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Ladder operators for grid interaction and Gates between Universes

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Abstract: *Starting from a discrete space-time model for a Multiverse, we find some rules that prevail when we obtain the energy spectrum and the wave functions for the discrete versions of Quantum Mechanics and Super Symmetric Quantum Mechanics (SUSYQM). We recall the probably most simple but significant problem that is the interaction betweenthe Harmonic oscillator and the space-time grid.The first step will be to remember the new discrete versions of the continuum equations. So we propose the new ladder operators and from these, we build some of the wave functions and their corresponding eigenvalues, and analyze their behavior and compare with the old continuous versions. Because it is convenient to use a common language we rewrite some published results in covariant form and show that hidden invariants can be observed and can also be used to show that a discrete Multiverse have symmetry transformation properties that are contained in those of a continuous one. Our central task is now to bring near the Quantum Mechanics dual concept of a particle, and the purely vibrating being of the Super Strings. The resultant thing interacts with the grid but at the same time is a vibrating mode of the later theory. Also we propose that the thing can get inertial mass by means of the classic relation between the elastic constant k and the frequency, and from this point, we give an alternative interpretation to the Higgs boson mechanism to getting mass at least for our interacting particles as a twist of the space-time which otherwise is a mechanism very near to the general theory of relativity where the mass folds the space, that is, in some sense, we give an inverse law of cause and effect.Even we do not need to exclude importanttopics like gravitational waves, black holes or Rainbow Universes from consideration we leave their discussion later. But we take advantage for introducing difference operators for the General Theory of Relativity, that is, the discrete equivalents for partial derivatives and covariant derivative for tensors, Riemann Christoffel tensor, d'Alambertian operator, metric tensor etc. At last we show that the presence of a discrete Multiverse generates a new point of view from Topology that may explain how the vibration modes of the grid can voyage from one Universe to another.*

Keywords:*Ladder operators, discrete space-time, superpotential, harmonic oscillator.*

I. Introduction

On a recent paper[1] we underline the convenience to think not only in a unique Universe but also in a more general concept: adiscrete Multiverse. From our own point of view, Symmetry is one of the principal basements that sustain the Multiverse[1,2-14]. On the other hand we have shown that the theories about the discreteness of the space could be generalized to the space-time with the benefit of fusethe relativity theory[16-19] with quantum mechanics (and SUSYQM[1]). Both proposals are linked by a common concept we named the relativetime[1]. Then we have explored the consequences in several physics fields, particularly in the Shrödinger equation and the SUSYQM. But then we need a systematic way to procedure in every physical problem. To this end we have proposed a kind of generalization to the correspondence Bohr principle we called the Extended Correspondence Principle (**ECP**) that basically consists into change the ordinary differential operators for difference ones that shows explicitly a grid parameter. This makes the necessary changes in the equations that describe the different physical systems. When we have the new discrete equations we must to face up with some similar problems like to solve them, but also with new challenges like explainthe mean of new types of solutions and the very central feature that entitle the present work, how we build the proper ladder operators in a fusion of Super Symmetric Quantum Mechanics (SUSYQM) and Relativity?. Next we ask how some properties of a discrete space-time can be approximated by a continuum topology and what kind of consequences have in our concept of Multiverse, for example the connection between two far points one of them in the same or in other Universe and the possible trip between different Universes.We first announce the steps we are going to follow along the different sections. On section 2 we remember the new faces of some traditional equations or statements in terms of the grid parameter and blueprint how to maneuver the correspondent difference equation of a four variable partial differentialone, or if the differential expressions are written in terms of the relative time. On section 3 we give a guide to the concept of proper time and define the discrete proper time and show how we can obtain the Lorentz invariants by two different ways: the special theory of relativity or the assumption of the space-time discreteness. On section 4 we apply the **ECP** to the Dirac equation as an example of covariant equation. On section 5 we discuss the application of **ECP** to covariant and non-covariant expressions and we review the uncertainty principle over a discrete space-time. On section 6 we propose the Superpotential for a system with a one dimension Harmonic Oscillator interacting with the grid and we propose the Stair operators for the last problem and find the energy levels, we also find explicit expressions for some of the SUSYQM states by applying the stair operators. On section 7 we discuss about general properties of ladder operators over discrete space-time. On section8 we propose a model of connection between Universes and the possible trip of particles among them; and also we introduce discrete versions for relevant operators on General Theory of Relativity. On section 9 we write our discrete version for the equation describing gravitational waves. On section10 we give our particular notion of Multiverse. On section 11we give our conclusions.

II. The new face of the old equations

For the equations we obtain in this section it is important to advise that we only follow the statement of a discrete space but the application of the **ECP** is not a warranty that they are the appropriate ones for relativistic conditions, this is because we only want to show how we can transform any known equations. We must be careful to ask for additional conditions like covariance. Also we must notice that the obtained expressions really accomplish with Lorentz transformations. The relativistic proper time and the conventional time bring to different expressions.As we said above we have developed a new kind of relativistic correspondence principle, that is, we now make a systematic use of the rule we named an "extended Bohr correspondence principle" that establish the accordance with the Lorentz transformations from a different starting point of view, but this mean that now we must demand that Lorentz transformations must be fulfilled for two (related) reasons, first because the space-time is discrete and second because relativity theory only can be observed if the equations are covariant . The application needs not only for the substitution of the differential operators for difference ones, but also for analyze any expression we take and decide if the relative time is already employed or not. As we said, also we must remember that special relativity theory requires that equations must be covariant. Then even as we have shown, the change to difference operators implies the relativity requirements about Lorentz transformationsare fulfilled, we must be careful for the equations that reallywere covariant. In this section we give the new version of some significant equations and expressions showing how some of them are written in terms of the relative time. But we do not demand that those equations were covariant.Before, we must give explicitly the relation between a one-dimensional difference and differential operators in terms of the limit when the grid parameter S approaches zero, that is:

$$\lim_{\mathrm{S}\to 0}\frac{1}{\mathrm{S}}\underset{\mathrm{S}}{\Delta}(\mathbf{p}) = D(\mathbf{p}) \quad (1)$$

Because one of the hard problems to unify relativity and quantum mechanics comes from their different asymptotic behavior we think that it is convenient to take from a previous work a simple equation like the one-dimensional Schrödinger equation transforming and solving them:

$$\frac{\partial\psi}{\partial t} = \gamma\frac{\partial^2\psi}{\partial x^2} \quad (2)$$

Where

$$\gamma = \frac{i\hbar}{2m} \quad (3)$$

By applying the **ECP**and equation (1) to equation (2) we get:

$$\underset{\mathrm{S}\,(t)}{\Delta}\psi = \gamma\frac{1}{\mathrm{S}}\left(\underset{\mathrm{S}\,(x)}{\Delta}\right)^2\psi \quad (4)$$

Now in this place we think it is important to reproduce the steps on solving equation (4) that we have followed on a first work [1] because we have separated the time dependence and for this reason we will use them as very near solutionsso we take permission for it and for write some other similar but not equal expressions. Now, in order to analyze the behavior at least in a subset of solutions so we write the function $\psi(x,t)$ as

$$\psi(x,t) = e^{i\omega t}\phi(x) \quad (5)$$

Next, we can define

$$x = \mathrm{S}u \quad (6)$$

And also

$$V(u) = \phi(\mathrm{S}u) \quad (7)$$

Substituting (5), (6) and (7) into (4) we obtain after some algebra

$$\mathrm{S}(e^{i\omega \mathrm{S}}-1)V(u)=(E^2-2E+1)V(u) \quad (8)$$

Equation (8) can be solved easily by finding the accepted values for E ; these values are

$$E=\begin{cases} 1\pm\sqrt{r}e^{i\frac{\varphi}{2}} \\ 1\pm\sqrt{r}e^{i\frac{(\varphi+2\pi)}{2}} \end{cases} \quad (9)$$

Where

$$r=\sqrt{\left(1-\mathrm{S}\left(1-\cos\left(\omega \mathrm{S}\right)\right)\right)^2+\mathrm{S}^2\sin^2\left(\omega \mathrm{S}\right)} \quad (10)$$

And

$$\varphi=\arctan\left(\frac{\mathrm{S}\sin\left(\omega \mathrm{S}\right)}{1-\mathrm{S}\left(1-\cos\left(\omega \mathrm{S}\right)\right)}\right) \quad (11)$$

Then we have

$$V(u)=C_1E_1^u+C_2E_2^u+C_3E_3^u+C_4E_4^u \quad (12)$$

The arbitrary constants C_1, C_2, C_3 and C_4 must be fixed with boundary conditions.
And because of (6) and (7), we have finally

$$\psi(x,t)=e^{i\omega t}\left(C_1E_1^x+C_2E_2^x+C_3E_3^x+C_4E_4^x\right) \quad (13)$$

Of course, if we take $\lim \mathrm{S}\to 0$, we recover the solution for a free particle when moving through the right direction

$$\psi(x,t)=e^{i(kx-\omega t)} \quad (14)$$

Where

$$k=\frac{2\pi}{\lambda} \quad (15)$$

The propagation number is defined.
With the last discussion we can extend the procedure to the case of a potential different to zero and three spatial dimensions:

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi+V(\bar{r},t)\psi \quad (16)$$

Equation (16) can thus be written as

$$i\hbar\frac{1}{\mathrm{S}}\underset{\mathrm{S}(t)}{\Delta}\psi=-\frac{\hbar^2}{2m\mathrm{S}^2}\underset{\mathrm{S}(\bar{r})}{\Delta^2}\psi+V(\bar{r},t)\psi \quad (17)$$

In equation (17) the square operator is defined as

$$\underset{\mathrm{S}(\bar{r})}{\Delta^2}\psi\equiv\underset{\mathrm{S}(x)}{\Delta}\left(\underset{\mathrm{S}(x)}{\Delta}\right)\psi+\underset{\mathrm{S}(y)}{\Delta}\left(\underset{\mathrm{S}(y)}{\Delta}\right)\psi+\underset{\mathrm{S}(z)}{\Delta}\left(\underset{\mathrm{S}(z)}{\Delta}\right)\psi \quad (18)$$

Because of the factors

$$\frac{1}{\mathrm{S}} \ \textbf{and} \ \frac{1}{\mathrm{S}^2} \quad (19)$$

We can ever recover equation (16) when we take $\lim \mathrm{S}\to 0$ .The solution of the three dimensional equation (16) for a free particle is then the same as equation (14) but with a vector wave number $\mathbf{k}$ parallel to the wave propagation direction, that is:

$$y(\mathbf{r},t) = e^{iwt}(C_1E_1^{\mathbf{k*}} + C_2E_2^{\mathbf{k*}} + C_3E_3^{\mathbf{k*}} + C_4E_4^{\mathbf{k*}}) \quad (20)$$

On section 8 we will substitute solutions (20) or (13) with the more easy to handle equation (14) in order to obtain the tunnel effect between two Universes.

From equations (2-19) we have shown how to proceed for converting differential equations into difference ones, but what about relativity? On the theory we gave in a previous work, we saw that the effect when we discretize space-time is the accordance with relativity theory, that is, the proper frame transformations are the Lorentz transformations. Now we only must complete the application of the **ECP**. The **ECP** avoids the building of equations and statements from zero. So we must introduce the concepts we need. For instance, we can assume that the mass is a relativistic one

$$m = m_0\sqrt{1-\frac{u^2}{c^2}} \quad (21)$$

And of course, the maybe most famous relativistic relation

$$E = mc^2 \quad (22)$$

Coming from the relation

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} \quad (23)$$

Where

$$\mathbf{P} = m_0\mathbf{V} \quad (24)$$

And

$$\mathbf{V} = \frac{d}{dt}(x_1, x_2, x_3, x_4) \quad (25)$$

On expressions (21-25) we have used the relative or proper time t and the notation for Minkowski space

$$\begin{aligned} x_1 &= x \\ x_2 &= y \\ x_3 &= z \\ x_4 &= -ict \end{aligned} \quad (26)$$

Now, because the discreteness of space-time equations (4) and (17)are in a natural way in accordance with the direct substitution of relativistic expressions like (20-24). Moreover we know that when we make $\lim \text{S} \to 0$ equations (4) and (17) describe properly the quantum mechanics original problem.

So we can say that the equations (4) and (17), plus the relativistic relations (21-26) describes simultaneously the quantum mechanics-relativity aspect of the original problem.That is, because the difference operator in (1) only acts over the function $\psi(x,t)$ and not over the mas, we don't need to modify the operators in equations (4) and (17), that is we can substitute directly the non-relativistic expressions for mas, energy and momentum for the corresponding relativistic ones and then we have for equation (4) and (17) the resulting expressions:

$$\sqrt{1-(\underset{S(t)}{\text{D}}x)^2 \big/ S^2c^2}\;\underset{S(t)}{\text{D}}y = \frac{i\hbar}{2m_0}(\underset{S(x)}{\text{D}})^2 y \quad (27)$$

$$i\hbar\sqrt{1-(\underset{S(t)}{\text{D}}\mathbf{r})^2 \big/ S^2c^2}\;\underset{S(t)}{\text{D}}y = -\frac{\hbar^2}{2Sm_\circ}\underset{S(\mathbf{r})}{\text{D}^2}y + V(\mathbf{r},t)y \quad (28)$$

Even when equations (27) and (28) have a more complicated form, when we take the limit $\lim \text{S} \to 0$ we recover equations (2) and (16) but with the relativistic mas. From the properties of the discrete space-time we have givenwe conclude that equations (27) and (28) could describe simultaneously a problem from a relativistic and for a quantum mechanics point of view. The general rule is to solve the difference equations as functions of the grid parameter S and if we want to compare with the known continuous solutions we must take the limit: $\lim \text{S} \to 0$ after. Also it is very important to take into account that when we take this last limit, we must take the non-relativistic expressions for mass, etc., whenever the original continuous equations where not relativistic as occurs with equations (2) and (16). The importance of equations (27) and (28) is to show how we must to act for applying the **ECP**but we have not hope that their solutions must describe properly a relativistic system because they are not in covariant form. If we want to obtain an appropriate covariant equation we can take Dirac equation and apply **ECP**,as we will do in the next section.

III. Proper time and relative time

We have used directly the more known proper time[1] than relative time[17,18] when we apply the **ECP**to an equation but from now we will be more clear about the relation between these two concepts. On a recent paper we have defined the relative time,but for convenience we use a slightly different notation, which satisfy the relativistic dilation relation:

$$T = \frac{T_0}{\sqrt{1 - \frac{u^2}{c^2}}} \quad (29)$$

But in (29) T_0 denotes the relative time measured in a moving frame with velocity u with respect a frame at rest meanwhile T the relative time in a frame at rest. But we have in addition a difference with relativity theory that is the frame at rest is conditioned for the local density of points; for simplicity we suppose that the density of points is homogeneous in the Universe we have choose. A different density could change the maximum velocity of light[20] c .In terms of time intervals we can write the proper time in different frames; that is, if we have a stationary frame S and a moving frame $S'$the relative time intervals measured in these different frames are:

$$DT = T_2 - T_1 \quad (30)$$

And

$$DT_0 = T_{02} - T_{01} \quad (31)$$

We can then after define an equivalent proper time in the same way as in relativity theory[16-19].
But first, we also have that for the distance:

$$d = d_0\sqrt{1 - \frac{u^2}{c^2}} \quad (32)$$

With the correspondent intervals:

$$Dd = d_2 - d_1 \quad (33)$$

And

$$Dd_0 = d_{02} - d_{01} \quad (34)$$

Then the natural consequence of equations (29-34) are the validity of invariance under Lorentz[16-19] transformations that is, we can go backwards by supposing we have make a Lorentz transformation and see that the quantity defined as

$$W^2 = DT^2 - \frac{1}{c^2}Dd^2 \quad (35)$$

Is an invariant because if we remember that we have introduced Minkowski space[16-18] (now we have a subset of Minkowski space because the space-time is a discrete one):

$$iWc = (Dd_1, Dd_2, Dd_3, icDT) \quad (36)$$

$W$is a vector and in consequence a Lorentz transformation invariant. So we have shown that W have exactly the same properties of the named proper time t , which satisfies:

$$t^2 = DT^2 - \frac{1}{c^2}D\mathbf{x}^2 \quad (37)$$

To distinguish between W and t we can name the former "discrete proper time".

IV. Applying ECP to a covariant equation

On this section we are going to demand that the resulting equation after applying **ECP** is a covariant one. So we hope that we obtain an equation that simultaneously take into account quantum mechanics and special relativity in the natural way we have proposed and of course shows explicitly the discreteness of the space-time. This equation is the free particle Dirac equation[19]:

$$i\frac{\partial\psi}{\partial t}=\left(\boldsymbol{\alpha}\cdot\frac{\nabla}{i}+\beta m\right)\psi \quad (38)$$

Where the 4×4 α_i and β matrices can be written in terms of the 2×2 σ_i Pauli matrices as

$$\alpha_i=\begin{pmatrix}0 & \sigma_i\\ \sigma_i & 0\end{pmatrix} \quad (39)$$

And

$$\beta=\begin{pmatrix}\mathbf{1} & 0\\ 0 & -\mathbf{1}\end{pmatrix} \quad (40)$$

With

$$\mathbf{1}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \quad (41)$$

Then, the difference version of equation (38) is

$$i\frac{1}{\mathrm{S}}\underset{\mathrm{S}(t)}{\Delta}\psi=\left(\boldsymbol{\alpha}\cdot\frac{\underset{\mathrm{S}(r)}{\Delta}}{\mathrm{S}i}+\beta m\right)\psi \quad (42)$$

Or

$$i\underset{\mathrm{S}(t)}{\Delta}\psi=\left(\boldsymbol{\alpha}\cdot\frac{\underset{\mathrm{S}(r)}{\Delta}}{i}+\mathrm{S}\beta m\right)\psi \quad (43)$$

When we take the limit $\lim \mathrm{S}\rightarrow 0$ we recover equation (38). We know that because (43) is in covariant form is completely compatible with relativity. The question about the necessity of the covariance of equations (42) and (43) is a matter of future consideration but we are not too worry for this reason since originally covariance was demanded over a continuum and not over a discrete space-time. Anyhow, we hope that covariance is ever preserved by application of the **ECP**.

V. ECP plus covariance

On the last section we follow two alternative ways to transform expressions with**ECP**, firstover non-covariant expressions and then demand covariance, the second is to apply **ECP**to a covariant expressions, like in the case of Dirac equation and we affirmed that the resultant equation is the desired equation we searched; but if we return to definition of relativity time, we expect that any equation transformed by application of **ECP**must agree with Lorentz transformations. The failure is in time definition. Special relativity uses a new concept of time named the proper time and only a covariant equation is in agreement with it, remember that when the proper time is defined we are saying that the differential element of displacement in Minkowski space is an invariant under Lorentz transformations. On the other hand **ECP**comes also from a different concept of time we called the relativity time, which is an alternative source for invariance under Lorentz transformations. So we can in general follow two alternative ways to transform expressions with **ECP**,the first is to apply over non-covariant expressions and then demand covariance (by introducing relativistic mas, energy etc.); the second is to apply **ECP** to covariant expressions.

When the expressions are explicitly depending on "*discrete proper time"* or "*proper time"*[17,18]is very easy to apply **ECP**, for example we can assign the following difference operators:

$$E\rightarrow i\hbar\frac{1}{\mathrm{S}}\underset{\mathrm{S}(t)}{\Delta} \quad (44)$$

And

$p \rightarrow -i\hbar \frac{1}{S} \underset{S(x)}{\Delta}$ (45)

So, the Heisenberg uncertainty principle remains unchanged

$\Delta x \Delta p \geq \hbar$ (46)

But now we have also that

$\Delta x \geq S$ (47)

And

$\Delta t \geq S$ (48)

And we can verify that with the operators (44) and (45) commutator bracket of x and p is

$[x, p] = i\frac{\hbar}{S}$ (49)

The relation seems to be quite different to the known conventional commutator but we can verify that because of the difference operators in (49), if we take $\lim S \rightarrow 0$ we recover

$[x, p] = i\hbar$ (50)

VI. The grid seeing from SUSYQM

Equation (17) can be returned to his differential form (16) in a very simple way by taking $\lim S \rightarrow 0$. Nevertheless, the operator defined by equation (18) can be seen in a qualitative very different sense to the conventional Laplacian operator ∇^2, because $\underset{S(\bar{r})}{\Delta^2}$ operates over a discrete space and then we can think the space-time as a grid subject to a kind of strengths that becomes to a particular space-time behavior that could be observed as an interaction of the physical particles that are living on it and the proper grid. So a particular distribution of mass and energy on the grid can be observed as a phantom particle that collides with some kind of particles that travels by a particular region of space. Indeed some solutions of the difference equation (17) could describe "vibrations" that travels through the proper space-time grid, as it was a solid. We then could observe the scatter of a massive particle with another massless but with an apparent mass becoming from the space-time grid. So we anticipate the observation of this last class of scattering when the linear scale is about S. The factorization of the Hamiltonian as in SUSYQM (Super Symmetric Quantum Mechanics), allows extending the applications more easily because we can set directly the correspondence $\lim_{S \to 0} \frac{1}{S} \underset{S}{\Delta}(\mathbf{p}) = D(\mathbf{p})$, for an example we propose the superpotential for a system with a one dimension harmonic oscillatorinteracting with the space-time grid (a slightly different but more appropriate expressionsthan those showed in (1)):

$H = \hbar\omega\left[\left(\frac{1}{S}\underset{S(y)}{\Delta} + W(y)\right)\left(-\frac{1}{S}\underset{S(y)}{\Delta} + W(y)\right) + \frac{1}{2}(1+S)\right]$ (51)

For the Hamiltonian, and where the super potential is

$W(y) = (1+S)y$ (52)

We propose the use ofthecreation and annihilation operators

$A^{+} = \frac{1}{\sqrt{2}}\left[-\frac{1}{S}\underset{S(y)}{\Delta} + W(y)\right]$ (53)

$A^{-} = \frac{1}{\sqrt{2}}\left[\frac{1}{S}\underset{S(y)}{\Delta} + W(y)\right]$ (54)

To determine that the lowest energy is

$$E_0 = \frac{\hbar\omega}{2}\left(1+S\right) \quad (55)$$

That is very near to the conventional result except for the term $\frac{\hbar\omega S}{2}$.

By using the ladder operators (53) and (54) we can obtain the complete spectrum of energies:

$$E_n = \frac{\hbar\omega}{2}\left(1+S\right)\left(1+n\right) \quad (56)$$

At this point it is convenient to underlying that in the two last examples (free particle and harmonic oscillator), we don't make use of relativity theory but only the need of discrete operators and in the secondone the potential can "see" the discreteness of the space when the scale is of order S.

By using the killing operator (54) we can obtain the base eigenfunction:

$$\psi_0(y) = C_0\Gamma\left\{\left[S\left(S+1\right)y\right]\right\} \quad (57)$$

And by using the creation operator A^+ from now on to the base state $\psi_0(y)$ we obtain all other states for example:

$$\psi_1(y) = \frac{C_0}{C_1 S\sqrt{2}}\left[\Gamma\left\{yS(S+1)+2\right\} - \frac{1}{yS(S+1)+1}\Gamma\left\{yS(S+1)+2+S\right\}\right] \quad (58)$$

And

$$\psi_2(y) = \frac{C_0}{C_2 C_1\sqrt{2}}\left[\left(1+S\right)y - \frac{1}{S}\Delta_S\right]\frac{1}{S\sqrt{2}}\left[\Gamma\left\{yS\left(S+1\right)+2\right\} - \frac{1}{yS\left(S+1\right)+1}\Gamma\left\{yS\left(S+1\right)2+S\right\}\right] \quad (59)$$

We can see very easy that by successive applications of the creation operator we can generate any higher order eigenfunction.

Now we can make an interpretation of our results.

First we can see that the eigenfunctions can bee obtained systematically as combinations of Gamma functions. From the properties of these last functions we know that are the limit expression for punctual description of more common functions like Hermite polynomials. So we are in agree with our expectations. Secondly, we can review the grid superpotential (equation 52) structure and take an special point of view: we can see these vibrations as a grid mode but a mode of that is something that have an equivalent of mass (that is a relativistic mass) which can be himself as the result of a change in the grid like a different density of points; that is, we think that a local density of points voyage through the entire grid. That is, the mass is indeed a manifestation of a local different density of space-time points.The interaction we describe is then a possible description of the effect of a background permanent force (the term Sy), acting on a grid mode (a particle). If the mass of the particle is observed as one that have a rest mass equal or different to zero is a matter of relativistic description, because we know that if the velocity of the particle is c the rest mass must be zero. The permanent background force also can be associated with theoretically proposed forces not yet observed directly but only through their entire Universe effects.

VII. A comment about Ladder operators acting on a discrete space

As we have showed in the past sections,we have now two sourcesfor Lorentz transformations, the first comes from the very well known relativity theory and the second one, the discreetness of the space-time, which is completely compatible with the former theory. But we have now the following problem: because Lorentz transformations beneath to continuous groups of transformations or Lie Groups we must talk now to a discrete sub-group of transformations. The answer to the problem is in the new uncertainty principle (equation 49), because S adjusts a sub-space to preserve the properties of the Lorentz transformations.

We want say that when we apply a Lorentz transformation to a point on the discrete space we have another point in the same sub-space. Then the Ladder operators we have defining on equations (53) and (54) acting on a discrete space brings another element of a discrete sub-space-time that belongs to a more general continuum space-time. Then we expect that any Ladder operator acting on a discrete space must have the same property but we must define them properly.

VIII. Topology continuum approach for a discrete Gate

Now we can return to the discrete Multiverse concept. On the past sections we have no restrictions about the number of dimensions in every Universe or the limit of the light velocity in each one. But we can go farther by noting that also we have no restrictions about the embedding of dimensions between two ore more Universes. This last concept remember the extra dimensions proposed by other authors that says that in our Universe could exists dimensions rolled in cylinders with a very little radius and for this reason we can not observe it easy; but now we propose that two or more Universes can share one or more dimensions and that each dimension can be seeing with the naked eyes or not. Suppose that we represent the sharing of one dimension between two closed Universes (we will show in other place that real discrete Universes must be closed even we can not see their limits or seems to grow for ever) by the figure 1 that is we make a sketch with a topological continuum approach. Then the region of the two Klein bottles (each bottle represents a Universe) that appears like a common wall is a sheet that represents a frontier. On one side of the frontier we are on the surface representing the Universe number 1. On the other side of the frontier we are on the surface representing the Universe 2. Any tangent to the frontier represents the common dimensions. Any change of direction non-parallel to the frontier signifies that the two Universes are separated.To preserve the Universe identity we cannot think that the frontier is a part of both Universes, but we can think that the frontier is like a sheet of paper with two faces each one belonging to a distinct Universe. Because the Multiverse is indeed discrete we have not the problem for assigning two Universes to the same surface because the "physical frontier" like a paper sheet has a thickness and two different physical faces, and the common dimensionsare only common directions (the tangents to the frontier). Each face of the paper sheet can be represented with mathematical continuous functions or surfaces but we must be careful for not replace the two approximate continuous Universe representations because they are locally identical. We can see how we can pretend to go from the point A in Universe 1 to the point A' in the Universe 2 by using a worm hole (we use the name wormhole even we propose that the topology of the Universes considered may not correspond to those proposed by other authors), but we can see that we can only can go to another point B in the same Universe 1. The rule to go from point A in the Universe 1 to point A' in Universe 2 is across the "brane" of thickness S. Because the most elementary particles (we name most elementary particles to the elementary bricks of any kind of matter like quarks etc., and the quanta of any field like photons or Higgs bosons etc.) can see the grid, we hope that at perpendicular incidence, waves with appropriate frequencies can make the trip without problem if knowing conservation rules are preserved when also consideration of fluctuations of the quantum fields are taken into account.Remember that with the aim to maintain the congruence about the symmetry considerations in paper number 1, we can supose (this is not necessary) that in Universe the time goes forward while in Universe 2 the time runs backward. So we can see eventually observe the pass of elementary particles from a Universe where time goes in reverse direction to our Universe and then we can observe that this particles voyages to the past. So the region where the two Universes share one or more dimensions can be properly named a Gate between Universes.

Because we have a real especific solution of tridimensional equation (17) in (20) that remains the wave function of a free particle, we can make a simulation of the crossing of a "particle" (20) through the brane of

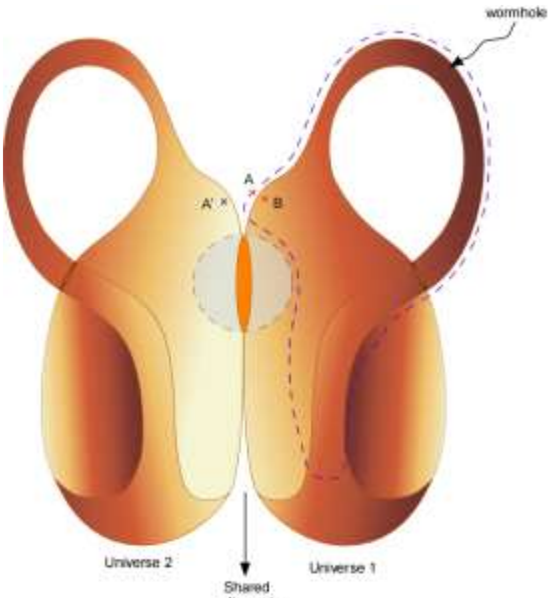

Figure1. Two discrete Universes communicated via a Gate

thicknes S. Because we have decided to use the **ECP**it means that we can use the known results about scattering across a surface (the brane).The only known dynamical properties of the two involved Universes is that in the neighboring region to the cross of the particle but inside each Universe, there is no potential, only in the very thin separating region of size S, so that we can assume that the relevant properties are the discreteness of the space-time and maybe that time follows opposite directions in each Universe. Let's describe the tresspasing of the "particle" trhough the region we name the Gate as a unidimensional problem of quantum mechanics. To avoid unnecessary problems, we will use the exponential solutions to the continuum equation that is solution (14) instead the solutions of the discrete equation that is solutions (13) or (20), also we suppose that $e > U_0$. Then we have three different regions where the normalized energy is greater than the three different potentials shown in figure 3. On region I we propose that the form of the solution is:

$e^{-i\sqrt{e}x} + \mathrm{R}e^{i\sqrt{e}x}$ that is for $x > S$ (60)

On region III only a solution that travels to the left:

$Qe^{-i\sqrt{e}x}$ that is for $x < 0$ (61)

On the midle region II we propose:

$Ae^{ikx} + Be^{-ikx}$ with $k = \sqrt{e - U_0}$ (62)

We then have a situation where the incident (normalized to 1), transmitted (Q) and refelected (R) coefficents are given by the conventional results of quantum mechanics that is we have that the transmission coefficient T is:

$$T = |Q|^2 = \frac{4e(e - U_0)}{4e(e - U_0) + U_0^2 \sin^2(kS)} \qquad (63)$$

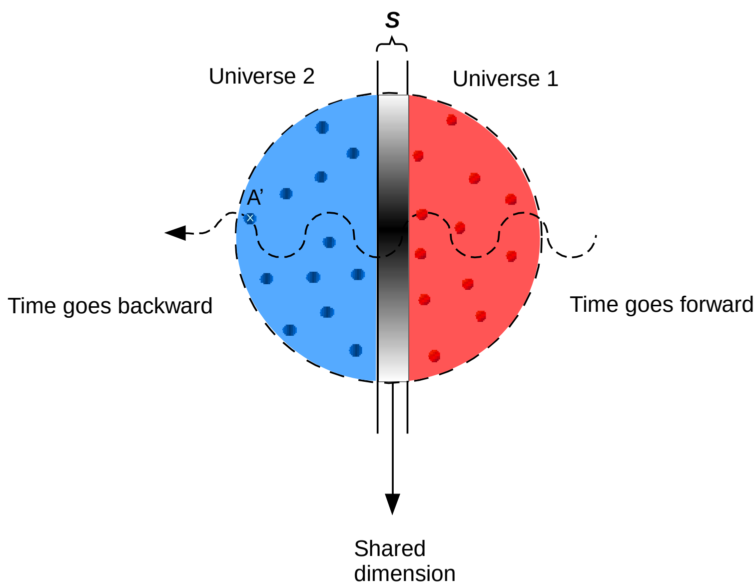

Figure 2. A particle travelling across the Gate of thikness S and seeing a contact potential U_0.

We have complete transmission ($T = 1$) for certain values of the energy, namely those for which kS is amultiple of p. As the energy increases, the transmission coefficient oscillates between this maximumvalue and a minimum value of the order of $4e(e - U_0) / (2e - U_0)^2$, where $E = \frac{\hbar^2}{2m} e$and $V(x) = \frac{\hbar^2}{2m} U(x)$, We can propose that when the conditions for complete transmission are given we can see the particle to travel beyond the Gate through another Universe. When the complete transmission is not obtained the probability for the particle travelling diminishes. The existense of a barrier potential U_0 between the two Universes also is an hypotetical one that may exists and that avoids a permanent flux of particles without any forbidense conditions.

$$U(x) = \begin{cases} 0 & x > S \quad (\text{region I}) \\ U_0(>0) & 0 < x < S \quad (\text{region II}) \\ 0 & x < 0 \quad (\text{region III}) \end{cases}$$

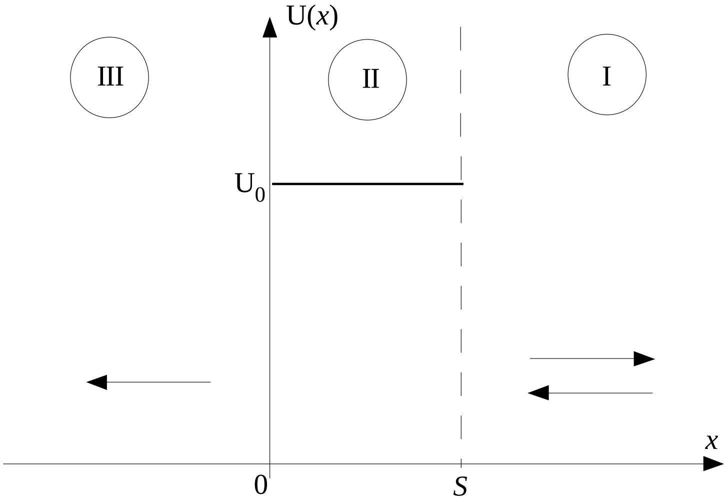

Fig. 3 Contact potential barrier.

All we said about the connection between Universes has been based in a continuity approximation that is we use the conventional Topoplogy for describe the model of a Universe. This is the sketch of the Klein bottles of figure 1, but we can not explain the complicated relation between the trayectories of a particle in both Universes. This is because this is only an aproach to reality. The real Multiverse can be seen as a finite but very large sea of space time "atoms" that can be represented as balls with a given size where many kinds of particles live. When we choose a subset of this sea we have the possibility to define a Universe when the properties of this subset complies some rules. The only known rules for defining a Universe are those observed in the own Universe. So we can only especulate about the corresponding rules on other Universes. For example in our Universe the observed velocity of light on space is every the same. But we think that many other rules are the effect of the occured in other Universe. The last is because we have supposed that the complete set of space-time "atoms" is finite and so the relation between them depends on wich Universe are. For example a sucession of points in Universe 1 must corresponde to another sucesion on Universe 2 but the first could be conditioned to be the time opossite to the other to preserve time symmetry. The Einstein's field equation is valid in its discrete version and the sign of the cosmological constant L has a different consequence on a finite discrete space-time. On figure 1 we can see that Universe expands apparently but only is travelling sliding over all the Klein bottle or better, over a Klein bottle with walls of finite thikness. The known Einstein's field equation is[16,17,18]:

$$R_{mn} - \frac{1}{2}Rg_{mn} + Lg_{mn} = \frac{8pG}{c^4}T_{mn} \quad (64)$$

Where G is the gravitational constant, also L is aconstant, T_{ij}^{k} is the energy-momentum tensor, R_{mn} is the contracted Riemann-Cristoffel tensor or Ricci tensor, and g_{mn} the metric tensor.Through equation (64) we can explain weird objects like black holes, but its difference version only kills the mathematical singularities and change them for a sharp but finite behavior. In order to avoid unnecessary complicated approach to Einstein's field equations we remember that the S parameter is physically the lenght of the space-atoms in any direction and determines the diameter which is directionally homogeneous, that is it doesn't matter in which direction of the Minkowski space we are the length of the space-atoms is the same. As we have seen the relativity phenomena of dilation of time and contraction of the space with the velocity comes from the connections between different space-atoms (This is a very different behavior respect to a continnum model). But the homogeneous property of

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S is the key that allow us to rewrite the partial derivatives in the General Theory of Relativityin a simple manner in terms of differences.
So we define the partial difference of a covariant tensor of the first rank as

$$\frac{\partial A_i}{\partial x^j} \Rightarrow {}_{(S)}A_{i,j} = \frac{1}{S}\underset{S(x^j)}{\Delta} A_i \quad (65)$$

and the covariant difference as

$$A_{i;j} \Rightarrow {}_{(S)}A_{i;j} = \frac{1}{S}\underset{S(x^j)}{\Delta} A_i - {}_{(S)}\Gamma^k_{ij} A_k \quad (66)$$

Where

$${}_{(S)}\Gamma^l_{ik} = \frac{1}{S^2}\underset{S(x^i,x^k)}{\Delta^2} y^j \underset{S(y^j)}{\Delta} x^l \quad (67)$$

And the discrete version of the Riemann-Cristoffel tensor can be defined as

$${}_{(S)}R^i_{jkl} = {}_{(S)}\Gamma^i_{rk}\,{}_{(S)}\Gamma^r_{jl} - {}_{(S)}\Gamma^i_{rl}\,{}_{(S)}\Gamma^r_{jk} + \frac{1}{S}\underset{S(x^k)}{\Delta}\,{}_{(S)}\Gamma^i_{jl} - \frac{1}{S}\underset{S(x^l)}{\Delta}\,{}_{(S)}\Gamma^i_{jk} \quad (68)$$

The metric tensor can then be writen as:

$$g_{il} = \frac{1}{S^2}\underset{x^i}{\Delta} x_k \underset{x^l}{\Delta} x_k \quad (69)$$

IX. Gravitational waves

We can now(we have omitted a lot of tensor algebra) use definitions (65) till (69) to write the discrete gravitational field equation (for a very weak gravitational field) in vacuum in the form:

$${}_S\Box h^k_l = 0 \quad (70)$$

Where the discrete d'Alambertian is defined as follows:

$${}_S\Box = \frac{1}{S^2}\left(\underset{S(x^2{}_\alpha)}{\Delta}{}^2 - \frac{1}{c^2}\underset{S(t^2)}{\Delta}{}^2\right) \quad (71)$$

But we recognize that equation (70) is the ordinary discrete wave equation. Despite the obvious discretness of equation (71) we know that the new point of view is that because of the shrink in the fabric of the discrete space-time, some kind of gravitational transporter travels trough the net with the light velocity c. That is the graviton is now another "elementary particle" (in the sense we have defined) voyaging like a photon or a neutrino. The remarkable difference is that now we do not make a gravitational class difference between "elementary particles" but it is obvious that description of the shrink in the discrete space-time fabric still very difficult.Anyway, we can think in the possible travel of a graviton as we described in section 8, that is we can describe the voyage between different Universes with the same tool.

What a Multiverse means?

We have taken a tooschematic definition of a Multiverse, but we need to give something better defined. That is, our sea of space-time atomsin addition with the particles and fields that fill it is not only a very large but finite set simply divided in subsets (Universes) with internal rules and frontiers. Our answer about what the real Multiverse is can be stated: the subsets must be considered as successions of collections of space-time atoms and the particles and fields that occupy them. Each element of the succession can be seen as an instant picture of somehow Universe. The complete succession can be considered as a specific Universe. So, in some Universes like ours the different elements of the succession taken in a specific order can be understood as the evolution in a time direction. But generally we can also have many time-like dimensions, and the time arrow is more a piece of the Multiverse symmetry and then remainsonly a sensation from their ancient exceptional and unique role. The rules relating the different pictures constitute the essence of each particular Universe. We find that it is possible to mix atoms in some regionso; in some neighborhood of that regionwe can find space-time atoms from two (or more) Universes. Symmetryis the guide to the order.Because we are not violating any relativistic or quantum physics law when applying to our Universe or contradicting all we stated above we think that is the better image of a Multiverse.

X. Conclusions

In a recent paper we have seen that even with a one-dimensional (bi-dimensional taking into account the time) model, extraordinarily simple, it is possible to reproduce results of the general relativity like the equations (13) and (18) under a scheme that never separates two fundamental concepts, first, that space-time is discrete and second, that relativity theory is related to this discreetness. In the present work we go farther and making use of the principle we named **ECP** we can follow two ways to apply them, that is when the equations taken are covariant or the case in which we start with a covariant expression. So we can see that the real problem is not

Figure 4. A glass Klein bottle.

The glass wall of the bottle represents the Universe: the complicated structure of the observed Universe that seems to be a spider net is embedded and live in the wall. The yellow liquid is outside the Universe but is part of the Multiverse. The mathematical surface with zero thickness is a non-oriented one, but the glass wall with non-zero thickness is an oriented volume. The primordial Universe is a collar on the thinnest part on the neck, but evolves to an expanded form.

To take simultaneously relativity principles and quantum mechanics, but to recognize that both theories have the same source. In this way, we must take the known concepts and tools and visualize from both points of view. For this reason we can say that one of the principle goals of the present paper is to give a road to follow when a ladder operator is used to determine the eigenfunctions and eigenvalues in many physical problems even when the unique example is a special version of the harmonic oscillator. Of course we take advantage of the fact that we are generating a new tool and then we propose an also new class of interaction with the hope that we are really describing physical problems that never in the past has been solved. We know that indeed we are trying toshow a very academic problem that maybe are not closely connected with phenomena that could be measured very easy but we also hope that soon we can make an application to the real world. Meanwhile we can make an advance on presenting the kind of ladder operators, superpotentials, grid interactions, eigenfunctions and eigenvalues we can wait (see equations (58)-(66)). Another goal is to show how we can have two sources for the Lorentz invariance but the answer comes from the also twice rigged uncertainty

principle so discreetness generates a subspace of the complete one in which relativity theory is in accordance with Lorentz invariance.Finally we would like that our modelgives two fundamental explanations of the connection between mass, superstrings, and the fundamental bricks on the Universe. The former is appearing when the grid interacts with vibrations (of himself); and the last two, as normal modes of the grid vibrations. The quantity of mass is proportional to the wave number k.We can tell in other words that mass appears when the space-time is twisted or shrinked. On the other hand, the normal modes are the elementary particles living in the grid like superstrings.Gravitational waves are not excluded of consideration because we are supposing that the fields are quantized and then their quanta are considered as elementary particles. Finally we can say that because the new tools are compatible with the existence of multi-Universes (That may include those known as Rainbow Universes or **RU**) under discrete space-time, we may suppose that elementary particles have no reasons to avoid visit another Universe eventually. As we can saw above, possibly two different Universes may share some dimensions and we expect that some elementary particles voyaging to the past in some Universe can visit ours and we could observe by a wee time interval. So, by extrapolation we can think about the possibility of a chain of Universes connected by "branes" (common dimensions).The region involved on the pass of particles between Universes can be named properly as a Gate.

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