

Estimating Stock Market Volatility Using Non-linear Models

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Abstract: Stock market price changes are negatively correlated with changes in volatility. A drop in the value of a stock (negative return) increases the financial leverage; this makes the stock riskier and thus increases its volatility. This effect is known as leverage effect. The volatility of asset returns can be seen as measurement of the risk for investment and provides essential information for the investors to make the correct decisions. The study focuses on developing a GARCH model for the daily closing price of S & P 500 stock price. The Akaike and Bayesian Information Criteria (AIC & BIC) techniques are used to estimate the p and q values of GARCH (p, q) model. The results show that GRCH (1, 1) is an appropriate model for the time series.

Keywords: Volatility, ARCH/GARCH, leptokurtic,

I. Introduction

In finance, volatility is a measure for variation of price of a security over time. Volatility is computed as the standard deviation of stock returns. Modeling volatility in financial market is imperative because it is often perceived as significant element for the evaluation of assets, the measurement of risk, the investment decision making, the valuation of stock and the monetary policy making. The stock market volatility is virtually time-varying. It is widely accepted that volatility changes in financial market are predictable.

The various models have been applied by extensive empirical studies for future volatility forecasting and measuring the predictability of volatility forecasts. However, there is little consensus in terms of which model or family of models is the best for describing assets returns. The two most popular approaches for future volatility forecasting are considered to be the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and the Risk Metrics approach introduced by Robert Engle (1982) and J. P. Morgan (1992), respectively till date. The forecasts of these two approaches are derived on the basis of historical data. The GARCH model is the natural extension of autoregressive conditional heteroskedasticity (ARCH) model which was thought to be the good description of stock returns and an efficient technique for estimating and analyzing time-varying volatility in stock returns.

II. Review of literature

Arowolo W B (1) in the study on the daily stocks prices of Zenith bank Plc in Nigeria Stock Exchange arrived at the result that GARCH (1, 2) is the model that best fits the series. The researcher has made an attempt to forecast properties of the series. They have reported leptokurtosis and considerable volatility in the series. The report concludes that the optimal value of p and q in GARCH (p, q) model depends on location, type of data and model order selected techniques used.

H Goudarzi et al (2) in an attempt to examine the volatility of in Indian market arrives at the conclusion that GARCH (1, 1) model fits well into a time series of returns of BSE 500. They have established the adequacy of the model by using ARCH LM test and LB Statistic. A Goyal (3) reported in the study on Exchange Rate Volatility using GARCH models found that quantitative credit restriction, higher interest differentials and policy lending rates depreciate the exchange rates probably due to reduced capital inflow.

A Goyal (4) in an attempt to predict stock return volatility for GARCH models, performed various GARCH models to deliver the volatility forecasts from stock returns data. Volatility is obtained from a variety of mean and variance specifications in GARCH models, they are compared to the proxy of actual volatility calculated. n -sample tests suggest that a regression of volatility estimates on actual volatility produces R^2 of less than 8%. An interesting by-product is evidence of significantly negative relation between unexpected volatility and stock returns. Finally, out-of-sample tests indicate that a simpler ARMA specification performs better than a GARCH-M model.

In an attempt to develop a nonparametric approach to GARCH models to the volatility of ISE 100 market data, Sebnem (5), found that this approach is less sensitive to model misspecification. They have compared the estimation capability of nonparametric and parametric GARCH models on volatility. They suggest that the findings by parametric methods might be misleading if the series has heavy tails and leptokurtosis. They have taken the non-parametric approach due to the two aspects, returns have heavy tails and leptokurtosis and changes in volatility over time. The study suggests value – at – risk approach is one of the most preferred non-parametric estimation method to measure market risk

Corradi et al (6) has examined the relative out of sample predictability of different GARCH models with particular emphasis on the predictive content of the asymmetric component. They have used squared returns as a proxy for the unobservable volatility process. A pairwise comparison of various models against GARCH (1, 1) model is carried out and this model is beaten by asymmetric GARCH models. The same findings remains true for longer forecast horizons also.

III. Methodology

Many relationships in finance are intrinsically non-linear. Most of the linear structural models fail to explain a number of important features like leptokurtosis, volatility clustering and leverage effects. Among the multitude of non-linear models, the most popular models are Autoregressive Conditionally Heteroskedastic (ARCH) model and Generalized Autoregressive Conditionally Heteroskedastic (GARCH) models. They are expedient in modelling and forecasting volatility. The assumption of CRLM is that the variance of the errors is constant, which is termed homoscedasticity. ARCH models do not assume that variance is constant. ARCH models are preferred to CLRMs since it is unlikely to financial time series to have constant variance for the errors over time. Another important feature that motivates the use of ARCH/ GARCH models is volatility clustering or volatility pooling. Volatility clustering describes the tendency of large changes in assets prices to follow large changes and small changes to follow small changes. Knowledge of volatility is important to make effective financial planning.

Most of the time series are random walk at level form, ie, non-stationary, but they are generally stationary in the first difference form. Therefore we model first difference instead of the time series. The squared returns are a very noisy measure but $r_t = \ln S_t - \ln S_{t-1}$, where S_t is the stock price, the continuously compounded return of the underlying asset can be used.(8) But if the first difference has varying variance instead if a constant, it is termed Autoregressive Heteroskedasticity (ARCH). ARCH models are capable of modeling and capturing volatility clustering. (6) An ARCH(q) model is usually represented as

$$Y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \dots + u_t; \quad u_t \sim N(0, h_t) \text{ where}$$

$$h_t = \alpha_0 + \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \alpha_3 u_{t-3} + \dots + \alpha_q u_{t-q}$$

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is one of the most popular ARCH model. It allows the conditional variance to be dependent upon previous own lags, so that the conditional variance equation can be represented as

$\sigma_{t-1}^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2$ which is the GARCH (1, 1) model. GARCH (p, q) model may be represented as follows:

$$\sigma_{t-1}^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \alpha_3 u_{t-3}^2 + \dots + \alpha_q u_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + \dots + \beta_p \sigma_{t-p}^2$$

Test for ARCH effect

Before estimating a GARCH type model, first we have to make sure that this model is appropriate. The most widely used test for non-linearity is BDS test. This test is also a model diagnostic. (7) It has a null hypothesis that the data are pure noises. If a proposed linear model is adequate, then the standardized residuals should be white noises, while if the postulates model in insufficient to capture all the relevant features of the data, BDS statistic for the standardized residuals will be statistically significant. Once ARCH effect is found then we are required to find the specification of the model using Akaike Information Criterion (AIC) and Schwartz Bayesian Information Criterion (SBIC).

IV. Results and Findings

The present study is on 2745 daily closing observations for S & P 500 index during the time period Jan 2002 to Dec 2012. E-views version 7 is used for the analysis of the series. The descriptive statistics and histogram of the series is given below.

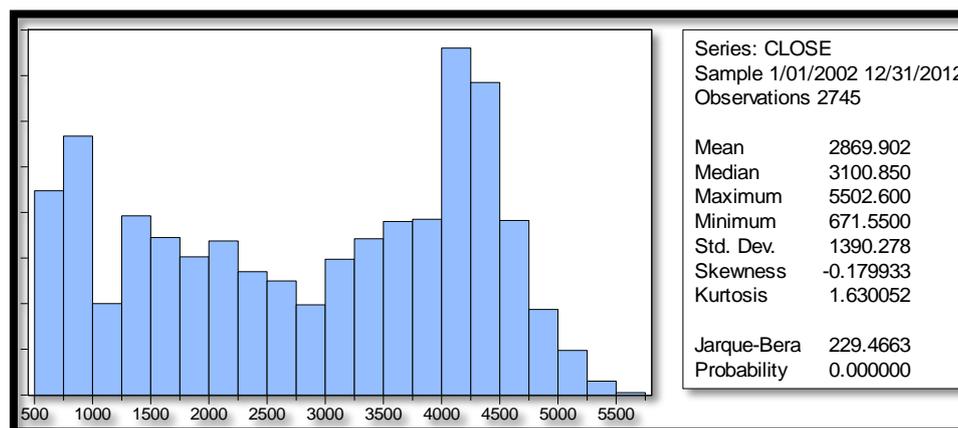


Figure 1: Daily closing observations of S & P 500 index

The series has an average closing price of 2869.9 with a standard deviation of 1390.28. The skewness coefficient of -0.18 indicates negative skewness. The kurtosis coefficient 1.63 indicates fatter tails of the distribution which is called leptokurtosis. The Jarque- Bera statistic value is 229.46 with p value 0.00 which rejects the hypothesis of normal probability distribution of the series.

The distribution of the series of returns $r_t = \log(p_t / p_{t-1})$ of closing price is given below

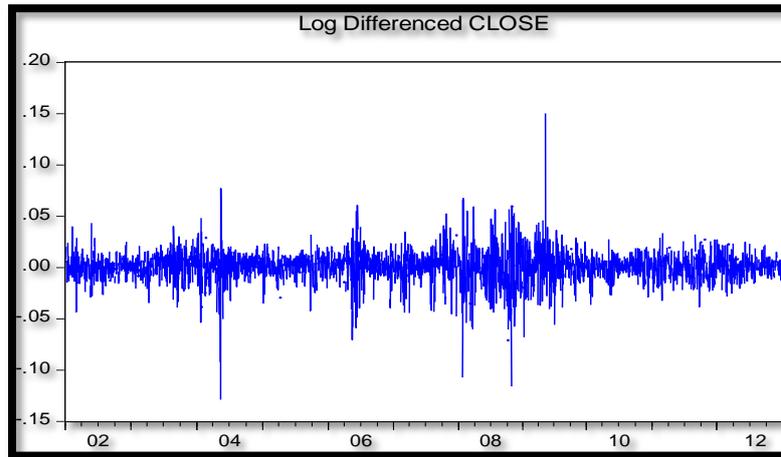


Figure 2: The returns of the S&P 500 index series.

The average returns is 0.000698. The volatility measured in terms of standard deviation is 0.01565. The returns series also indicates negative skewness (coefficient = -0.4968) with leptokurtosis (kurtosis 8.9112 > 3). The normality assumption is rejected (Jarque Bera = 229.4663, p value 0.00 < 0.05) significantly due to negative skewness and high kurtosis.

The inspection of the returns series reveals that the returns fluctuates around the mean value that is close to zero. The important feature exhibited in the figure is that volatility occurs in bursts. There appears to have a period of relative tranquility which indicated relatively small positive and negative returns in the beginning quarter and also at the last quarter of the time period. The inter-quartile time period depicts far more volatility where large positive and large negative returns can be observed. This motivates us to develop a non- linear model for analyzing the situation and to forecast.

ADF test is carried out and the result is in Table 1. The results shows that the returns series does not have unit root and is not stationary. These properties are in consistency with other financial time series.

Table 1. Unit Root Results

Table 1. ADF, PP and KPSS test results for returns

| | Augmented Dickey Fuller Test | | Philip Perrons Test | | KPSS test | |
|------------------|------------------------------|---------|---------------------|---------|--------------------|-------------------|
| | t statistic | p value | t statistic | p value | LM Statistic value | 1% critical value |
| At level | -1.1613 | 0.6932 | -1.1550 | 0.6968 | 5.6329. | 0.7390 |
| First difference | -46.9282 | 0.0001* | -46.8694 | 0.0001* | 0.0511 | 0.7360 |

* Significant at 1% level.

The statistics of the test results shows that the series is non- stationary at level, but attains stationarity at first difference. Non stationarity implies instability in the series and makes the series unpredictable. Emphasis on stationarity is because some kind of stability over time in a variable is essential to analyze the series.

The result of ARCH-LM test is given in table 2. At 5 lags, the p value indicates the presence of ARCH effect in the residuals of mean equation.

Table 2. ARCH LM test at 5 lags

| | Value | p value |
|--------------|----------|---------|
| F Statistics | 88.5952 | 0.0000* |
| R- squared | 382.0272 | 0.0000* |

Significant at 1% level.

The ARCH effect tested shows that ARCH (1) is appropriate than the other models. The results of this model is given in table 3. ML - ARCH (Marquardt) assuming Normal distribution is the method adopted to estimate the parameters.

Table 3. ARCH(1) model

| Variable | Coefficient | Standard error | Z – statistic | Probability |
|-------------------|-------------|----------------|---------------|-------------|
| C | 2843.067 | 43.8115 | 64.8932 | 0.0000* |
| AR(1) | 0.8878 | 0.0041 | 216.837 | 0.0000* |
| Variance equation | | | | |
| C | 1269045. | 28649.00 | 44.29629 | 0.0000* |
| RESID(-1)^2 | -5.913718 | 0.448941 | -13.17260 | 0.0000 |

Significant at 1% level

Thus the model can be written as $y_t = 2843.067 + 0.8878 \beta_{(t-1)} + \epsilon_t$

The GARCH model: Using AIC and SBC criterion the value of p and q in GARCH (p, q) model is obtained. The most appropriate model turns out to be GARCH (1, 1) by comparing the value of AIC for various other combinations of p and q. The estimates of the parameters of GARCH (1, 1) model is represented in table 4

Table 4. GARCH (1, 1) model

| Variable | Coefficient | Standard error | Z – statistic | Probability |
|-------------------|-------------|----------------|---------------|-------------|
| C | 2883.454 | 353.2378 | 8.162926 | 0.0000* |
| AR(1) | 1.046008 | 0.010471 | 99.89948 | 0.0000* |
| Variance equation | | | | |
| C | 1255252. | 48843.44 | 25.69951 | 0.0000* |
| ARCH (1) | -2.241762 | 0.260895 | -8.592599 | 0.0000* |
| GARCH(1) | -0.991778 | 0.001772 | -559.6004 | 0.0000* |

Significant at 1% level

The representation of the model developed is given below:

$$y_t = 2883.454 + 1.046 y_{(t-1)} + \epsilon_t$$

$$\sigma_t^2 = 1255252 + -2.241762 \alpha_{(t-1)}^2 + -0.991778 \sigma_{t-1}^2 + \epsilon_t$$

This model seems to be an appropriate fit for the data on closing observations for S & P 500 index.

V. Conclusion

ARCH and GARCH models are invariably applied in financial time series. It helps in making financial decisions which are generally influenced by the trade off between risk and return; hence the econometric analysis of risk plays an integral role. The present study has presented a GARCH model to the time series data on S & P 500 stock returns daily data of 10 years. The log returns of the closing price of the stock is used to develop the model. The returns exhibit leptokurtosis and volatility clustering. The most appropriate model found was GARCH(1, 1). This result is in agreement with the many literature available that for closing price of stock prices GARCH (1, 1) appear to be a good fit. This non-linear model allows us to capture volatility and serial correlation in the series.

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