Fuzzy-Genetic Algorithm based inventory model for shortages and inflation under hybrid & PSO

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Abstract: The purpose of this article is to evaluate the value of integrating inventory decision. Therefore in this paper a inventory model for deteriorating items is considered under assumption that the Inventory cost (including Fuzzy holding cost and Fuzzy deterioration cost), Fuzzy Transportation cost is incurred and is function of distance irrespective of number of items and the capacity of vehicle. Four kinds of meta-heuristic algorithms or meta-heuristic Fuzzy hybrid algorithms: Fuzzy harmony search, Fuzzy particle swarm optimization, Fuzzy genetic algorithms, Fuzzy simulated annealing. A numerical example is presented to illustrate and validate the model and sensitivity analysis is carried out in case-1 for different values of a parameter keeping rest unchanged.

I. Introduction

Fuzzy-Genetic Algorithm

Genetic algorithm is a potential tool for global optimization, fuzzy logic is a powerful tool for dealing with imprecision and uncertainty and neural network is an important tool for learning and adaptation. However each of these tools has its inherent limitations. Combined techniques are developed to remove the limitations of constituent tools and at the same time to utilize their strengths. A large number of combined techniques (also known as soft computing techniques) have been developed to solve a variety of problems. These techniques include fuzzy-genetic algorithm, genetic-fuzzy systems, genetic-neural systems and other.

In a fuzzy-genetic algorithm a fuzzy logic technique is used to improve the performance of a genetic algorithm. A GA is found to be an efficient tool for global optimization but its local search capability is seen to be poor. On the other hand an FLC is a powerful tool for local search. Therefore the global search power of a GA may be combined with the local search capability of an FLC to develop an FGA. Moreover the performance of a GA depends on its parameters such as probability of crossover, probability of mutation, population size etc. And an FLC may be used to control these parameters.

Hybrid Genetic Algorithms

The three main pillars of soft computing are fuzzy logic artificial neural networks and evolutionary search particularly genetic algorithms all of these have been successfully applied in isolation to solve practical problems in various fields where conventional techniques have been found inadequate. However these techniques are complementary to each other and they may work synergistically combining the strengths of more than one of these techniques as the situation may occasionally demand. Hybrid systems are systems where several techniques are combined to solve a problem. Needless to say that such amalgamation must be used only if it returns better results than any of these techniques in isolation. Based on how or more systems are combined hybrid systems have been classified into three broad categories viz sequential, auxiliary and embedded hybrid systems. Three primary types of hybrid systems are briefly discussed in this paper these are neuro-genetic, neuro-fuzzy and fuzzy-genetic hybrid systems. Hybrid systems integrating all three techniques, viz, fuzzy logic neural network, and genetic algorithms are also implemented in several occasions.

Particle swarm optimization Algorithm

Particle Swarm Optimization (PSO) introduced by Kennedy and Eberhart in 1995 is a population based evolutionary computation technique. It has been developed by simulating bird flocking fish schooling or sociological behaviour of a group of people artificially. Here the population of solution is called swarm which is composed of a number of agents known as particles. Each particle is treated as point in d-dimensional search space which modifies its position according to its own flying experience and that of other particles present in the swarm. The algorithm starts with a population (swarm) of random solutions (particles). Each particle is assigned
a random velocity and allowed to move in the problem space. The particles have memory and each of them keeps track of its previous (local) best position.

\[ U^b \text{ particle’s velocity} \]
\[ Y^b \text{ particle’s position} \]
\[ b \text{ number of elements in a particle,} \]
\[ q \text{ inertia weight of the particle,} \]
\[ m \text{ generation number,} \]
\[ a_1, a_2 \text{ acceleration constants,} \]
\[ e \text{ and } \text{random value between 0 and 1} \]
\[ R_{pbest}^b \text{ local best position of the particle,} \]
\[ R_{gbest}^b \text{ global best position of particle in the swarm} \]

\[ U_n^b = q \times U_{n-1}^b + a_1 \times e \times d_1 \times [U_{pbest}^b - Y_{n-1}^b] + a_2 \times e \times d_2 \times [U_{gbest}^b - Y_{n-1}^b] \]
\[ Y_n^b = Y_{n-1}^b + U_n^b \]

Concept of modification of a searching point by PSO

II. Inventory

In the classical inventory it is assumed that all the costs associated with the inventory system remains constant over time. Since most decision makers think that the inflation does not have significant influence on the inventory policy and most of the inventory models developed so far does not include inflation and time value of money as parameters of the system. But due to large scale of inflation the monetary situation in almost of the countries has changed to an extent during the last thirty years. Nowadays inflation has become a permanent feature in the inventory system. Inflation enters in the picture of inventory only because it may have an impact on the present value of the future inventory cost. Thus the inflation plays a vital role in the inventory system and production management though the decision makers may face difficulties in arriving at answers related to decision making. At present, it is impossible to ignore the effects of inflation and it is necessary to consider the effects of inflation on the inventory system.

III. Related work

Buzacott (1975) developed the first EOQ model taking inflationary effects into account. In this model, a uniform inflation was assumed for all the associated costs and an expression for the EOQ was derived by minimizing the average annual cost. Misra (1975, 1979) investigated inventory systems under the effects of inflation. Bierman and Thomas (1977) suggested the inventory decision policy under inflationary conditions. An economic order quantity inventory model for deteriorating items was developed by Bose et al. (1995). Authors developed inventory model with linear trend in demand allowing inventory shortages and backlogging. The effects of inflation and time-value of money were incorporated into the model. Hariga and Ben-Daya (1996) then discussed the inventory replenishment problem over a fixed planning horizon for items with linearly time-varying demand under inflationary conditions. Ray and Chaudhuri (1997) developed a finite time-horizon deterministic economic order quantity inventory model with shortages, where the demand rate at any instant
depends on the on-hand inventory at that instant. The effects of inflation and time value of money were taken into account. The effects of inflation and time-value of money on an economic order quantity model have been discussed by Moon and Lee (2000). The two-warehouse inventory models for deteriorating items with constant demand rate under inflation were developed by Yang (2004). The shortages were allowed and fully backlogged in the models. Some numerical examples for illustration were provided. Models for ameliorating / deteriorating items with time-varying demand pattern over a finite planning horizon were proposed by Moon et al. (2005). The effects of inflation and time value of money were also taken into account. An inventory model for deteriorating items with stock-dependent consumption rate with shortages was produced by Hou (2006). Model was developed under the effects of inflation and time discounting over a finite planning horizon. Jaggi et al. (2007) presented the optimal inventory replenishment policy for deteriorating items under inflationary conditions using a discounted cash flow (DCF) approach over a finite time horizon. Shortages in inventory were allowed and completely backlogged and demand rate was assumed to be a function of inflation. Two stage inventory problems over finite time horizon under inflation and time value of money was discussed by Dey et al. (2008).

The concept of soft computing techniques (fuzzy logic) first introduced by Zadeh (1965). The invention of soft computing techniques (fuzzy set theory or fuzzy logic) by the need to represent and capture the real world problem with its fuzzy data due to uncertainty. Instead of ignoring or avoiding uncertainty, Zadeh developed a set theory to remove this uncertainty. It is to use hybrid intelligent methods to quickly achieve an inexact solution rather than use an exact optimal solution via a big search. Since Genetic Algorithms are good for adaptive studies and fuzzy logic can be used to solve complex problems using linguistic rule-based techniques. Silver and Peterson (1985) discussed on decision systems for inventory management and production planning. Zimmermann (1985) gives a review on fuzzy set theory and its applications. Bard and Moore (1990) discussed a model for production planning with variable demand. Avraham (1999) presented a review on enterprise resource planning (ERP).


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IV. Assumptions and Notations

The notations and basic assumptions of the model are as follows:

- $\bar{a}_1$ is Fuzzy the ordering cost per cycle.
- $\bar{b}_1$ is Fuzzy the constant deterioration cost per unit item.
- $(a + b) + \gamma t$ is Fuzzy the time dependent inventory holding cost per unit per unit time.
- $\bar{c}_1$ is Fuzzy the constant purchase cost per unit item.
- $\bar{d}$ is Fuzzy the backordering cost per unit per unit time.
- $\bar{e}$ is Fuzzy the opportunity cost per unit.
- $s$ is Fuzzy the selling price per unit item.
- $D(s)$ represents Fuzzy the price-dependent demand rate, where $D(s) = (u + 1)s^{-\gamma(v+1)}$, $(u + 1) > 0$, $(v + 1) > 1$ mark up elasticity.
- $\theta(t)$ is Fuzzy the time-proportional decay rate of the stock defined as $\theta(t) = (a + \beta)t$, $0 < (a + \beta) < 1$. Since $(a + \beta) > 0$, $(d\theta(t)/dt) = (a + \beta) > 0$. Hence the decay rate increases with time at a rate $\beta$.
- $r$ is Fuzzy the inflation rate.
- $e^{-\omega t}$ is Fuzzy the time dependent backlogging rate with $\delta \geq 0$.

Let $I(t)$ be the inventory level at time $t$ ($0 \leq t \leq t_2$ ). During the time interval $[0, t_1]$ inventory level decreases due to the combined effect of Fuzzy demand and Fuzzy deterioration both and at $t_1$ inventory level depletes up to zero. The Fuzzy differential equation to describe immediate state over $[0, t_1]$ is given by

$$\frac{dI(t)}{dt} = -(a + \beta)t I(t) - (u + 1) s^{-(v+1)}$$

(1)

Now, during time interval $[t_1, t_2]$ Fuzzy shortages stars occurring and at $t_2$ there are Fuzzy maximum shortages, due to Fuzzy partial backordering some sales are lost. The Fuzzy differential equation to describe instant state over $[t_1, t_2]$ is given by

$$\frac{dI(t)}{dt} = -e^{-\omega (t_2 - t)} (u + 1) s^{-(v+1)}$$

with boundary condition $I(t_1) = 0$

(2)

The solutions of the Fuzzy differential Equations 1, 2 are as follows:

$$I(t) = \left[\left(u + 1\right) s^{-(v+1)} \right] \left[\left(t_1 + \frac{(a+\beta)t_1}{6}\right) - \left(t + \frac{(a+\beta)t^3}{6}\right)\right] e^{-\frac{(a+\beta)t^2}{2}}$$

(3)

$$I(t) = \left[\left(u + 1\right) s^{-(v+1)} \right] \left[ e^{-\omega (t_2-t_1)} - e^{-\omega (t_2-t)} \right]$$

(4)

The inventory level at time $t = 0$ is $I(0)$ and is given by

$$I(0) = \left[\left(u + 1\right) s^{-(v+1)} \right] \left[ t_1 + \frac{(a+\beta)t_1}{6} \right]$$

(5)

The maximum Fuzzy shortages occurs at time $t = t_2$ is $I(t_2)$ and is given by

$$I(t_2) = \left[\left(u + 1\right) s^{-(v+1)} \right] \left[ e^{-\omega (t_2-t_1)} - 1 \right]$$

(6)

The total Fuzzy order quantity is $Q$ and is given by

$$Q = I(0) + [-I(t_2)]$$

(7)

Now, total Fuzzy average cost consists of the following costs

A. Fuzzy ordering cost is

$$\bar{O}_C = \bar{a}_1$$

(9)

B. Fuzzy Deteriorating cost is DC and is given by

$$\bar{D}_C = \bar{b}_1 \int_0^{t_1} (a + \beta)t I(t)e^{\gamma t} dt$$

(10)

C. Fuzzy Holding cost is HC and is given by

$$\bar{H}_C = \int_{t_1}^{t_2} [(a + \beta) + \gamma t] I(t)e^{\gamma t} dt$$

(11)

D. Fuzzy Purchasing cost is PC and is given by

$$\bar{P}_C = \bar{c}_1 \left[ \left(u + 1\right) s^{-(v+1)} \left(t_1 + \frac{(a+\beta)t_1}{6}\right) - \left\{\left(u + 1\right) s^{-(v+1)} \left(e^{-\omega (t_2-t_1)} - 1\right)\right\} e^{\gamma t} \right]$$

(12)

E. Fuzzy Shortage cost is SC and is given by

$$\bar{S}_C = \bar{d} \int_{t_1}^{t_2} \left[-I(t)\right] e^{\gamma t} dt$$

(13)

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F. Fuzzy Lost sales cost is LC and is given by
\[ \tilde{L}_c = \tilde{e} \int_{t_1}^{t_2} \left\{ (u + 1) s^{-v} \left( 1 - e^{-w(t_2-t_1)} \right) \right\} e^{rt} dt \]  \hspace{1cm} (14)

G. Fuzzy Total Cost and is given by
\[ \bar{T}_C = \frac{1}{s} \left[ C_0 + D_C + P_C + S_C + L_C \right] \]  \hspace{1cm} (15)

Our objective is to minimize the total cost function \( \bar{T}_C(t_1, t_2) \). The necessary conditions for minimizing the total cost are
\[ \frac{\partial \bar{T}_C(t_1, t_2)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial \bar{T}_C(t_1, t_2)}{\partial t_2} = 0 \]

Using the software Mathematica-8.0, from these two equations we can determine the optimum values of \( t_1^* \) and \( t_2^* \) simultaneously and the optimal value \( \bar{T}_C(t_1^*, t_2^*) \) of the total average cost can be determined by (15) provided they satisfy the sufficiency conditions for minimizing \( \bar{T}_C(t_1^*, t_2^*) \) are
\[ \frac{\partial^2 \bar{T}_C(t_1, t_2)}{\partial t_1^2} > 0, \quad \frac{\partial^2 \bar{T}_C(t_1, t_2)}{\partial t_2^2} > 0 \]
And
\[ \frac{\partial^2 \bar{T}_C(t_1, t_2)}{\partial t_1 \partial t_2} - \left( \frac{\partial^2 \bar{T}_C(t_1, t_2)}{\partial t_1^2} \right) \left( \frac{\partial^2 \bar{T}_C(t_1, t_2)}{\partial t_2^2} \right)^2 > 0 \]

Fuzzy the ordering cost=25, Fuzzy the constant deterioration cost =1.0, Fuzzy the time dependent inventory holding cost = 0.6, Fuzzy the constant purchase cost = 30 Fuzzy the backordering cost = 16, Fuzzy the opportunity cost = 40, Fuzzy the inflation rate = 0.30, Fuzzy the time dependent backlogging rate =0.6, \( \alpha = 7.2, u + v = 1014, v + 1 = 7.2, \gamma = 0.16, s=1.2 \)

Solving equations given in (A) we get \( t_1^* =0.846464, t_2^* =0.944693, Q^* =62.8884 \) and \( TC(t_1^*, t_2^*) =816.485 \).

Table, columns represent the number of products, the products, the average of 7 times running for Particle Swarm Optimization and fuzzy simulation, the average of 7 times running for Genetic Algorithm and fuzzy simulation, the standard division of 7 times running for Particle Swarm Optimization and fuzzy simulation, the standard division of 7 times running for Genetic Algorithm and fuzzy simulation, the running time for Simulated Annealing, Harmony Search, Particle Swarm Optimization and Genetic Algorithm and fuzzy simulation, respectively.

Since the model in 1 is integer in nature, reaching an analytical solution (if any) to the problem is difficult (Gen and Cheng, 1997). So we need to use meta heuristic algorithms. The proposed model for six products is solved using meta-heuristic approach, four hybrid intelligent algorithms of harmony search-fuzzy simulation (HS-FS) Taleizadeh and Niaki (2009), particle swarm optimization-fuzzy simulation (PSO-FS) Taleizadeh et al. (2009), simulated annealing-fuzzy simulation (SA-FS) Taleizadeh et al. (2009), and genetic algorithm-fuzzy simulation (GA-FS) Taleizadeh et al. (2009).

A comparison of the results in Table 1 & 2 shows the PSOFS hybrid algorithm performs better than the GA-FS and SA-FS algorithms in terms of the objective function values, and the proposed HS-FS method of this research performs the best. In the term of CPU time the expected values of SA-FS, GA-FS, PSO-FS and HS-FS are respectively 67, 59, 59 and 52 seconds showing HS-FS performs better than other do.

<table>
<thead>
<tr>
<th>Hybrid Algorithms</th>
<th>Product's Maximum inventory Level</th>
<th>Maximum profit (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated Annealing and fuzzy simulation</td>
<td>1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>Genetic Algorithm and fuzzy simulation</td>
<td>650.5 964 4078 3747.5 3073.5 3623.5 8224</td>
<td>504578.5</td>
</tr>
<tr>
<td>Particle Swarm Optimization and fuzzy simulation</td>
<td>734.2 871 5071 5073 4073 4621.5 8223</td>
<td>406916.5</td>
</tr>
<tr>
<td>Harmony Search and fuzzy simulation</td>
<td>675 962 4074 4072 3025 3601 8216</td>
<td>612595</td>
</tr>
</tbody>
</table>

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Table: 2 Best results of purpose function by fuzzy based different algorithms for decreasing demand.

<table>
<thead>
<tr>
<th>Hybrid Algorithms</th>
<th>Product’s Maximum inventory Level</th>
<th>Maximum profit (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated Annealing and fuzzy simulation</td>
<td>39.5  8.5  50.5  43.5  27.5  39.5  52.5</td>
<td>79160</td>
</tr>
<tr>
<td>Genetic Algorithm and fuzzy simulation</td>
<td>45  42  70  60  30  211  64</td>
<td>81135</td>
</tr>
<tr>
<td>Particle Swarm Optimization and fuzzy simulation</td>
<td>65  32  70  60  30  126.5  62</td>
<td>97822</td>
</tr>
<tr>
<td>Harmony Search and fuzzy simulation</td>
<td>55  52  70  60  30  19.5  72</td>
<td>202475</td>
</tr>
</tbody>
</table>

V. Conclusion
In this study, we have proposed a deterministic inventory model for fuzzy deteriorating items fuzzy time varying demand and fuzzy holding cost varying with respect to ordering cycle length with the objective of minimizing the total inventory cost. Fuzzy Shortages are allowed and fuzzy partially backlogged. Furthermore the proposed model is very useful for fuzzy deteriorating items. This model can be further extended by incorporation with other deterioration rate probabilistic demand pattern. To solve the models, four kinds of fuzzy meta-heuristic algorithms or fuzzy meta-heuristic hybrid algorithms: harmony search, fuzzy particle swarm optimization, fuzzy genetic algorithms, fuzzy simulated annealing. Computational results showed that HS, hybrid of HS and hybrid method of HS performed the best, based on objective function values respect to other kind of the fuzzy algorithms. Other applications of HS in inventory and supply chain managements can be extended to consider pricing problems.

References:


