Design of Model Predictive Controller for Level Process

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Abstract: The objective of this study is to investigate the Model predictive control (MPC) strategy, analyze and compare the control effects with Proportional-Integral-Derivative (PID) control strategy in maintaining a water level system. An advanced control method, MPC has been widely used and well received in a wide variety of applications in process control, it utilizes an explicit process model to predict the future response of a process and solve an optimal control problem with a finite horizon at each sampling instant. In this project, we first designed and built up a closed-loop water level system. Next, we modeled the system and linearized the model for simplification in the analysis and design. Then, we implemented the model in a simulation environment based on MATLAB. We tried both MPC and PID control methods to design the controller for the water level system, and compared the results in terms of peak time, settling time, overshoot, and steady-state error under various operational conditions including time delays. The results showed the advantage of MPC for dealing with the system dynamic over PID and could be designed for more complex and fast system dynamics even in presence of constraints.

Keywords: Model Predictive Control (MPC), FOPDT (First Order Plus Delay Time), Proportional Integral Derivative (PID), SOPDT (Second Order Plus Delay Time).

I. Introduction

Due to the fast development of process industry, the requirements of higher product quality, better product function, and quicker adjustments to the market change have become much stronger, which lead to a demand of a very successful controller design strategy, both in theory and practice. As a closed loop optimal control method based on the explicit use of a process model, model predictive control has proven to be a very effective controller design strategy over the last twenty five years and has been widely used in Process industry such as oil refining, chemical engineering and metallurgy. The purpose of this work is to study the theory of model predictive control method, analyze and indentify the characteristics and the performance of model predictive controller compared with PID controller when being implemented in the water level control system. PID controller is relatively simple in structure which can be easily implemented in practice. Therefore, it is widely used in process control industry. In this report, simple methods proposed by Ziegler-Nichols [1], Astrom Hagglund [8] is implemented for the real time measurement of laboratory Level control system. System model for laboratory level control system using system identification toolbox of MATLAB 7.1 version is determined and this level loop is configured with SCADA. Controller performance is determined on the basis of time domain specification. Existing control loop uses PID controller more than 90%. Since 1940’s many methods are proposed to tune PID controller but every method have some limitations. As a result, the design of PID controller still remains a challenge before researchers and engineers.

II. Proportional – Integral – Derivative (PID) Control

A proportional–integral–derivative controller (PID controller) is a controller which is popularly used in industrial control systems. It is fed with the error signal, that is, the difference between the reference, or the desired output and the actual output (which is obtained as a feedback). The controller then attempts to bring the actual output to track the reference. The structure of PID controller is showed in fig 1 [2].

![Fig. 1 Structure of PID Control](image-url)

The PID controller algorithm involves three separate constant parameters (proportional, integral and derivative) which are typically adjusted to achieve the desired performance characteristics.
Model Predictive Control (MPC) includes a class of control algorithms that utilize an explicit process model to predict the future response of a plant. At each control interval an MPC algorithm attempts to optimize future plant behavior by computing a sequence of future manipulated variable adjustments. The first input in the optimal sequence is then sent into the plant, and the entire calculation is repeated at subsequent control intervals. The following is a figure2 shows the basic idea of predictive control based on a single-input, single output plant.
The objective of model predictive control law is to drive future plant outputs $y(t+k|t)$ as close as $w$ as shown in figure 2.

**Objective Function Optimization Problem**

The term optimization implies a best value for some type of performance criterion. This performance criterion is known as an objective function. Here, we first discuss possible objective functions, then possible process models that can be used for MPC.

Here, there are several different choices for objectives functions. The first one that comes to mind is a standard least-squares or "quadratic "objective function. The objective function is a "sum of squares " of the predicted errors (differences between the set points and model-predicted outputs) and the control moves (changes in control action from step to step)

A quadratic objective function for a prediction horizon of 3 and a control horizon of 2 can be written

$$
\Phi = (R_k + 1 - \hat{y}_k + 1)^2 + (R_k + 2 - \hat{y}_k + 2)^2 + (R_k + 3 - \hat{y}_k + 3)^2 + w \Delta u_k^2 + w \Delta u_k + 1^2
$$

(2)

Where $\hat{y}$ represents the model predicted output, $r$ is the set point, $\Delta U$ is the change in manipulated input from one sample to the next, $w$ is a weight for the changes in the manipulated input, and the subscripts indicate the sample time ($k$ is the current sample time ). For a prediction horizon of $P$ and a control horizon of $M$, the least squares objective function is written

$$
\Phi = \sum (R_k + 1 - \hat{y}_k + 1)^2 + w \sum \Delta u_k + 1^2
$$

(3)

Another possible objective function is to simply take a sum of the absolute values of the predicted errors and control moves. For a prediction horizon of 3 and a control horizon of 2, the absolute value objective function is

$$
\Phi = |R_k + 1 - \hat{y}_k + 1| + |R_k + 2 - \hat{y}_k + 2| + |R_k + 3 - \hat{y}_k + 3| + w |\Delta u_k| + w |\Delta u_k + 1|
$$

(4)

Which has the following general form for a prediction horizon of $P$ and a control horizon of $M$:

$$
\Phi = \sum |(R_k + 1 - \hat{y}_k + 1)| + |\Delta u_k + 1|
$$

(5)

The optimization problem solved stated as a minimization of the objective function, obtained by adjusting the M control moves, subject to modeling equations (equality constraints), and constraints on the inputs and outputs.

**Min $\Phi$**

Least-squares formulations are by far the most common objective functions in MPC. Least squares yields analytical solutions for unconstrained problems and penalizes larger errors (relatively) more than smaller errors. The absolute value objective function has been used in a few algorithms because linear programming (LP) problem results. LPs are routinely solved in large-scale scheduling and allocation problems. For example, an oil company often uses an LP to decide how to distribute oil to various refineries.
and to decide how much and what product to produce at each plant. The LP approach is not useful for model predictive control, because the manipulated variable moves often “hop” from one extreme constraint to another.

IV. Description Of Level Control Process

Level control process is designed for understanding the basic principles of level control. The process setup consists of supply water tank fitted with pump for water circulation. The level transmitter used for level sensing is fitted on transparent process tank. The process parameter (level) is controlled by microprocessor based digital indicating controller which manipulates pneumatic control valve through I/P converter. A pneumatic control valve adjusts the water flow in to the tank. These units along with necessary piping are fitted on support housing designed for tabletop mounting. The controller can be connected to computer through USB port for monitoring the process in SCADA mode. Fig. 3 explores the system schematic arrangement of Level Control System.

![System schematic arrangement of level control system](image)

**Figure. 3.** System schematic arrangement of level control system

**Determination of Process Model**

A process model is a system of mathematical equations and constants that are usually solved on a computer to make quantitative predictions about some aspect(s) of a real process. The specific variables required as input data and generated as output predictions are important features of the model. The equations often stem from a numerical solution to one or more differential equations and their boundary conditions. In the design of model based controller, system model is an important element. White box model requires complete and correct physical data of the system under consideration. But this data is not available for the system described. Hence, system model is determined through system identification. We used time domain step test data from the system for determination of model. We considered FOPDT model [4] [8].

This step response locates the system parameters like steady state gain, time delay and the time constant of the process from which model obtained is of general form as,

\[
G_s = \frac{Kp e^{-\tau_d s}}{1 + \tau s}
\]

(6)

Where, \(Kp\) is steady state gain of system, \(\tau\) is time constant of system, \(\tau_d\) is dead time of system.

![Step response of FOPDT system](image)

**Fig. 4.** A typical step response of FOPDT system

Figure 5 shows a typical step response of SOPTD system. This step response locates the system parameters like peak overshoot, settling time, dead time of the system from which the model can be obtained as,
Where, $\omega_n$ is natural frequency of system, $\xi$ is damping ration of system, $t_d$ is dead time of system.

$$G(s) = \frac{\omega_n^2 e^{-t_d s}}{s^2 + 2\xi \omega_n s + \omega_n^2}$$  \hspace{2cm} (7)

The system parameters $\omega_n$ and $\xi$ are calculated from peak overshoot $M_p$ and settling time (2\% criterion) $t_s$ by solving (3) and (4) given by,

$$M_p = e^{\sqrt{1-\xi^2}} \hspace{2cm} (8)$$

$$t_s = \frac{4}{\xi \omega_n} \hspace{2cm} (9)$$

In the given case, from the open loop response of the Level Control System, it is seen that by measuring input–output data we can create the mathematical models of dynamic systems from measured input-output data by using system Identification Toolbox in MATLAB. The following estimate of the plant is obtained by using system Identification Toolbox:

$$G_p(s) = -0.22 \frac{e^{-1.69s}}{(1 + 26.43s)} \hspace{2cm} (10)$$

V. Simulation Results & Discussion

The simulation results for PID controller tuning by Ziegler-Nichols & Astrom Hagglund methods for FOPDT model (10) obtained for Level control system is shown in figure 7
The step response of the proposed MPC controller with the control horizon $M=2$, prediction horizon, $P=10$ without manipulated variable constraint and output variable constraint is shown in Fig. 8.

Table 1 shows the result of response of controller we have taken for analysis, using simulation process. These controllers have different responses for the input taken as Step. After simulation we have find that these entire controller have different value of parameters such as peak time $t_p$, settling time $t_s$, maximum overshoot ($M_p$), and steady state error $e_{ss}$. In the analysis we have seen that more accurate result came using Astrom Hagglund PID Controller over Ziegler-Nichols PID controller, further better result got in case of MPC Controller. Table 1 show that MPC controller gives better time domain specifications than PID Controller.
VI. Conclusion

A high performance Model based Predictive Control algorithm is proposed for the level Control process. The MPC control algorithm is compared with conventional PID control in terms of time domain specifications like settling time, overshoot, Peak time, steady state error. The Model Predictive Controller gives better performance than PID Controller for the level control system. MPC controller can adjust the control action before a change in the output set point actually occurs. Hence from the results we conclude that MPC is better than PID controller.

References