Influence of Compressive and Rarefactive Ion-Acoustic Shock Waves in a Multi-Component Degenerate Dense Plasma

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The properties of ion-acoustic shock waves associated with the nonlinear propagation of compressive and rarefactive electrostatic perturbations in an unmagnetized collisionless degenerate dense plasma have been investigated theoretically. Our considered model contains degenerate electron, positron, and ion fluids, which are valid for both of the non-relativistic and ultra-relativistic limits. The Burgers equation has been derived by employing the reductive perturbation method and by taking the effect of viscous force in the ion fluid into account. The stationary shock wave solution of Burgers equation is obtained, and numerically analyzed in order to identify the basic properties of ion-acoustic shock structures. It has been shown that depending on plasma parametric values, the degenerate plasma under consideration supports compressive or rarefactive shock structures. It has been also found that the effects of degenerate pressures of electrons, ions, and positrons significantly modify the basic features of the shock waves that are found to exist in such a degenerate plasma. The relevance of our results in astrophysical objects like white dwarfs and neutron stars, which are of scientific interest, are briefly discussed.

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I. INTRODUCTION

During the last few decades, significant progress has been taken place to study the linear and nonlinear ion-acoustic (IA) waves among the researchers. Interesting nonlinear features of these quantities in different plasma environments as well as their abundant occurrence in nature has grown the researcher’s attention over the past few years. One of the important nonlinear features of the kind is the ion-acoustic shock waves (IASWs). The discovery of remarkable shape-preservation feature of these waves during their interactions by Zubursky et al. in 1965 made their first important applications in communication technology Washimi et al. showed that such waves, in a weakly nonlinear regime, can be mathematically modeled by the well known shock wave equations. The propagation of the ion-acoustic waves are very important from both the academic point of view and from the view of its vital role in understanding the electrostatic disturbances in space and laboratory plasma. The physics of quantum plasmas, rapidly grown beyond conventional plasmas found in space or laboratory for many years [1, 2]. This is mainly due to the potential applications of quantum plasmas in different areas of scientific and technological importance [3–6]. It is a common idea that electron-positron plasmas have presumably appeared in the early universe [7, 8] and are frequently encountered in active galactic nuclei [9] and in pulsar magnetospheres [10, 11]. This electron-positron plasma is usually characterized as a fully ionized gas consisting of electrons and positrons of equal masses. Recently, there has been a great deal of interest in studying linear as well as nonlinear wave motions in such plasmas [12, 13]. The nonlinear studies have been focused on the nonlinear self-consistent structures [12–14] such as envelope solitons, vortices, etc. However, most of the astrophysical plasmas usually contains ions, in addition to the electrons and positrons. Clearly, the properties of wave motions in an electron-positron-ion plasma should be different from those in two-component electron-positron plasmas. For example, Rizzuto [15] and Berezhiani et al. [16] have investigated envelope solitons of electromagnetic waves in three-component electron-positron-ion plasmas. The electron-positron plasmas are thought to be generated naturally by pair production in high energy processes in the vicinity of several astrophysical objects as well as produced in laboratory plasmas experiments with a finite life time [17]. Because of the long life time of the positrons, most of the astrophysical [18] and laboratory plasmas [19] become an admixture of electrons, positrons, and ions. It has also been shown that over a wide range of parameters, annihilation of electrons and positrons, which is the analog of recombination in plasma composed of ions and electrons, is relatively unimportant in classical, [20] as well as in dense quantum plasmas [21] to study the collective plasma oscillations. The ultra-dense degenerate electron-positron plasmas with ions are believed to be found in compact astrophysical bodies like neutron stars and the inner layers of white dwarfs [21–24] as well as in intense laser-matter interaction experiments [25, 26]. Therefore, it seems important to study the influence of quantum effects on dense e-p+ plasmas. Several authors have theoretically investigated the collective effects in dense unmagnetized and magnetized e-p+ quantum plasmas under the assumption of low-phase velocity (in comparison with electron/positron Fermi velocity) [27–29]. In these studies, the authors have focused on the lower order quantum corrections appearing in the

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well known classical modes.

A dense plasma is usually characterized as cold and degenerate such as that encountered in metals and semiconductors. However, it has been remarked that a hot fusion plasma such as that found in dense stellar objects (e.g., white dwarfs) may also be considered as quantum degenerate plasma [1]. In such environments the production of positrons is and a degenerate plasma of electron-positron-ion can be expected. The main objection to the existence of dense electron-positron-ion plasma may be high electron-positron annihilation rate which is naturally expected where the electron and positron density are very high. In a typical white dwarf star the electron density can be as high as $10^{26} \text{cm}^{-3}$, however, for massive stars [30] such as that for a collapsing white dwarf, this value can even be much higher [24]. The propagation and collision of small-amplitude ion-acoustic waves in ultra relativistic plasma have been already investigated [31, 32].

Now-a-days, a number of authors have become interested to study the properties of matter under extreme conditions [33-36]. Recently, a number of theoretical investigations have also been made of the nonlinear propagation of electrostatic waves in degenerate quantum plasma by a number of authors [37-39] etc. However, these investigations are based on the electron equation of state valid for the non-relativistic limit. Some investigations have been made of the nonlinear propagation of electrostatic waves in a degenerate dense plasma based on the degenerate electron equation of state valid for ultra-relativistic limit [40, 41, 43]. We are interested to study the dissipation relation of the ion-acoustic waves in a degenerate e-p-i plasma system where we added positrons for the rather long lifetime of positrons, most of the astrophysical [9, 11-13, 18, 24, 44, 45] as we have mentioned in the introductory chapter. The ion-acoustic waves are longitudinal oscillations of the ions (and the electron-positron) a dusty e-p-i plasma. The linear dispersion relation (DR) will give the relation between the wave frequency $\omega$ and the wave number $k$. The number density of the degenerate electron and positron in a compact object is so high (e.g. in white dwarfs, the degenerate electron number density can be order of $10^{26} \text{cm}^{-3}$ even more, and order of $10^{29} \text{cm}^{-3}$ even more in neutron stars) that the non-relativistic limit is practically applicable for degenerate fermions (electron-positrons). The equation of state for degenerate particles in astrophysical compact objects is the ultra-relativistic limit mathematically explained by S. Chandrasekhar [24, 46]. It is very important to note that the degenerate pressure depends only on the fermion number density, but not on its temperature.

To the best of our knowledge, no theoretical investigation has been developed to study the extreme condition of matter for both non-relativistic and ultra-relativistic limits in a degenerate plasma system with the nonlinear propagation and formation of the ion-acoustic shock waves. Therefore, in our present investigation, we consider a degenerate dense plasma system in absence of the magnetic field containing non-relativistic degenerate cold ion fluid, both non-relativistic and ultra-relativistic degenerate electrons and positrons where the ion is the heavier element among all other elements. To study the basic features of the ion-acoustic nonlinear structures of shock wave in such unmagnetized three component degenerate dense plasma, we have studied the Burgers equation and the numerical solution of Burgers equation. The model is relevant to compact interstellar objects (e.g., white dwarf, neutron star, etc.).

II. GOVERNING EQUATIONS

We consider an unmagnetized collisionless three component degenerate dense plasma system consisting of non-relativistic degenerate cold degenerate ion fluid and both non-relativistic and ultra-relativistic degenerate electrons and positrons fluids. We assume that the ion is the heavier element among all other considering elements. The dynamics of the one dimensional ion-acoustic waves in such a three component degenerate dense plasma system is governed by

$$
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(u_i n_i) = 0, \quad (i = e, \text{ion, p for positron})
$$

$$
\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial}{\partial x} \left( n_i \frac{\partial n_i}{\partial x} - \frac{\eta}{2} \frac{\partial^2 u_i}{\partial x^2} \right) = 0, \quad (i = e, \text{ion, p for positron}).
$$

$$
\frac{\partial \phi}{\partial t} + u_e \frac{\partial \phi}{\partial x} + \frac{\partial}{\partial x} \left( n_e \frac{\partial \phi}{\partial x} - \frac{\eta}{2} \frac{\partial^2 \phi}{\partial x^2} \right) = 0, \quad (i = e, \text{ion, p for positron}).
$$

where $n_i$ is the plasma number density of the species $s$ ($s = e$ for electron, $i$ for ion, and $p$ for positron) normalized by its equilibrium value $n_{eq}(s) = (m_e c^2/m_i)^{1/2}$ with $m_i$ being the electron (ion) rest mass mass and $c$ being the speed of light in vacuum, $\phi$ is the electrostatic wave potential normalized by $m_e c^2/e$ with $e$ being the magnitude of the charge of an electron, the time variable $t$ is normalized by $t_{eq} = (4 \pi n_{eq}^2/m_e)^{1/3}$, and the space variable $x$ is normalized by $\lambda_{eq} = (m_e c^2/4 \pi \rho_{eq}^2)^{1/3}$. The coefficient of viscosity $\eta$ is a normalized quantity given by $\omega_i \lambda_{eq} n_{eq}^2$, and $\omega_i$ is the ratio of the number density of electron and ion ($n_e/n_i$). The constants $K_1 = n_{eq}^{1/3} k_i/m_e c^2$ and $K_2 = n_{eq}^{1/3} k_i/m_e c^2$ are normalized by $n_{eq}^{1/3} k_i/m_e c^2$. The equations of state used here for the degenerate pressures of electrons, ions and positrons are given by

$$
P_i = K_i n_i^2,
$$

where

$$
\alpha = \frac{5}{3}, \quad K_1 = \frac{3}{5} \left( n_{eq} \right)^{1/3} \frac{\pi \rho_{eq}^2}{m} \approx \frac{3}{5} \lambda_{eq} c, \quad (i = e, \text{ion, p for positron}).
$$
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for the non-relativistic limit (where $\lambda_e = \pi \hbar/mc = 1.2 \times 10^{-10}$ cm, and $\hbar$ is the Planck constant divided by $2\pi$).

While for the electron fluid,

$$P_e = K_e n_e^2,$$

(8)

and while for the positron fluid

$$P_p = K_p n_p^2,$$

(9)

where for non-relativistic limit [33-35, 40, 43]

$$\gamma = \alpha_e K_e = K_p = K,$$

(10)

and for the ultra-relativistic limit [33-35, 40, 43]

$$\gamma = \frac{4}{3} K_e = K_p = \frac{3}{4} \left(\frac{\pi^2}{9}\right) \frac{1}{\hbar c} \simeq \frac{3}{4} \frac{1}{\hbar c},$$

(11)

III. DERIVATION OF BURGERS EQUATION

Now we derive a dynamical equation for the nonlinear propagation of the ion-acoustic shock waves by using Eqs.(1 - 5). To do so, we employ a reductive perturbation technique to examine electrostatic perturbations propagating in the relativistic degenerate dense dusty plasma due to the effect of dissipation, we first introduce the stretched coordinates [47]

$$\zeta = \epsilon (x - V_p t),$$

(12)

$$\tau = \epsilon t,$$

(13)

where $V_p$ is the wave phase speed ($\omega/k$ with $\omega$ being angular frequency and $k$ being the wave number of the perturbation mode), and $\epsilon$ is a smallness parameter measuring the weakness of the dispersion ($0 < \epsilon < 1$). We then expand $n_i, n_e, u_i, u_e$, and $\phi$ in power series of $\epsilon$:

$$u_i = 1 + \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \cdots,$$

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots,$$

$$n_e = 1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \cdots,$$

$$u_e = \epsilon u_e^{(1)} + \epsilon^2 u_e^{(2)} + \cdots,$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots,$$

and develop equations in various powers of $\epsilon$. To the lowest order in $\epsilon$, using equations (12)-(18) into equations (1) - (5) we get as, $u_i^{(1)} = V_p \phi^{(1)}/(V_p^2 - K^2)$, $n_i^{(1)} = \phi^{(1)}/(V_p^2 - K^2)$, $n_e^{(1)} = \phi^{(1)}/(V_p^2 - K^2)$, and $V_p = \sqrt{(n_e - n_i) + K^2}$, where $K_1 = n_e^{(1)}/m_i C_i^2$ and $K_2 = n_e^{(1)}/m_e C_e^2 = n_i^{(1)}/K_p m_e C_e^2$. The relation $V_p = \sqrt{(n_e - n_i) + K^2}$ represents the dispersion relation for the ion-acoustic type electrostatic waves in the degenerate plasma under consideration.

We are interested in studying the nonlinear propagation of these dissipative ion-acoustic type electrostatic waves in a three components degenerate plasma. To the next higher order in $\epsilon$, we obtain a set of equations

$$\frac{\partial n_i^{(1)}}{\partial \tau} - V_p \frac{\partial n_i^{(2)}}{\partial \zeta} - \frac{\partial}{\partial \zeta} [n_i^{(2)} + n_e^{(1)} n_i^{(1)}] = 0,$$

(19)

$$\frac{\partial n_i^{(1)}}{\partial \tau} - V_p \frac{\partial n_i^{(2)}}{\partial \zeta} + n_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \zeta} + \frac{\partial u_i^{(2)}}{\partial \zeta} - \eta \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} = 0,$$

(20)

$$+ K_1 \frac{\partial}{\partial \zeta} \left[n_i^{(2)} - \frac{(\gamma - 2)(\eta_i^{(1)})^2}{2} \right] = 0,$$

(21)

$$\frac{\partial \phi^{(2)}}{\partial \zeta} = K_2 \frac{\partial}{\partial \zeta} \left[n_e^{(2)} + \frac{(\gamma - 2)(\eta_i^{(1)})^2}{2} \right],$$

(22)

$$0 = \alpha_e n_e^{(2)} - n_i^{(2)} - \alpha_p n_p^{(2)}.$$  

(23)

Now, combining (19-23) we deduce a Burgers equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \frac{\partial \phi^{(1)}}{\partial \zeta} = C \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2},$$

(24)

where the value of $A$ and $C$ are given by

$$A = \frac{(V_p^2 - K^2)}{2V_p} \left[3V_p^2 + K^2 (\alpha_e - 2) \right],$$

(25)

$$C = \frac{\eta}{2}.$$  

(26)

The shock wave solution of (24) is

$$\phi^{(1)} = \phi_m [1 - \tanh \left(\frac{\xi}{\delta}\right)],$$

(27)

where the special stretched coordinates, $\xi = \zeta - \eta_0 \tau$, the amplitude, $\phi_m = u_0^2/4$, the width, $\delta = 2C/V_p$. $u_0$ is the wave speed and the parameter $\eta$ was chosen from standard value [48] for the system under consideration.

IV. NUMERICAL ANALYSIS

We have numerically solved the Burgers equation (24), and have studied the effects of $u_0$, $\eta$, $\alpha_e$, and $\alpha_p$ with $\xi$ on ion-acoustic nonlinear structures of shock waves in both non-relativistic and ultra-relativistic degenerate electrons and positrons who ions always being non-relativistic degenerate. It is obvious from figures 1 - 16 that the degenerate plasma system under consideration supports compressive and reductive ion-acoustic shock waves which are associated with both positive and negative potential and the amplitude of these nonlinear structures of shock waves depend on the Chandrasekhar limits [33], i.e. non-relativistic and ultra-relativistic limits.

In figures 1 - 8 we have observed the effect of $u_0$, $\eta$, $\alpha_e$, and $\alpha_p$ on the potential structure with the variation
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FIG. 1: The effect of $u_0$ on shock wave for positive potential when e-i-p being non-relativistic degenerate.

FIG. 2: The effect of $u_0$ on shock wave for positive potential when e-p being ultra-relativistic and i being non-relativistic degenerate.

FIG. 3: The effect of $\eta$ on shock wave for positive potential when e-i-p being non-relativistic degenerate.

FIG. 4: The effect of $\eta$ on shock wave for positive potential when e-p being ultra-relativistic and i being non-relativistic degenerate.

of $\xi$ in case of both non-relativistic and ultra-relativistic limits. It has been observed when the values of $\alpha_a$ is greater than 0.62 ($\alpha_a > 0.62$) then we observe the formation of positive potential for the nonlinear structures of shock wave in our considered degenerate plasma system. Again we consider the values of $\alpha_a$ is less than or equal to 0.62 ($\alpha_a \leq 0.62$) then we observe the negative potential for the nonlinear structures as shown in figures from 7 to 16. It should be noted here that in both cases we keep all the parameters same so that it could be easy to analysis the effects of every parameter.

To observe the nature of the positive potential in our considered degenerate plasma system, we have studied the effect of the increasing value of $u_0$, $\eta$, $\alpha_e$, and $\alpha_p$ on the potential, $\phi^{(1)}$ with the variation of $\xi$ in both limits from figures 1 to 8. The interesting point is that with the increasing values of $u_0$, $\alpha_e$, and $\alpha_p$, the values of potential increases smoothly, but the increasing values of $\eta$ the potential decreases smoothly. It should be noted that the formation of the compressive ion-acoustic shock waves depends only on the values of the ratio of the electron number density and the ion number density ($\alpha_e$) and does not depend on the values of $u_0$ and $\alpha_p$ (the ratio of the electron number density and the positron number density). And the figures 1 - 8 also show us that the potential for electron-positron (e-p) being ultra-relativistic and ion (i) being non-relativistic degenerate is always greater than for electron-ion-positron (e-i-p) being non-relativistic degenerate.

From the study of the negative potential in our three components degenerate plasma system, we have observed that with the increasing value of $u_0$ and $\eta$ the potential
also increases. But it has been also noted that with the increasing values of $\alpha_e$ and $\alpha_p$, the potential, $(\phi^{(1)})$ also decreases very smoothly in both limits from figures 7 to 16. It should be noted here that in this case, the values of $\alpha_e$ is always less than or equal to 0.62 ($\alpha_e \leq 0.62$). It needs to be pointed here that the formation of the reductive ion-acoustic shock waves (shown in figures 7 to 16) depends only on the values of the ratio of the electron number density and the ion number density ($\alpha_e$) and does not depend on the values of $\omega_0$ and $\alpha_p$ (the ratio of the electron number density and the positron number density). From the analysis of the negative potential structures for $\alpha_e \leq 0.62$ (shown in figures 7 - 16) it has been again pointed out that the potential for electron-positron (e-p) being ultra-relativistic and ion (i) being non-relativistic degenerate is always greater than for electron-ion-positron (e-i-p) being non-relativistic degenerate.

It is to be noted here that we have taken all the parameters in normalized form, so all the ranges of parameters are taken arbitrarily.

V. DISCUSSION

We have considered an unmagnetized degenerate dense plasma containing non-relativistic degenerate cold ions fluid and both non-relativistic and ultra-relativistic degenerate electrons and positrons fluid, and have examined the basic features of the electrostatic nonlinear structures that are found to exist in such degenerate
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dense plasma. In our present investigation all the degenerate constitutents of the considered dense unmagnetized plasma system follow relativistic limits of pressure. The nonlinear IA shock waves and its propagation have been described thoroughly with Burgers equation (24) and its solution (27). The effects of different plasma parameters on the nonlinear propagation of IA shocks waves have been graphically shown (figures 1 - 12).

The profiles of shock wave, caused by the balance between nonlinearity and dissipation, are depicted in figures 1-12. And the potential of the ion-acoustic shock waves profiles for non-relativistic degenerate ions fluid and ultra-relativistic degenerate electrons and positron fluid is different from that when all the particles follow the same limit. From the mathematical calculation and the numerical solution of Burgers equation we have found that for a certain value of $\alpha_p$ we get both positive and negative potential when all other parameters are kept same, i.e. when the values of $\alpha_p$ is always greater than 0.62 ($\alpha_p > 0.62$) then we obtain positive potential and when the values of $\alpha_p$ is less than or equal to 0.62 ($\alpha_p \leq 0.62$) then we obtain negative potential. It has made the great interest in the study of the ion-acoustic nonlinear structures of shock waves in an unmagnetized degenerate dense plasma system to analyze the existence conditions for the positive and negative potential where the elements, electrons, ions, and positrons are always being degenerate.

Our present investigation is different from the related investigations [31, 40, 41, 43, 49] in the way that we have considered the pressure of all the constituent particles (electrons and ions), as the whole system is degener-
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FIG. 13: The effect of $\alpha_e$ on shock wave for negative potential when $e-i-p$ being non-relativistic degenerate.

FIG. 15: The effect of $\alpha_e$ on shock wave for negative potential when $e-i-p$ being non-relativistic degenerate.

FIG. 14: The effect of $\alpha_e$ on shock wave for negative potential when $e-p$ being ultra-relativistic and $i$ being non-relativistic degenerate.

FIG. 16: The effect of $\alpha_e$ on shock wave for negative potential when $e-p$ being ultra-relativistic and $i$ being non-relativistic degenerate.

crate and all the particles should follow the equation of state (6)–(11) whatever the limit is (non-relativistic or ultra-relativistic). This is obvious that the shock profiles obtained from our present investigation are also quite differ-ent from the previous investigation [31, 40, 41, 43, 50] in the sense that we have had electrostatic shock profiles while the others have got to support electrostatic solitary profiles [40, 41, 43] and double layers [40, 41]. The degenerate dense plasma is found to support shock structures whose basic features depend on the plasma number density. From this point of view our present investigation is more acceptable and the system constituents have made the validity of our investigations unique.

We note that in our numerical analysis we have used a wide range of the degenerate plasma parameters [31–33, 40, 43], which are relevant for many cosmic environments and compact astrophysical objects. The results of the present investigation is, therefore, expected to be useful in understanding the dispersion properties of the electrostatic shock waves in such cosmic environments [9, 11, 18], compact astrophysical objects [12, 13, 24] and interstellar compact objects [44, 45]. The electrostatic waves in an ultra-relativistic and non-relativistic degenerate dense plasma, which is relevant to interstellar compact objects like white dwarfs, have been investigated. The results, which have been found from this investigation, represent ion acoustic-type of electrostatic waves in which the restoring force comes from the electron-ion degenerate pressure and inertia is provided by the ion mass density.

It can be expected that the basic features and the underlying physics of the IA shock waves with the exis-
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VII. REFERENCES

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