A Study on Learning Factor Analysis – An Educational Data Mining Technique for Student Knowledge Modeling

S. Lakshmi Prabha¹, Dr. A.R. Mohamed Shanavas²

¹Ph.D Research Scholar, Bharathidasan University & Associate professor, Department of Computer Science, Seethalakshmi Ramaswami College, Tiruchirappalli, Tamilnadu, India,
²Associate professor, Department of Computer Science, Jamal Mohamed College, Tiruchirappalli, Tamilnadu, India,

Abstract: The increase in dissemination of interactive e-learning environments has allowed the collection of large repositories of data. The new emerging field, Educational Data Mining (EDM) concerns with developing methods to discover knowledge from data collected from e-learning and educational environments. EDM can be applied in modeling user knowledge, user behavior and user experience in e-learning platforms. This paper explains how Learning Factor Analysis (LFA), a data mining method is used for evaluating cognitive model and analyzing student-tutor log data for knowledge modeling. Also illustrates how learning curves can be used for visualizing the performance of the students.

Keywords: e-learning, Educational Data Mining (EDM), Learning Factor Analysis (LFA)

I. Introduction

Educational Data Mining is an inter-disciplinary field utilizes methods from machine learning, cognitive science, data mining, statistics, and psychometrics. The main aim of EDM is to construct computational models and tools to discover knowledge by mining data taken from educational settings. The increase of e-learning environments such as interactive learning environments, learning management systems (LMS), intelligent tutoring systems (ITS), and hypermedia systems, as well as the establishment of school databases of student test scores, has created large repositories of data that can be explored by EDM researchers to understand how students learn and find out models to improve their performance.

Baker [1] has classified the methods in EDM as: prediction, clustering, relationship mining, distillation of data for human judgment and discovery with models. These methods are used by the researchers [1][2] to find solutions for the following goals:

1. Predicting students’ future learning behavior by creating student models that incorporate detailed information about students’ knowledge, meta-cognition, motivation, and attitudes.
2. Discovering or improving domain models that characterize the content to be learned and optimal instructional sequences.
3. Studying the effects of different kinds of pedagogical support that can be provided by learning software, and
4. Advancing scientific knowledge about learning and learners through building computational models that incorporate models of the student, the software’s pedagogy and the domain.

The application areas [3] of EDM are: 1) User modeling 2) User grouping or Profiling 3) Domain modeling and 4) trend analysis. These application areas utilize EDM methods to find solutions. User modeling [3] encompasses what a learner knows, what the user experience is like, what a learner’s behavior and motivation are, and how satisfied users are with online learning. User models are used to customize and adapt the system behaviors’ to users specific needs so that the systems ‘say’ the ‘right’ thing at the ‘right’ time in the ‘right ‘way [4]. This paper concerns with applying EDM method Learning factor Analysis (LFA) for User Knowledge Modeling. This paper is organized as follows: section 2 lists the related works done in this research area; section 3 explains LFA method used in this research; section 4 describes methodology used, section 5 discusses the results and section 6 concludes the work.

II. Literature Review

A number of studies have been conducted in EDM to find the effect of using the discovered methods on student modeling. This section provides an overview of related works done by other EDM researchers.

Newell and Rosenbloom[5] found a power relationship between the error rate of performance and the amount of practice. Corbett and Anderson [6] discovered a popular method for estimating students’ knowledge is knowledge tracing model, an approach that uses a Bayesian-network-based model for estimating the probability that a student knows a skill based on observations of him or her attempting to perform the skill. Baker et.al [7] have proposed a new way to contextually estimate the probability that a student obtained a correct answer by guessing, or an incorrect answer by slipping, within Bayesian Knowledge Tracing. Koedinger

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et. al [8] demonstrated that a tutor unit, redesigned based on data-driven cognitive model improvements, helped students reach mastery more efficiently. It produced better learning on the problem-decomposition planning skills that were the focus of the cognitive model improvements. Stamper and Koedinger [9], presented a data-driven method for researchers to use data from educational technologies to identify and validate improvements in a cognitive model which used Knowledge or skill components equivalent to latent variables in a logistic regression model called the Additive Factors Model (AFM). Brent et. al [10] used learning curves to analyze a large volume of user data to explore the feasibility of using them as a reliable method for fine tuning adaptive educational system. Feng et. al[11], addressed the assessment challenge in the ASSISTment system, which is a web-based tutoring system that serves as an e-learning and e-assessment environment. They presented that the on line assessment system did a better job of predicting student knowledge by considering how much tutoring assistance was needed, how fast a student solves a problem and how many attempts were needed to finish a problem. Saranya et. al [12] proposed system regards the student’s holistic performance by mining student data and Institutional data. Naive Bayes classification algorithm is used for classifying students into three classes – Elite, Average and Poor. Koedinger, K.R.,[13] Professor, Human Computer Interaction Institute, Carnegie Mellon University. Pittsburgh has done lot to this EDM research. He developed cognitive models and used students interaction log taken from the Cognitive Tutors, analyzed for the betterment of student learning process Better assessment models always result with quality education.

Assessing student’s ability and performance with EDM methods in e-learning environment for math education in school level in India has not been identified in our literature review. Our method is a novel approach in providing quality math education with assessments indicating the knowledge level of a student in each lesson.

III. Learning Factor Analysis

User modeling or student modeling identifies what a learner knows, what the learner experience is like, what a learner’s behavior and motivation are, and how satisfied users are with e-learning. Item Response Theory and Rasch model [20] is Psychometric Methods to measure students’ ability. They lack in providing results that are easy to interpret by the users. This paper deals with identifying learners’ knowledge level (knowledge modeling) using LFA in an e-learning environment.

LFA is an EDM method for evaluating cognitive models and analysing student-tutor log data. LFA uses three components: 1) Statistical model – multiple logistic regression model is used to quantify the skills.

2) Human expertise- difficulty factors (concepts or KCs) defined by the subject experts (teachers): a set of factors that make a problem-solving step more difficult for a student and


A good cognitive model for a tutor uses a set of production rules or skills which specify how students solve problems. The tutor should estimate the skills learnt by each student when they practice with the tutor. The power law [5] defines the relationship between the error rate of performance and the amount of practice, depicted by equation (1). This shows that the error rate decreases according to a power function as the amount of practice increase.

\[
Y = aX^b \quad \text{(1)}
\]

Where

\[Y\] = the error rate
\[X\] = the number of opportunities to practice a skill
\[a\] = the error rate on the first trial, reflecting the intrinsic difficulty of a skill
\[b\] = the learning rate, reflecting how easy a skill is to learn

While the power law model applies to individual skills, it does not include student effects. In order to accommodate student effects for a cognitive model that has multiple rules, and that contains multiple students, the power law model is extended to a multiple logistic regression model (equation 2)[24].

\[
\ln[P_{ij}/(1-P_{ij})] = \alpha + \sum \beta_j Y_j + \sum \gamma_T T \quad \ldots \ldots \text{(2)}
\]

Where \(P_{ij}\) is the probability of getting a step in a tutoring question right by the ith student’s tth opportunity to practice the jth KC; \(X\) = the covariates for students; \(Y\) = the covariates for skills(knowledge components); \(T\) = the number of practice opportunities student i has had on knowledge component j; \(\alpha\) = the coefficient for each student, that is, the student intercept; \(\beta\) = the coefficient for each knowledge component, that is, the knowledge component intercept; \(\gamma\) = the coefficient for the interaction between a knowledge component and its opportunities, that is, the learning curve slope. The model says that the log odds of \(P_{ij}\) is proportional to the overall “smarts” of that student \((a)\) plus the “easiness” of that KC \((\beta)\) plus the amount gained \((\gamma)\) for each practice opportunity. This model can show the learning growth of students at any current or past moment.

A difficulty factor refers specifically to a property of the problem that causes student difficulties. The tutor considered for this research has metric measures as lesson 1 which requires 5 skills (conversion, division,
multiplication, addition, and result). These are the factors (KCs) in this tutor (Table 1) to be learnt by the students in solving the steps. Each step has a KC assigned to it for this study.

| Table 1. Factors for the Metric measures and their values |
|---------------------------------|----------------|
| **Factor Names** | **Factor Values** |
| Conversion | Correct formula, Incorrect |
| Addition | Correct, Wrong |
| Multiplication | Correct, Wrong |
| Division | Correct, Wrong |
| Result | Correct, Wrong |

The combinatorial search will select a model within the logistic regression model space. Difficulty factors are incorporated into an existing cognitive model through a model operator called Binary Split, which splits a skill a skill with a factor value, and a skill without the factor value. For example, splitting production Measurement by factor conversion leads to two productions: Measurement with the factor value Correct formula and Measurement with the factor value Incorrect. A* search is the combinatorial search algorithm [25] in LFA. It starts from an initial node, iteratively creates new adjoining nodes, explores them to reach a goal node. To limit the search space, it employs a heuristic to rank each node and visits the nodes in order of this heuristic estimate. In this study, the initial node is the existing cognitive model. Its adjoining nodes are the new models created by splitting the model on the difficulty factors. We do not specify a model to be the goal state because the structure of the best model is unknown. For this paper 25 node expansions per search is defined as the stopping criterion. AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) are two estimators used as heuristics in the search.

AIC = -2*log-likelihood + 2*number of parameters. .... (3)

BIC = -2*log-likelihood + number of parameters * number of observations. ..... (4)

Where log-likelihood measures the fit, and the number of parameters, which is the number of covariates in equation 2, measures the complexity. Lower AIC & BIC scores, mean a better balance between model fit and complexity.

IV. Methodology

In this paper the LFA methodology is illustrated using data obtained from the Metric measures lesson of Mensuration Tutor Maths Tutor[18] . Our dataset consist of 2,247 transactions involving 60 students, 32 unique steps and 5 Skills (KCs) in students exercise log. All the students were solving 9 problems 5 in mental problem category, 3 in simple and one in big. Total steps involved are 32. While solving exercise problem a student can ask for a hint in solving a step. Each data point is a correct or incorrect student action corresponding to a single skill execution. Student actions are coded as correct or incorrect and categorized in terms of “knowledge components” (KCs) needed to perform that action. Each step the student performs is related to a KC and is recorded as an “opportunity” for the student to show mastery of that KC. This lesson has 5 skills (conversion, division, multiplication, addition, and result) correspond to the skill needed in a step. Each step has a KC assigned to it for this study. The table 2 shows a sample data with columns: student name of the student; Step – problem 1 Step 1; Success – Whether the student did that step correctly or not in the first attempt. 1- success and 0-failure; Skill – Knowledge component used in that step; Opportunities – Number of times the skill is used by the same student computed from the first and fourth column.

| Table 2. The sample data |
|-------------------------|----------------|---------|--------|
| Student | Step | Success | Skill |
| X | P1s1 | 1 | conversion | 1 |
| X | P1s2 | 1 | result | 1 |
| X | P2s1 | 0 | conversion | 2 |

To find fitness of the model logistic regression values are calculated with Additive Factor Model (AFM)[26]. The values are present in Table 3.Number of parameters and number of observations in equation 3 and 4 is 60 (students) and 1920 (32unique steps x 60 students) respectively. Lower values of AIC, BIC and Root Mean Squared Error (RMSE) indicate a better fit between the model’s predictions and the observed data. Two types of cross validation are run for each KC model in the dataset. These types are a 3-fold cross validation of the Additive Factor Model’s (AFM)[25] error rate predictions. In student stratified, data points are grouped by student, the full set of students is divided into 3 groups. 3-fold cross validation is then performed across these 3 groups. In Item stratified, data points are grouped by step, the full set of steps is divided into 3 groups. 3-fold cross validation is then performed across these 3 groups. The Slope parameter represents how quickly students will learn the knowledge component. The larger the KC slope, the faster students learn the knowledge...
component. The conversion KC has 0 slope representing no learning takes place to be attended by the teacher. The addition KC has higher value indicating that students find it easier to solve. This table shows that this model best fitted the current tutor dataset with lower AIC, BIC, and RMSE values for the KC models used.

### Table 3. Logistic Regression Model values

<table>
<thead>
<tr>
<th>KC Model</th>
<th>AIC</th>
<th>BIC</th>
<th>Log likelihood</th>
<th>RMSE (student stratified)</th>
<th>RMSE (item stratified)</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>1,189.43</td>
<td>1,545.18</td>
<td>-530.72</td>
<td>0.302511</td>
<td>0.288114</td>
<td>0.732</td>
</tr>
<tr>
<td>Conversion</td>
<td>1,155.22</td>
<td>1,511.02</td>
<td>-513.61</td>
<td>0.298859</td>
<td>0.284091</td>
<td>0.000</td>
</tr>
<tr>
<td>Division</td>
<td>1,190.19</td>
<td>1,546.03</td>
<td>-513.09</td>
<td>0.301930</td>
<td>0.289071</td>
<td>0.623</td>
</tr>
<tr>
<td>Multiplication</td>
<td>1,193.94</td>
<td>1,549.76</td>
<td>-532.97</td>
<td>0.301943</td>
<td>0.287855</td>
<td>0.112</td>
</tr>
<tr>
<td>Result</td>
<td>1,197.65</td>
<td>1,553.49</td>
<td>-534.82</td>
<td>0.301916</td>
<td>0.287417</td>
<td>0.075</td>
</tr>
</tbody>
</table>

Learning curves [10] have become a standard tool for measurement of students’ learning in intelligent tutoring systems. Here in our study we used learning curve to visualize the student performance over opportunities. Slope and fit of learning curves show the rate at which a student learns over time, and reveal how well the system model fits what the student is learning. We used learning curves to measure the performance of tutoring system domain or student models. Measures of student performance are described below in table 3. Regardless of metric, each point on the graph is an average across all selected knowledge components and students.

### Table 3. Measures of student performance

<table>
<thead>
<tr>
<th>Measure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assistance Score</td>
<td>The number of incorrect attempts plus hint requests for a given opportunity</td>
</tr>
<tr>
<td>Error Rate</td>
<td>The percentage of students that asked for a hint or were incorrect on their first attempt. For example, an error rate of 45% means that 45% of students asked for a hint or performed an incorrect action on their first attempt. Error rate differs from assistance score in that it provides data based only on the first attempt. As such, an error rate provides no distinction between a student that made multiple incorrect attempts and a student that made only one.</td>
</tr>
<tr>
<td>Number of Incorrect</td>
<td>The number of incorrect attempts for each opportunity</td>
</tr>
<tr>
<td>Number of Hints</td>
<td>The number of hints requested for each opportunity</td>
</tr>
<tr>
<td>Step Duration</td>
<td>The elapsed time of a step in seconds, calculated by adding all of the durations for transactions that were attributed to the step.</td>
</tr>
<tr>
<td>Correct Step Duration</td>
<td>The step duration if the first attempt for the step was correct. The duration of time for which students are &quot;silent&quot;, with respect to their interaction with the tutor, before they complete the step correctly. This is often called &quot;reaction time&quot; (on correct trials) in the psychology literature. If the first attempt is an error (incorrect attempt or hint request), the observation is dropped.</td>
</tr>
<tr>
<td>Error Step Duration</td>
<td>The step duration if the first attempt for the step was an error (hint request or incorrect attempt). If the first attempt is a correct attempt, the observation is dropped.</td>
</tr>
</tbody>
</table>

Learning curve is categorised as follows:
- **low and flat**: The low error rate shows that students mastered the KCs but continued to receive tasks for them.
- **no learning**: the slope of the predicted learning curve shows no apparent learning for these KCs.
- **still high**: students continued to have difficulty with these KCs. Consider increasing opportunities for practice.
- **too little data**: students didn't practice these KCs enough for the data to be interpretable.
- **good**: these KCs did not fall into any of the above "bad" or "at risk" categories. Thus, these are "good" learning curves in the sense that they appear to indicate substantial student learning.

The above categorisations assist the teacher in knowing about the students’ knowledge level in specific concepts to be mastered by the students.

### V. Results And Discussions

To analyse the performance of student(s), we used Datashop[13] analysis and visualization tool for generating learning curves by uploading our dataset. The fig. 1 shows the problem steps involved in the first problem and number of correct/incorrect attempts done by 60 students.
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The following chart (Fig. 2) shows that the KC-conversion had maximum error rate compared with other KCs. This explains that the students struggled in conversion step (converting from one unit to other unit in metric measures lesson).

From Fig. 3 it is identified that average number of hints requested by the students for conversion KC is greater than other KCs. The difficulty level of Conversion KC is greater than other KCs. It indicates that conversion KC has to be explained by the teacher in the class or more practice has to be given to the students. The Fig. 4 shows the assistance score made the students in all the 9 problems they solved. Though the fourth problem is defined in mental problem category requires 2 or 3 steps to find the solution, the students made maximum number of incorrect attempts and requested for hints. This indicates that the problem is tough for the learners and they did not understand the concept. Students took more time for solving the conversion KC than other KCs (Fig. 5). This indicates the difficulty level of that skill.

The empirical learning curve give a visual clue as to how well a student may do over a set of learning opportunities, the predicted curves allow for a more precise prediction of a success rate at any learning opportunity. The predicted learning curve is much smoother. It is computed using the Additive Factor Model (AFM)[25], which uses a set of customized Item-Response models to predict how a student will perform for each skill on each learning opportunity. The predicted learning curves are the average predicted error of a skill over each of the learning opportunities. The blue line in learning curves shows the predicted value and category is defined using the predicted value. The learning curve has some blips depending on error rate but the predicted line is very smooth.
From the predicted learning curve for conversion KC (Fig. 6) we can infer that ‘no learning’ took place while practicing. There were 11 opportunities for conversion and 4th conversion has maximum error rate 33.3%. We understood that no conversion was at 0% error rate. The teacher can better guide the students in that area. He can do changes in domain modeling by adding new problems in examples and providing more exercises. Learning curves shown in Fig. 7 and 9 are in the category ‘Low and Flat’ explains that students likely received too much practice for these KCs. This shows that the students were mastered in these skills and do not require any more practice. Fig.8 and 11 are in the category ‘good’ indicate that the students got sufficient learning in that. Single-KC model in Fig. 11 shows the overall performance of the students in all the 32 unique steps are good. In 32 steps only 2 steps used addition so fig. 10 shows ‘too little data’. We can add problems for this KC or it can be merged with other KCs.

VI. Conclusion

Student knowledge models can be improved by mining students’ interaction data. This paper analyzed the use of LFA in student knowledge modeling in maths education with learning curves by mining the students log data. This method assists the teacher in: 1) measuring the difficulty and learning rates of Knowledge Components (KCs). 2) predict student performance in practicing each KC. 3) identify over-practiced or under-practiced KCs. The learners can understand what they know and do not know. The students with poor performance can be given with more problems for practicing. This method provides more insight into the performance of skills in every step for each student. The next step of this research is to provide a personalized tutoring environment for the students by incorporating the results into the tutor and providing automated suggestion to improve their performance. Clustering algorithms can be used to suggest the teacher in grouping the students according to their performance.

References