A Comparison of Hsu & Wang Model and Cost of Carry Model: The case of Stock Index Futures.

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Abstract: The study empirically tests and compares the pricing performance of two alternative futures pricing models; the standard Cost of Carry Model and Hsu & Wang Model (2004) for three futures indices of National Stock Exchange (NSE), India – CNX Nifty futures, Bank Nifty futures and CNX IT futures. It is found that, the Hsu & Wang Model with an argument of incomplete arbitrage mechanism and real capital markets are imperfect, provides much better pricing performance than the standard Cost of Carry Model for all the three futures markets. On the basis of Mean Absolute Pricing Error (MAPE), CNX Nifty Futures contract with highest trading history and trading volume is preferred, followed by Bank Nifty futures and CNX IT futures contract for both the pricing models. This result implies that Indian futures markets are imperfect and arbitrage process cannot complete. Degree of market imperfection might influence the pricing error. Therefore, investors should know the degree of market imperfection of the futures markets in which they would like to participate.

Key words: Degree of Market Imperfection, Futures Indices, Futures pricing models, pricing Performance

I. Introduction

Right from launch of Index futures and individual stock futures on June 12 2000 and November 2001 respectively, the futures market in India constantly growing on annual basis in terms of number of contracts traded and turnover. According survey conducted by World Federation of Exchanges (23, July 2013) on the performance of world stock exchanges during first half of 2013, National Stock Exchange of India has ranked No 4 for number of single stock futures contracts traded, No 8 for number of stock index futures contracts traded in first half of 2013. Thus, it is clearly indicating that more number of investors attracting and educating towards Indian futures markets. Pricing performance of stock index futures markets has triggered a substantial volume of research by finance academicians. A number of researchers have made an extensive effort to predict stock index futures price under various assumptions and economic conditions. Literature shows that many researchers used two important pricing models to determine future pricing performance – Standard Cost of Carry Model (CCM) and Hsu and Wang model (HWM) (2004).

The cost of carry model has been considered as the standard model for pricing stock index futures. The difference between index futures price and spot index futures will reflect the carrying cost. Cornell and French (1983a, b) used an arbitrage argument to develop a pricing model of stock index futures under the following assumptions:

(a) Capital markets are perfect - No transaction costs and taxes and, no restrictions on short sales, and divisibility of securities. (b) No limits exist on borrowing or lending at the same risk-free rate. (c) The risk-free interest rate is known with certainty.

Many researchersAndreou and Pierides (2008) examined Athens futures market, Phil Holmes (2002) studied on UK stocks and index futures market, Fung and Draper (1999) examined affect of mispricing of futures contracts using Cost of carry model. Similarly Gay & Jung, (1999), Brailsford and Cusack (1997) studied individual shares on Australian Stock Exchange, Wolfgang, Buhler & Alexander Kemp (1995) examined German market. Brenner, Subrahmanyam, Uno, Jun (1990), studied on Japanese Stocks and futures market] has been documented the existence of mispriced futures contract i.e. the spot price of futures was persistently below the theoretical value of futures estimated by the cost of carry model.

Hsu- Wang (2004) includes the factor of price expectation (Expected growth rate) and uses an argument of the incomplete arbitrage mechanism and developed a pricing model of stock index futures in imperfect markets (here after Hsu- Wang model). Hsu – Wang states that capital markets are imperfect. First, index arbitrage involves transaction costs, including commissions, bid-ask spread, and taxes. Second, there are constraints on short sales and securities are not perfectly divisible. Third, price changes in securities and constant and continuous dividends cannot be expected always. Fourth, it’s not always possible to purchase and sale exact number of the underlying index simultaneously. Fifth, there is a limitation on borrowing or lending at the same risk-free rate. Finally, traders may have asymmetric information. Further Hsu - Wang (2004) argues that in perfect markets if actual futures price deviates from its theoretical value predicted by the cost of carry model, then the arbitrages can form a riskless arbitrage profit making no investment. Under the assumption of
perfect markets, Suppose if any deviation of actual futures price from its theoretical ‘Fair Price’ estimated by CCM then this deviation will be adjusted back to equilibrium simultaneously and risk less profitable arbitrage opportunity will be eliminated. Thus arbitrage mechanism completes. However literature found that real capital markets are imperfect and arbitrage mechanism not possible to complete and thus, arbitrage process will expose to a very large risk in imperfect markets.

The number of previous studies, Gay and Jung (1999), Brenmer, Subrahmanyam and Uno, (1990, Garry J. Twite (1998) and Panayiotis C. Andreou and Yiannos A. Pierides (2008) support Hsu-Wang arguments and found real capital markets are imperfect. Additionally Hsu-Wang states that CCM model cannot reasonably explain the negative basis (Difference between actual futures price and the underlying value). According to CCM the basis should reflect the carrying cost and this carrying cost must be positive (Actual futures price > Spot price). Unless the dividend yield is higher than risk free interest rate this seldom occurs. Further the author’s states that Hsu – Wang (1999) examined Taiwan stock index futures during Asian crisis (1998-1999) and observed that a significant relationship appears to exist between investors bear market expectation and negative basis i.e Investors considers that the expected growth rate of stock is negative or a negative basis will not occur. From these Hsu-Wang claims that Price expectation is one of the important factors in determining stock index futures prices. Finally, Hsu and Wang (2004) developed a futures pricing model for stock index futures in imperfect markets by incorporating the factor of price expectation (Expected growth rate) with an argument of incompleteness of arbitrage mechanism.


1.1 Futures Indices: History and Institutional background

<table>
<thead>
<tr>
<th>Particulars</th>
<th>CNX NIFTY Futures</th>
<th>BANK NIFTY Futures</th>
<th>CNX IT Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening Date</td>
<td>June 12, 2000</td>
<td>June 2005</td>
<td>August 2003</td>
</tr>
<tr>
<td>Underlying Index</td>
<td>CNX NIFTY</td>
<td>BANK NIFTY</td>
<td>CNX IT</td>
</tr>
<tr>
<td>Contract Size</td>
<td>The value of the futures contracts on Nifty may not be less than Rs. 2 lakhs at the time of introduction. Lot Size: 50</td>
<td>The value of the futures contracts on BANK Nifty may not be less than Rs. 2 lakhs at the time of introduction. Lot Size: 25</td>
<td>The value of the futures contracts on CNX IT may not be less than Rs. 2 lakhs at the time of introduction. Lot Size: 25</td>
</tr>
<tr>
<td>Contract Months</td>
<td>The near month (one), the next month (two) and the far month (three). at any point in time, there will be 3 contracts available for trading in the market</td>
<td>The near month (one), the next month (two) and the far month (three). at any point in time, there will be 3 contracts available for trading in the market</td>
<td>The near month (one), the next month (two) and the far month (three). at any point in time, there will be 3 contracts available for trading in the market</td>
</tr>
<tr>
<td>Minimum price change</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Price limits</td>
<td>+/- 10% LTP</td>
<td>+/- 10% LTP</td>
<td>+/- 10% LTP</td>
</tr>
<tr>
<td>Last trading Day</td>
<td>Last Thursday of delivery month</td>
<td>Last Thursday of delivery month</td>
<td>Last Thursday of delivery month</td>
</tr>
<tr>
<td>Settlement</td>
<td>Cash</td>
<td>Cash</td>
<td>Cash</td>
</tr>
</tbody>
</table>

Source: Retrieved & Adapted from http://www.nseindia.com

Table 2: Descriptive Statistics of daily trading Volume and frequency of negative basis of futures indices

<table>
<thead>
<tr>
<th>Contract</th>
<th>N</th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>Number of Negative Basis</th>
<th>Number of Negative Basis (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNX Nifty Futures</td>
<td>1741</td>
<td>442.9262</td>
<td>1338598</td>
<td>1933</td>
<td>550</td>
<td>31.59</td>
</tr>
<tr>
<td>Bank Nifty Futures</td>
<td>1741</td>
<td>52007.03</td>
<td>256601</td>
<td>7</td>
<td>599</td>
<td>34.40</td>
</tr>
<tr>
<td>CNX IT Futures</td>
<td>1741</td>
<td>305.26</td>
<td>3037</td>
<td>1</td>
<td>640</td>
<td>36.76</td>
</tr>
</tbody>
</table>

Source: Collected and Compiled by the Authors

NSE is India’s leading Stock Exchange incorporated in the year 1992. Index value calculates based on Free Float market capitalization Method (After 2008). Currently about 1500 securities listed on NSE. Table 1 and 2 lists the main features of the three futures contracts. Currently there are 10 futures indices trading in NSE. Only three indices (S&P CNS Nifty futures, CNXIT futures & CNX Bank futures) have selected for the study. Indices selected based on number of years their trading in NSE. The CNX Nifty Index futures contract is based on popular underlying index and market bench mark CNX Nifty Index, constitutes 50 major stocks and began trading on NSE on 12 June 2000. Average daily trading volume during the period of the study was 442492 contracts. The importance of CNX Nifty Index cannot be under rated as it constitutes 66.85% of free float market capitalization of NSE. This data is collated as on June 30, 2014. The CNXIT Index futures contract are
based on the underlying index of CNXIT Index, constitutes 20 major stocks from IT sector which trade on the National Stock Exchange and began trading on August 2003. Average daily trading volume during the period of the study was 305 contracts. Since CNX IT Index represents only the IT industry the overall representation to NSE is much lower than CNX Nifty. CNX IT index indicates 11.27% of the free float market capitalization of NSE and 97.25% of the free float market capitalization of the stocks constituting part of the IT sector as on June 30, 2014. The Bank Nifty Index futures contract based on the underlying index of CNX Bank Nifty Index constitutes 12 stocks from the banking sector which trade on the National Stock Exchange. As for the Bank Nifty index futures market the history is relatively short compared CNX Nifty Index futures. Began trading on June 2005 and Average daily trading volume during the period of the study was above 52007 contracts. Since CNX Bank Nifty index represents only the Bank industry, the overall representation to NSE is too much lower than CNX Nifty index. The CNX Bank Index represents about 15.55% of the free float market capitalization of the stocks listed on NSE and 89.90% of the free float market capitalization of the stocks constituting part of the Banking sector as on June 30, 2014.

Additionally as shown in the table 2, Nifty futures index having lowest frequency of negative basis (31.59%) during the sample period, followed by, bank nifty futures index having next lowest frequency of negative basis (34.40%) after Nifty futures and CNXIT futures index having highest frequency of negative basis (36.76%).NSE futures contracts have a maximum of 3-month trading cycle - one month (near), the two month (next) and the three month (far). A new futures contract is introduced on the immediate next trading day of the expiry of the near month contract. The new contract will be introduced for three month duration. This way, at any point in time, there will be 3 contracts available for trading in the market i.e., one near month, one second month and one far month duration respectively. All the three futures contracts mature on the last Thursday of every month. If the last Thursday of every month is happened to be a trading holiday, the contracts expire on immediate previous trading day. The futures contract is cash settle only.

1.2 Futures pricing Models
Two alternative futures pricing models are compared in the present study. i.) Cost of Carry Model (CCM) ii.) Hsu & Wang Model (HWM).

i.) Cost of Carry Model (CCM)

If dividend yield is non-stochastic, Cornell and French (1983) show that the index futures price can be estimated by

\[ F_t = S_t e^{(r-q)(T-t)} \]  \hspace{1cm} (1)

Where \( F_t \) is the theoretical futures price at time t for a contract that matures at a time T, \( S_t \) is the current stock price at time t; \( r \) is the annualized risk free interest rate (Cost of financing); \( q \) is constant annual dividend yield, \( T-t \) represents time to maturity.

ii.) Hsu & Wang Model (HWM),

Hsu & Wang (2004) incorporated price expectation parameter (\( u_0 \)) and developed futures pricing model in imperfect markets.

This study uses the following assumptions to derive a pricing model of stock index futures in imperfect markets: 
1. The underlying stock index pays a continuous constant dividend yield, \( q \), during the life of the futures contract. 2. The instantaneous degree of market imperfection remains constant throughout the life of the futures contract. 3. The underlying stock index price, \( S \), follows a geometric Wiener process, as follows:

\[ \frac{dP}{P} = (w_s u_t + u) \, dt + (w_s \sigma_t + \sigma) \, dZ \]  \hspace{1cm} (2)

Where \( P \) is the hedged portfolio, \( w_s \), \( S \) represents the price of the underlying stock index , \( F \) denotes the price of the futures index, \( U \) & \( \sigma \) represents constant expected growth rate and constant volatility of the underlying stock index (\( S \)) respectively. \( u_t \) & \( \sigma_t \) denotes the instantaneous expected return on future and instantaneous standard deviation of return on future respectively and \( dz \) is a geometric wiener process.

Further, If \( w_s \) then \( w_t \sigma_t + \sigma = 0 \). \( u_t \) & \( \sigma_t \) remain same but second part in equation 1 become zero. It indicates that, the hedged portfolio (\( P \)) can expected certainly and hedged portfolio becomes riskless. However in order to keep this portfolio risk free, it’s necessary to rebalance \( w_t \) continuously until expiration of the futures.
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Figlewski (1989) found that forming riskless portfolio hedge and continuously rebalancing hedged positions is only possible in perfect markets. Because of incomplete arbitrage mechanism and arbitrage process is exposed to heavy risk, the hedged portfolio is not possible to riskless at any point of time.

Let \( u_p \) & \( \sigma_p \) represents the instantaneous expected rate of return of the hedged portfolio \( P \) & the coefficient of variance process \( dz \) in the equation 1 respectively. This can be obtained as follows.

\[
\begin{align*}
\text{w}_1 u_p + u &= u_p \\
\text{w}_2 \sigma_p + \sigma &= \sigma_p
\end{align*}
\]

From equation 2 & 3 the result of partial differential equation can be obtained as follows

\[
\frac{1}{2} \sigma^2 S^2 F_{01} + u_0 SF_t + F_t = 0
\]

Where \( u_0 \) is the Hsu & Wang’s price expectation parameter \( u_0 = [u_p - q] - (u_q) \frac{P_0}{\sigma^2} \) / (1. \frac{P_0}{\sigma^2})

The second order partial differential equation 4 along with the following futures index price terminal condition at expiry date \( T \), fully characterize the futures index price.

\[
F(S, T) = S_t
\]

Finally the solution of this PDE is given by

\[
F(S, T) = S_t e^{u_\alpha(T - t)}
\]

Equation (6) is known as Hsu & Wang Futures pricing Model.

II. Data and Methodology

For the CNX Nifty futures, CNX IT futures and Bank Nifty futures contract, only near month (one month) contracts were considered for this study because the nearest maturity contracts have significant trading volume compared to next month (two months) & far month (three months) contracts. Daily closing prices were obtained from NSE for all the three futures indices for the period from 1 April 2007 to 31 March 2014. The 364-day government of India Treasury bill rates were used as proxy for risk free interest rates and obtained from RBI database. The annualized daily dividend yield obtained from NSE and has been used to estimate theoretical futures price from Cost of Carry model for all the three futures indices. Implied method is used to estimate price expectation parameter for Hsu & Wang model.

2.1 Hypothesis

\( H_0 \): There is no significant difference in MAPE statistics generated form Cost of Carry Model and Hsu & Wang Model.

Independent t test is used to test whether the MAPE statistics generated from each model is significantly different.

2.2 Measuring the pricing performance for the two models

Following Hsu & Wang (2004), pricing performance between Cost of Carry Model (CCM) and Hsu & Wang Model (HWM) can be measured by Calculating the mean absolute error (MAE), the mean percentage error (MPE) and mean absolute percentage error (MAPE) are illustrated as follows.

Pricing Error \( (\varepsilon) = AF_t - F_t \)

\[
\begin{align*}
\text{MAE} &= \frac{1}{N} \sum_{t=1}^{N} \left| AF_t - F_t \right| \\
\text{MPE} &= \frac{1}{N} \sum_{t=1}^{N} \left( \frac{AF_t - F_t}{AF_t} \right) \times 100 \\
\text{MAPE} &= \frac{1}{N} \sum_{t=1}^{N} \left( \frac{AF_t - F_t}{AF_t} \right) \times 100
\end{align*}
\]

Where \( AF_t \) is the actual price of stock index futures at time \( t \) and \( F_t \) is the theoretical price of stock index futures at time \( t \). Further, to compare the futures pricing error statistics between Hsu & Wang Model (HWM) and Cost of Carry Model (CCM).

2.3 Estimation of Price expectation parameter for Hsu & Wang model

Implied method: For Hsu & Wang Model in imperfect markets, only price expectation parameter \( u_\alpha \) cannot be estimated directly. This parameter \( u_\alpha \) can be estimated same as implied volatility in the black-scholes model using the actual futures prices. The spot index that pays constant dividend yield, the implied \( u_n \) time at \( t-1 \) can be obtained from eq (Hsu and Wang model)

\[
u_{t-1} = \frac{1}{t-(t-1)} \ln \frac{P_{t-1}}{S_t}
\]
III. Empirical Results

Table 3: Descriptive statistics of pricing errors of both the models for all the three futures indices.

<table>
<thead>
<tr>
<th>Futures Index</th>
<th>N</th>
<th>Mean (%)</th>
<th>SD(%)</th>
<th>Mean (%)</th>
<th>SD (%)</th>
<th>Mean (%)</th>
<th>SD (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CCM</td>
<td>1741</td>
<td>12.0680</td>
<td>7.92646</td>
<td>11.7802</td>
<td>-0.1484</td>
<td>0.0093</td>
</tr>
<tr>
<td></td>
<td>HWM</td>
<td>1740</td>
<td>7.701437</td>
<td>0.0093</td>
<td>0.2262</td>
<td>0.2262</td>
<td>0.1611</td>
</tr>
<tr>
<td></td>
<td>CCM</td>
<td>1741</td>
<td>23.77</td>
<td>15.8919</td>
<td>24.0362</td>
<td>-0.1460</td>
<td>0.0088</td>
</tr>
<tr>
<td></td>
<td>HWM</td>
<td>1740</td>
<td>15.2893</td>
<td>0.0088</td>
<td>0.2505</td>
<td>0.2505</td>
<td>0.1811</td>
</tr>
<tr>
<td></td>
<td>CCM</td>
<td>1741</td>
<td>15.34</td>
<td>10.6948</td>
<td>15.7949</td>
<td>-0.1620</td>
<td>0.0075</td>
</tr>
<tr>
<td></td>
<td>HWM</td>
<td>1740</td>
<td>11.4074</td>
<td>0.0075</td>
<td>0.3357</td>
<td>0.3357</td>
<td>0.2032</td>
</tr>
</tbody>
</table>

Source: Collected and Compiled by the Authors
Note: OP - Over Price, UP – Under Price; OP= -ve (Ft > AF), UP = +ve ; Ft < AF

Table 4: Results of statistical tests for difference in MAPE between the futures pricing models

<table>
<thead>
<tr>
<th>Futures Index</th>
<th>Pricing Models</th>
<th>N</th>
<th>t-value</th>
<th>Sig (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CCM vs HWM</td>
<td>1741 - 1740</td>
<td>59.839***</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1741 - 1740</td>
<td>107.064***</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1741 - 1740</td>
<td>96.998***</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note. *** Significant at the 1% Level.

3.1 Pricing performance of both the pricing models for all three futures indices

According to table 3, from the percentage error, CCM overprices all the three futures indices – Nifty futures, bank nifty futures and IT futures contract by an average of -0.1484%, -0.1460% and -0.1620% respectively. The largest overestimate of CCM is an average of -0.1620% for IT futures index. Further, HWM underprices all the three futures indices Nifty futures, Bank nifty futures and IT futures index by an average of 0.0093%, 0.0088% & 0.0075% respectively. The MAPE of HWM 0.1611%, 0.1811%, & 0.2032% is the lowest compare to MAPE of CCM for the three futures indices - Nifty, Bank & IT futures index respectively. Overall, on the basis of mean percentage error (MPE) & MAPE, the best model preferred is HWM than CCM. This result supports Hsu & Wang (2006) and Jianchung Wang (2009) for Taiwan Futures Exchange. Further the table 3 reports pricing performance statistics of two pricing models. The pricing performance of CNX Nifty futures contract is significantly better than that of Bank Nifty futures and CNX IT futures contract for both the pricing models. CNX Nifty futures contract with highest trading history and average trading volume has smallest pricing errors than Bank Nifty futures and CNX IT futures. Pricing performance statistics of two pricing models clearly indicates that the MAPE of both the pricing models is lowest for CNX Nifty futures index having highest average trading volume during the sample period (4, 42,492), followed by Bank nifty futures index having next highest average trading volume after Nifty futures index (52,007) and then highest MAPE for CNXIT futures index having the lowest average trading volume of only 306.

From table 4, Independent t test is used to test whether the MAPE statistics generated from each model is significantly different. It’s clearly indicates that for all the three futures indices – CNX Nifty, Bank nifty and CNX IT futures index, the MAPE statistics generated from each model is statistically significant at 1%. Thus, The Mean Absolute Percentage Error (MAPE) generated from each model is statistically different.

Additionally as shown in the table 2 and 3, the MAPE of CCM is lowest for Nifty futures index having lowest frequency of negative basis (31.59%) during the sample period, followed by, bank nifty futures index having next lowest frequency of negative basis (34.40%) after nifty futures and then highest MAPE for CNXIT futures index having highest frequency of negative basis (36.76%). This result implies that frequency of negative basis might influence performance of CCM for all the three futures indices.
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Figure 1: Percentage Errors Cost of Carry Model and Hsu & Wang Model for CNX Nifty Futures Index

Figure 2: Percentage Errors Cost of Carry Model and Hsu & Wang Model for Bank Nifty Futures Index

Figure 3: Percentage Errors Cost of Carry Model and Hsu & Wang Model for CNX IT Futures Index

Figures 1 to 3, plot the percentage errors Cost of Carry Model and Hsu & Wang Model for all the three futures indices. It clearly shows that Percentage errors of Cost of Carry Model much higher than Hsu & Wang Model for all the three futures Indices. Finally, from table 3 and figures 1 to 3, indicate that Cost of Carry model overprices and Hsu & Wang Model underprices all the three futures markets.

IV. Conclusion

The study pertains to predict index futures prices using two alternative pricing models – Cost of Carry Model and Hsu & Wang Model (2004) for three futures indices of National Stock Exchange (NSE), India – CNX Nifty futures, Bank Nifty futures and CNX IT futures. Overall the Hsu & Wang Model with an argument of incomplete arbitrage mechanism and real capital markets are imperfect, provides much better pricing performance than the standard Cost of Carry Model with an assumption of completion of arbitrage mechanism and capital markets are perfect for all the three futures markets. This result implies that Indian futures markets are imperfect and arbitrage process cannot complete. Degree of market imperfection might influence the pricing error. Therefore, investors should know the degree of market imperfection of the futures markets in which they
would like to participate. CNX Nifty futures contract with highest trading history and average trading volume
has smallest pricing errors than Bank Nifty futures and CNX IT futures.CNX Nifty futures contract has lowest
MAPE, followed by Bank Nifty and CNXIT futures contract for both the futures pricing models. It implies that
trading volume influences pricing performance of the futures market. The study suggests further research of
investigating degree of market imperfection derived by Hsu and Wang (2004) for Indian futures market and
impact of degree of market imperfection on pricing performance of Indian futures markets.

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