^3} = 0.............(35)$$

The coefficient of cubic nonlinearity $\rho$ is as follows

$$\rho = \frac{15}{2}\beta\left[1 + m^3\left(\frac{Z_2}{Z_1}\right)^3\left(1+\mu_{ce}+\mu_{se}-\mu_H\right)\right]$$

We have derived the Sagdeev potential corresponding to mKdV equation by adopting same method used previously for the soliton. The potential is obtained as

$$V(\psi,U)=\frac{\psi^2}{2\beta}\left(\frac{\rho}{6}\psi^2+\frac{\alpha}{3}\psi-U\right)\text{............}(36)$$

For the existence of double layers, the Sagdeev potential has to satisfy the following conditions [6, 26]

(i). $V(\psi,U)\big|_{\psi=0}=0=V(\psi,U)\big|_{\psi=\psi_D}$

(ii). $\left.\frac{dV(\psi,U)}{d\psi}\right|_{\psi=0}=0=\left.\frac{dV(\psi,U)}{d\psi}\right|_{\psi=\psi_D}$ ………………(37)

(iii). $\left.\frac{d^2V(\psi,U)}{d\psi^2}\right|_{\psi=0,\psi_D}<0$

Where $\psi=0,\ \psi_D$ are the two extreme points of Sagdeev potential V ( $\psi$ , U). The above conditions give

$$U=-\frac{\rho}{6}\psi_D{}^2$$

$$\psi_D=-\frac{\alpha}{\rho}$$

and

$$V(\psi,U)=\frac{\psi^2\rho}{12\beta}\left(\psi_D-\psi\right)^2$$

The double layer solution of mKdV equation is given by [9]

$$\psi=\phi^{(1)}=\frac{\psi_D}{2}\left[1-\tanh(\frac{2\eta}{\Delta})\right]\quad\text{................}(38)$$

Where $\Delta$ is the thickness of the double layer and is given by

$$\Delta=\frac{4\sqrt{\frac{6\beta}{-\rho}}}{|\psi_D|}\quad\text{.............}(39)$$

From the (39), we arrive at the conclusion that the ion acoustic double layer will exist only if

$-\frac{\rho}{\beta}>0\Rightarrow\rho<0$ as $\beta>0$

The above condition is satisfied if and only if

$$m^3\left(\frac{Z_2}{Z_1}\right)^3\left(\mu_H-\mu_{ce}-\mu_{se}-1\right)>1\quad\text{.............}(40)$$

The nature of the double layer i.e. whether the system will support a compressive or rarefactive double layer depends on the sign of the coefficient of quadratic nonlinearity. The condition for the existence of both compressive and rarefactive double layer is the same as in the case of the soliton.

## V. Results

Though our equations (22) and (35) are applicable to any plasma we are interested, in this paper, on parameters relevant to comet Halley. The observed value of the density of hydrogen ions was $n_H=4.95\,\text{cm}^{-3}$; their temperature was $T_H=8\times10^4K$. The temperature of the solar (or hot) electrons was $T_{se}=2\times10^5\text{K}$ [27]. The temperature of the second component of photo-electron was set at $T_{ce}=2\times10^4\text{K}$ . Negatively charged oxygen ions with an energy ~ 1eV and densities ≤ 1 $\text{cm}^{-3}$ was unambiguously identified by Chaizy et al [25].

We thus set the densities of positively charged oxygen ions at $n_{20} = 0.5\,\text{cm}^{-3}$ and that of negatively charged oxygen ions at $n_{10} = 0.05\,\text{cm}^{-3}$ [27, 25].

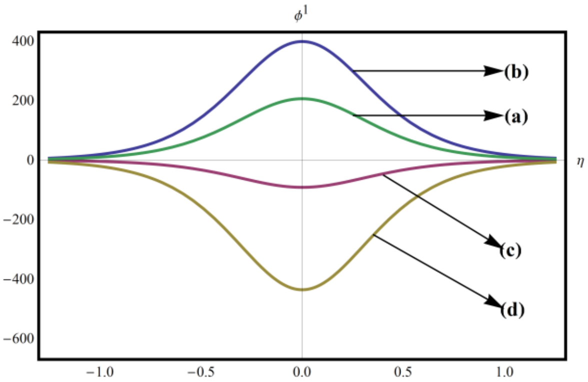


Figure 1
Soliton profile as a function of kappa index $\kappa_H$

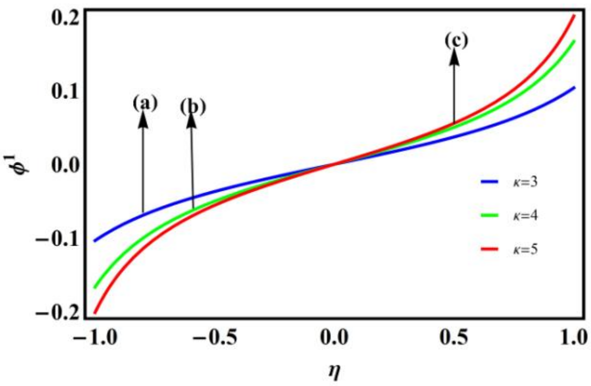


Figure 2
Double layer profile as a function of kappa indices

We numerically analysed the existence condition (27) for solitons. Keeping $\kappa_{ce} = \kappa_{se} = 5$ fixed at 5, figure 1 depicts the variation of soliton profile with respect to the spectral index $\kappa_H$. Curve (a) is for $\kappa_H = 1.91$, curve (b) is for $\kappa_H = 2.71$, curve (c) is for $\kappa_H = 2.72$ and curve (d) is for $\kappa_H = 2.73$. It is found that, the compressive soliton becomes rarefactive after a critical value $\kappa_H = 2.71$ and it continues to remain as a rarefactive soliton for the higher values of spectral indices.

Figure 2 depicts the variation of strength of double layer profile as a function of the spectral index. The parameters for the figure are $\lambda = 1$, $n_{10} = 0.05\,\text{cm}^{-3}$, $n_{20} = 0.5\,\text{cm}^{-3}$, $n_H = 4.95\,\text{cm}^{-3}$, $T_{ce} = 2\times10^4\,\text{K}$, $T_{se} = 2\times10^5\,\text{K}$, $T_1 = T_2 = 1.16\times10^4\,K$ and $Z_1 = Z_2 = 1$. Curve (a) (blue colour) is for the spectral index $\kappa$ = 3, curve (b) (green colour) is for $\kappa$ = 4 and curve (c) (red colour) is for $\kappa$ = 5. We find that the strength of double layer profile increases with an increase of the spectral index kappa.

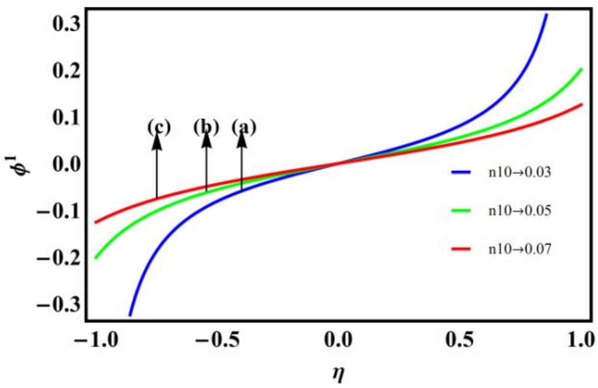


Figure 3
Double layer profile as a function of O[-] ion density

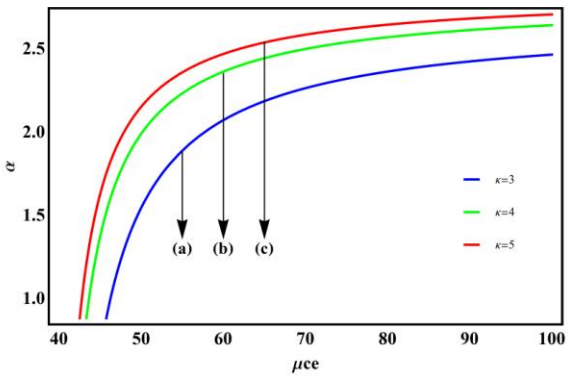


Figure 4
α Vs $\mu_{ce}$ as a function of kappa indices

Figure 3 shows the variation of the strength of the double layer structure as a function of negatively charged oxygen ion densities. The parameters used in this case are $n_{20} = 0.5\,\text{cm}^{-3}$, $n_H = 4.95\,\text{cm}^{-3}$, $T_{ce} = 2\times10^4\,\text{K}$, $T_{se} = 2\times10^5\,\text{K}$, $T_1 = 1.16\times10^4\,K$, $T_2 = 2.32\times10^4\,K$, $Z_1 = Z_2 = 1$ and $\kappa_{ce} = \kappa_{se} = \kappa_H = 5$. Curve (a) is for $n_{10} = 0.03\,\text{cm}^{-3}$, curve (b) is for $n_{10} = 0.05\,\text{cm}^{-3}$ and curve (c) is for $n_{10} = 0.07\,\text{cm}^{-3}$. We find that the strength of double layer decreases with an increase of negatively charged oxygen ion density.

Figure 4 depicts the variation of the quadratic nonlinearity coefficient (α) versus normalized cometary electron density ( $\mu_{ce}$ ) as a function of the spectral index kappa. The parameters used are the same as in Figure 2. Curve (a) (blue colour) is for the spectral index $\kappa$ = 3, curve (b) (green colour) is for $\kappa$ = 4 and curve (c) (red colour) is for $\kappa$ = 5. We find that the quadratic nonlinearity increases with an increase of the spectral index kappa.

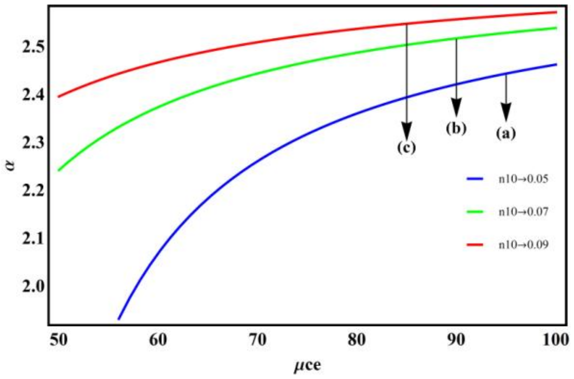


Figure 5

α Vs $\mu_{ce}$ as a function of O$^-$ ion density

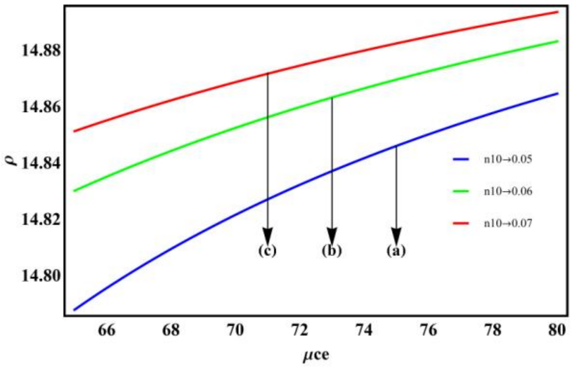


Figure 6

$\rho$ Vs $\mu_{ce}$ as a function of O$^-$ ion density

The dependence of the same coefficient on negatively charged oxygen ion densities is studied next. Figure 5 thus depicts the variation of α versus $\mu_{ce}$ for three values of the negatively charged oxygen density. Curve (a) is for $n_{10} = 0.05\,\text{cm}^{-3}$ (blue), curve (b) is for $n_{10} = 0.07\,\text{cm}^{-3}$ (green) and curve (c) is for $n_{10} = 0.09\,\text{cm}^{-3}$ (red). The other parameters for the figure are: $\kappa_{ce} = \kappa_{se} = \kappa_H = 3$, $n_{20} = 0.5\,\text{cm}^{-3}$, $\lambda = 1$, $Z_1 = Z_2 = 1$, $T_1 = 1.16\times10^4 K$, $T_2 = 2.32\times10^4 K$, $T_{se} = 2\times10^5\,\text{K}$ and $T_{ce} = 2\times10^4\,\text{K}$. It is clear that the quadratic nonlinearity increases with an increase of negatively charged oxygen ion densities.

The variation of the cubic nonlinearity is depicted next. Figure 6 thus depicts the variation of cubic nonlinearity coefficient $\rho$ versus $\mu_{ce}$ for three values of the negatively charged oxygen density. Curve (a) is for $n_{10} = 0.05\,\text{cm}^{-3}$ (blue), curve (b) is for $n_{10} = 0.06\,\text{cm}^{-3}$ (green) and curve(c) is for $n_{10} = 0.07\,\text{cm}^{-3}$ (red). The other parameters for the figure are the same as in Figure 5. We find that the cubic nonlinearity $\rho$ also increases with an increase of negatively charged oxygen ion densities.

## VI. Conclusion

We have studied the nonlinear propagation characteristics of ion acoustic waves in a five component plasma of positively and negatively charged oxygen ions, lighter hydrogen ions and hot and cold electrons by deriving the KdV and modified KdV equations. The condition for the existence of both solitary waves and double layers are obtained. It is found that, at fixed values of $\kappa_{ce}$ and $\kappa_{se}$, there exists a critical value for the spectral index of hydrogen ion ( $\kappa_H$ ), beyond which the compressive soliton becomes rarefactive. The impact of spectral indices kappa, and density of negatively charged oxygen ions (O$^-$) on the strength of the double layer profile is studied. The strength of double layer profile increases with an increase of spectral index kappa while, it decreases with an increase of negatively charged oxygen ion densities. In our investigation we find that the density of negatively charged oxygen ions significantly affects the nonlinearity coefficients (both quadratic and cubic).


## Acknowledgements

Financial assistance from Kerala State Council for Science, Technology & Environment, Thiruvananthapuram, Kerala, India (JRFs for MM and SG) and the University Grants Commission (EF) is gratefully acknowledged.

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