Properties of black holes and their searches at the LHC

Tetiana Obikhod, Ievgenii Petrenko
Department of High Energy Physics, Institute for Nuclear Research NAS of Science, Kiev 03680, Ukraine

Abstract: We presented the main properties of black holes and their evolution from macro to micro black holes. We considered Kruskal-Szekeres diagram representing the evolution of time in the form of hypersurfaces and stressed that the transition between the Universes is connected with the so-called Einstein-Rosen bridge. This fact was used for the consideration of the models of Randall-Sundrum, large extra dimensions and multidimensional theory of relativity for the description of scattering of two particles with the formation of a multidimensional brane space. For experimental searches of these exotic objects at the LHC we have studied their different properties: numbers of micro black holes produced at the LHC with energy 14 TeV, momentum distribution of black hole decay products and formation cross sections of rotating and non-rotating black holes for different models of extra dimensions. These results are of interest for future experimental searches of new physics beyond the Standard Model.

Keywords: Micro black hole, Models of extra dimensions, Cross sections of rotating and non-rotating black holes

I. Introduction

Among the objects of nature, Black Holes (BH) are the most mysterious objects in the cosmos. These massive objects with great gravity are described by Einstein’s theory of relativity as the curvature of space-time: the Einstein field equations describe the relation between the stress–energy tensor and the curvature of space-time:

\[ G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \]

where \( G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \) (the Ricci tensor, \( R \) is the scalar curvature) is the Einstein tensor, \( \Lambda \) is the cosmological constant, \( G \) is the gravitational constant. At first we’ll consider the definition and properties of astrophysical BH and then smoothly move on to the quantum BH and to the studying of their properties.

II. Astrophysical black holes

As is known, BH is space-time formation with so great gravity that even light can’t escape it. The mass of BH, \( M \), its charge, \( Q \) and angular momentum, \( J \) are expected to satisfy the relation

\[ Q^2 + \left( \frac{J}{M} \right)^2 \leq M^2. \]

BHs are the most mysterious objects in the Universe with their uncertainty inside the event horizon. Let’s consider the most interesting properties of BHs:

1. Stars located near the BH lose some weight in the form of stellar winds;
2. The existence of BH was predicted by Karl Schwarzschild who coined the term “the Schwarzschild radius”;
3. The singularity of BH cancels the usual laws of physics and can produce the new Universe;
4. BHs stretch the objects that are close to them;
5. BHs not only absorb the stellar wind but also are evaporated;
6. BHs slow the time;
7. BHs are the most advanced power plants;
8. BHs curve space.

Stars pass several stages in their evolution. The final stage of this evolution is the formation of a BH. At the beginning is formed a white dwarf, where the force of gravity balanced by the electron pressure inside the star. If mass of the star is above the the Chandrasekhar limit, \( M = 1.4 M_\odot \) (\( M_\odot \) - mass of the Sun), gravitational forces exceed the pressure of electron gas and due to the reaction \( e^- + p^+ \rightarrow n + \nu \) is formed a neutron star, where the neutrons balance the gravitational forces. At the same time, the mass of the star does not exceed \( M_{\text{max}} \approx 3 M_\odot \).
More massive stars continue to collapse into a BH, the ideal form of which is spherically symmetrical ball. According to the theorem of Birkhoff the geometry of star is described by the Schwarzschild metric (for a BH without rotation and without electric charge).

The Schwarzschild metrics

\[ ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \]

where \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \). \( G = 1, \ c = 1 \). In Eddington-Finkelstein coordinates \((\nu, r, \theta, \phi)\) the metrics takes the form

\[ ds^2 = -\left(1 - \frac{2M}{r}\right) d\nu^2 + 2drd\nu + r^2 d\Omega^2. \]

It has no singularity because the relation between \( \nu = t + r^* (r) \) is defined for \( r > 2M \). Using the Kruskal-Szekeres coordinates

\[ U = -e^{-u/4M}, \ V = e^{v/4M} \text{ for } r > 2M \]

Schwarzschild metrics can be rewritten as follows [1]

\[ ds^2 = -\frac{32M^3}{r} e^{-r/2M} dUdV + r^2 d\Omega^2, \]

where \( UV = \left(\frac{r-2M}{2M}\right)^2 e^{r/2M} \).

The geometry can be represented by the following Kruskal-Szekeres diagram, Fig. 1 (URL: http://www.astronet.ru/db/msg/1174703/kaufman-09/kaufman-09.html)

![Kruskal-Szekeres diagram](image1)

**Fig. 1** Kruskal-Szekeres diagram

Here lines of constant distance are space-like under the event horizon, and lines of constant time have time-like direction. So the distance behaves like the time inside the event horizon. Space-like pictures of the evolution over time are shown in Fig. 2 (URL: http://www.astronet.ru/db/msg/1174703/kaufman-09/kaufman-09.html)

![Kruskal-Szekeres diagram](image2)

**Fig. 2** Kruskal-Szekeres diagram representing the evolution of time in the form of hypersurfaces
Then, it is possible to imagine the transition of our universe (left) to another universe (right) using the Penrose diagram, Fig. 3 (URL: https://en.wikipedia.org/wiki/Penrose_diagram)

![Penrose diagram of a BH](image)

**Fig. 3** Penrose diagram of a BH

At the middle point (the 2-sphere) of the transition between the Universes, takes place the so-called Einstein-Rosen bridge, shown in Fig. 4

![Einstein-Rosen bridge](image)

**Fig. 4** Einstein-Rosen bridge, connecting the two universes, from [1]

### III. Microscopic black holes

BHs do not have to be monstrous. The theoretical physicists claim [2, 3] that the BH size can vary within very wide limits up to a size smaller than the size of elementary particles. At the same time they should blow up almost immediately after formation.

Theorists have hypothesized [2, 3] that microscopic BHs could be created in collisions of particles in the Universe. However, the energy required for this is too big. But if space has extra dimensions with the desired properties, the energy threshold for the birth of BHs is much lower and they could be produced at the Large Hadron Collider (LHC, CERN). Physicists could use BHs for investigations of extra dimensional space through their generation of radiation and decay products.

If a proton energy at the LHC is about 7 TeV, then, in accordance with famous Einstein’s relation \( E = mc^2 \), this energy is equivalent to the weight of \( 10^{-23} \) kg. When two such particles collide and approach each other at a small distance, their energy is concentrated in a small region of space. Therefore, it is possible that occasionally colliding particles can form a BH. However, the weight of \( 10^{-23} \) kg is small compared to the Planck mass \( 10^{-8} \) kg, which is the minimum possible mass of the BH.

String theory, which is the main candidate for the quantum theory of gravity [4, 5] predicts, that space has additional extra dimensions to the usual three spatial dimensions. Gravity can penetrate into these extra dimensions and become stronger at short distances. Besides the string theory could be the following theories of extra dimensions: large extra dimensional model, Randall–Sundrum model [6, 7, 8]. In the centre-of-mass energy about 10 TeV LHC could allow the production rate of BH about one per second. The cross-section for the BHs creation can be estimated by the geometric approximation \( \sigma \sim \pi R_H^2 \) [9].
IV. Models of extra dimensions

Let us consider the models of extra dimensions for determination of the Schwarzschild radius or the size of the extra dimensions for cross-section calculation. The Randall – Sundrum (RS1) model is a model in a five-dimensional space-time with the extra dimension compactified to the orbifold $S^1/Z_2$.

Metrics of this nonfactorizable geometry is the following

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,$$

where the function $\sigma(y)$ in the warp factor

$$\sigma(y) = k |y|, \ (k > 0),$$

$k$ is dimensional parameter, $\eta_{\mu\nu}$ - Minkowski metrics. Due to this geometry Planck scale $M_P$ is related to the fundamental scale of quantum gravity, $M$ by the hierarchy formula

$$M_P = M e^{\pi R},$$

where $R$ is the size of extra dimensions.

Two 3-branes are located at the points $y = 0$ and $y = \pi R$ of the orbifold, with $R$ - the radius of the circle $S_1$. The brane 1 is at $y = 0$, whereas the brane 2 is at $y = \pi R$, that is presented in Fig. 5.

According to these theories, our Universe consists of zero modes of Standard Model (SM) fields located in the SM brane (brane 1), gravity is focused on another brane (brane 2), while the remaining sets of particles states (Kaluza-Klein modes of gravitons and gauge bosons, BHs) are located in the space of extra dimensions between branes. Experimental searches for BHs are related to the formation of BH in the bulk, which decay to the particles detected in brane 1, that is presented in Fig. 6.
The Large extra dimensional model or ADD (N. Arkani-Hamed, S. Dimopoulos and G. Dvali) relates Planck scale to the fundamental scale of quantum gravity, $M$ by the hierarchy formula

$$M_{Pl}^2 = R^d M^{d+2},$$

where the size of the extra dimensions $R$ is defined by the formula

$$R \sim \frac{1}{M} \left( \frac{M_{Pl}}{M} \right)^{2/d} \sim 10^{-17} \text{ cm}.$$  

Multidimensional theory of relativity [10] is connected with Schwarzschild solution, which gives the following expression for the radius of BH horizon for n+3 dimensions

$$R_h^{(n)} = \frac{1}{\sqrt{\pi} M_p} \left[ \frac{M_{BH}}{M_p} \left( \frac{8 \Gamma \left( \frac{n+3}{2} \right)}{n+2} \right) \right]^{1/n+1}.$$  

V. Calculations of cross section of the microscopic black hole formation

We worked with a computer program BlackMax [11]. In our calculations we used two versions of BH model:
1) non-rotating BH;
2) rotating BH.

Two versions of extra dimensional model were used:
1) ADD and RS model;
2) User defined convention connected with multidimensional theory of relativity.

We also used Mass_loss_factor = 0.5 connected with the fact that the scattering of the incoming particles is not completely inelastic. Since BHs can have angular momentum, the correction factor of angular momentum in our calculations is taken 0.5.

Among the hypotheses of formation and decay of the BH an important place occupies the hypothesis of dijet decay [12, 13], presented in Fig. 7.

![Fig. 7 The formation of micro BH and its decay to dijet](image)

As we study the experimental searches for microscopic BHs at the LHC, it was interesting to calculate the mass distribution of BHs with energy 14 TeV. These calculations are of particular relevance in connection with the experimental searches for dijet invariant mass distributions [14], which are formed as a result of decay of new states predicted by physics beyond the SM [15, 16].

In Fig. 8 are presented the mass distributions of microscopic BHs, calculated with BlackMax program [11].
Properties of black holes and their searches at the LHC

Fig. 8 Number of micro BHs produced at the LHC with energy in cm 14 TeV, as a function of the BH mass with parameter $M_{min}$: a) 1 TeV, b) 2 TeV, c) 3 TeV, d) 6 TeV, f) 6.9 TeV.

The 8 TeV ATLAS analysis [17] of the track multiplicity in same-sign dimuon events depending on details of the model and on the number of extra dimensions, provides lower mass limits for micro black holes of 5.1 - 5.7 TeV for the fundamental quantum-gravity energy scale 1.5 TeV. A CMS analysis [18] of multi-object final states using 12 fb$^{-1}$ of 8 TeV data provides similar limits for $M = 5$ TeV.

For the studying of momentum distribution of the products of micro BH decay we have calculated distribution $dN/dp_z$ as a function of momentum of emitted by BH particles, presented in Fig. 9.

Fig. 9 Momentum distribution of emitted by micro BH particles for non-rotating BH model at the LHC with cm =14 TeV, $n = 4$ and $M =$10 TeV

From our calculations we can conclude that the BH decay products are directed generally along the axis of proton-proton collisions at the LHC. This fact is essential for experimental searches of micro BH at the LHC. As we study the experiments with the deviation from the SM in processes with fermion-fermion interactions, it would be interesting to look at the character of the cross-section of micro BH formation on the LHC energy, presented in Fig. 10.
Fig. 10 Cross section of micro BH formation for ADD model (n=4, M=10 TeV, non-rotating BH model) in the energy range 13-14 TeV

Fig. 10 shows that the cross section of micro BH formation increases with increasing energy at the LHC, but no resonances in the given energy scale is not observed. Such behavior of micro BH cross section for ADD model may be a signal of its decay in our (3 + 1) - dimensional space, but not in the space of extra dimensions, as is shown in Fig. 6.

It was interesting to compare the results of cross sections calculation for ADD model of rotating micro BH with the results of the calculation according to the multidimensional theory of relativity [19]. We have performed the comparison of the results of two models for different numbers of extra dimensions. The results of the cross sections calculation for micro BH formation of these two models are presented in Fig. 11.

Fig. 11 Cross sections of micro BH formation on the energy at the LHC for two models: a) model of multidimensional theory of relativity b) ADD model for rotating micro BH
From Fig. 11 it is seen that the behavior of the cross section for two cases is the same, though it has significant difference at high energies at the LHC. Thus it is important to emphasize that the cross section for the case of ADD model of rotating micro BH with \( n = 2 \) is the largest of all the presented calculations, and the cross section of the model, connected with multidimensional theory of relativity is significantly less. It is interesting to make a comparison of the cross sections of the BH formation in the energy range of 13 - 13.5 TeV for two cases:

a) a non-rotating BH (Fig. 10);

b) a rotating BH (Fig. 11b)).

A comparison of two models for the number of extra dimensions \( n = 4 \) demonstrates that the cross section of micro BH formation of a rotating BH exceeds the cross-section of a non-rotating BH. This fact confirms the theoretical assumptions [2], according to which the cross section is determined by the formula:

\[
\sigma_{ij \to BH}(s) = F(s) \sigma_{R}^{2},
\]

where

\[
F(s) = \int dM dJ \frac{d^2 F(s, M, J)}{dM dJ}
\]

is the form factor coefficient of the BH cross section, which generally depends on the mass and angular momentum of the micro BH formed in proton-proton collisions. Since the cross section depends on the impact parameter \( b \leq r_0 \) and the order of the angular momentum is connected with the order of the impact parameter, it is clear that the usual micro BHs, located at SM brane, should be formed with large components of the angular momentum, though within general theory of relativity there are the correlations of mass and angular momentum in the high-energy limit.

It is also necessary to notice the different behavior of the cross sections for two models

a) model of the multidimensional theory of relativity, Fig. 11 a);

b) ADD model for the rotating micro BH, Fig. 11 b)

which depend on the number of extra dimensions. In the first case (a) the cross section increases with the number of extra dimensions, in the second case (b) the cross section of micro BH decreases. Such behavior is due to the fact that with increasing of the number of extra dimensions \( n \), decreases the size of the extra dimensions \( R \), and thus increases the fundamental scale of quantum gravity, \( M \) in accordance with the hierarchy formula of ADD model. In addition, it should be stressed that the multidimensional semiclassical approximation of general relativity is not adequate at high energies, so we need to use ADD model.

Fig. 12 shows the cross-section distribution of non-rotating micro BH formation depending on the different values of the fundamental quantum-gravity energy scale, \( M \) for different numbers of extra dimensions, \( n \) in the ADD model.

![Graph showing cross section of micro BH formation as a function of fundamental quantum-gravity energy scale, M for different numbers of extra dimensions, n](image)

From Fig. 12 it is seen that cross section of a BH is the largest when the number of extra dimensions \( n = 6 \) and this value by eight orders of magnitude smaller than the cross section, calculated in [12] for a model of quantum BH. Comparison of the obtained results with the cross sections calculations for the other two cases (Fig. 13):
A) model of multidimensional theory of relativity;
B) model of rotating BH

shows that the cross section is maximal for non-rotating BH, and minimal for the model of multidimensional theory of relativity with the number of extra dimensions \( n = 3 \).

![Graph](image)

**Fig. 13** Cross section of micro BH formation as a function of fundamental quantum-gravity energy scale, \( M \) for two models: a) model of multidimensional theory of relativity; b) model of rotating BH

**VI. Conclusions**

Many extensions of the SM predict the existence of new massive particles of micro BH type that can be detected as resonances. The searches for micro BH have been performed using pp collisions at the LHC. Here we report a search for these massive exotic particles that are compared with the experimental data. We also presented searches for micro BHs with masses between 1 TeV and 6.9 TeV at \( \sqrt{s} = 14 \) TeV and the momentum distribution of its decay products, that is actual for future experimental searches at the LHC.

We presented different models for searches of non-rotating and rotating micro BHs: 1) ADD and RS models; 2) model of multidimensional theory of relativity.

We calculated the cross sections for BH formation in large energy scale at the LHC and received the best results for ADD model at \( n = 2 \) for rotating BH. In the paper was studied the behavior of the cross sections for the models of the multidimensional theory of relativity and ADD of the rotating micro BH on the number of extra
dimensions. The comparison of these results shows that while in the first case the cross section increases with the number of extra dimensions, the second cross section of micro BH decreases. It was studied the cross section of micro BH formation as a function of fundamental quantum-gravity energy scale, $M$ for different numbers of extra dimensions, $n$ and was found the largest value for the number of extra dimensions $n = 6$. Comparison of the obtained results for the model of multidimensional theory of relativity and model of rotating BH shows that the cross section is maximal for non-rotating BH, and minimal for the model of multidimensional theory of relativity with the number of extra dimensions $n = 3$.

Thus, we carried out calculations for different models of extra dimensions at energies close to the energy of the LHC. These results are important for searches of black holes, as they give the possibility to select the optimal model of extra dimensions for maximal cross-section value. In addition, these calculations are based on a thorough physical basis and are of theoretical interest for high-energy physics.

References