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Profit Analysis Of Computer System With S/W Redundancy Priority To H/W PM Over S/W Up-Gradation

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Abstract
The authors concentrate in this paper on the stochastically modelling of a computer system with software redundancy by introducing the concept of priority to hardware preventive maintenance (PM) over software up-gradation and hardware maximum repair time (MRT). The system fails independently from normal mode. All the repair activities such as hardware repair, software up-gradation, hardware preventive maintenance before failure and hardware replacement after maximum repair time are carried out by a single server immediately, if required. All random variables are statistically independent. The negative exponential distribution is taken for the failure time of the component while the distributions of repair time, up-gradation time, preventive maintenance and replacement time are assumed arbitrary with different probability density functions. Semi-Markov process and regenerative point technique are used for obtaining the values of various parameters. The behaviour of some important performance measure has been examined. The profit comparison of the present model has also been made with that of the model analyzed by Munday et. al (2019).

Keywords: *Computer System, Software Redundancy, Priority, Up-gradation, Repair, Preventive Maintenance, Replacement, and Profit Analysis.*

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I. Introduction

The use of computers system and stochastic modelling in our daily lives is essential. Every day, people (such as engineers, doctors, students, teachers, investors) calculate, evaluate and test computer system modelling. In these decades, computers have made life easier with the help of different types of programming. A computer system consists of hardware components that have been carefully chosen so that they work well together and software components or programs that run in the computer. It is a set of integrated devices that input, output, process, and store data and information. The unit wise redundancy technique has been considered as one of these in the development of stochastic models for computer systems. Malik and Anand (2010), Malik and Sureria (2012) and Kumar et al. (2013) examined computer systems with cold standby redundancy under different failures and repair policies. Also, Malik and Munday (2014, 15, 16) analysed a stochastic model for a computer system by providing component wise redundancy in cold standby. Munday et al. (2019) and Munday and Permila (2023) developed a computer system with software redundancy in cold standby subject to hardware preventive maintenance and maximum repair time.

In this paper, the authors evaluate profit analysis of a computer system with software redundancy by introducing the concept of priority to hardware component preventive maintenance (PM) over software up-gradation and hardware maximum repair time (MRT). The system fails independently from normal mode. All the repair activities such as hardware repair, software up-gradation, hardware PM and hardware replacement are carried out by a single server immediately on need basis. The failed hardware component undergoes for repair. All random variables are statistically independent. The negative exponential distribution is taken for the failure time of the component while the distributions of repair time, up-gradation time and replacement time are assumed arbitrary with different probability density functions. Semi-Markov process and regenerative point technique are used for obtaining the values various performance measures. The behaviour of some important performance measure has been examined for different parameters and costs. The profit comparison of the present model has also been made with that of the model analyzed by Munday et al. (2019).

II. Notations

E	:	Set of regenerative states
$\bar{E}$	:	Set of non-regenerative states

O : Computer system is operative
Scs : Software is in cold standby
PM : Preventive Maintenance
MRT : Maximum Repair Time
a/b : Probability that the system has hardware / software failure
α_0/β_0 : The rate by which hardware component undergoes for replacement/preventive maintenance
λ_1/λ_2 : Hardware/Software failure rate
HFUr /HFWr : The hardware is failed and under repair/waiting for repair
SFUg/SFWUg : The software is failed and under/waiting for up-gradation
HFURp /HFWRp : The hardware is failed and under replacement/waiting for replacement
HFUPm /HFWPm : The hardware is failed and under replacement/waiting for Preventive maintenance
HFUR/HFWR : The hardware is failed and continuously under repair / waiting for repair from previous state
SFUG/SFWUG : The software is failed and continuously under up-gradation /waiting for up- gradation from previous state
HFURP/HFWRP : The hardware is failed and continuously under replacement / waiting for replacement from previous state
HFUPM/HFPM : The hardware is continuously under/waiting for Preventive maintenance from previous state
g(t)/G(t) : pdf/cdf of hardware repair time
f(t)/F(t) : pdf/cdf of software up-gradation time
r(t)/R(t) : pdf/cdf of hardware replacement time
m(t) : pdf/cdf of hardware preventive maintenance time
$q_{ij}(t)/Q_{ij}(t)$: pdf / cdf of first passage time from regenerative state S_i to a regenerative state S_j or to a failed state S_j without visiting any other regenerative state in (0, t]
$q_{ij.k}(t)/Q_{ij.k}(t)$: pdf/cdf of direct transition time from regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting state S_k once in (0, t]
$M_i(t)$: Probability that the system up initially in state $S_i \epsilon E$ is up at time t without visiting to any regenerative state
$W_i(t)$: Probability that the server is busy in the state S_i up to time't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
μ_i : The mean sojourn time in state S_i which is given by

$$\mu_i = E(T) = \int_0^\infty P(T > t)\, dt = \sum_j m_{ij} \,,$$

where T denotes the time to system failure.
m_{ij} : Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that

$$\mu_i = \sum_j m_{ij} \text{ and } m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$$

& /© : Symbol for Laplace-Stieltjes convolution/Laplace convolution
*/** : Symbol for Laplace Transformation (LT)/Laplace Stieltjes Transformation (LST)
P : Profit of the Model as shown in Munday et al. (2019)
P1 : Profit of the present model

III. Transition Probabilities And Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements.

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$$

$p_{01} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\beta_0}$, $p_{02} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\beta_0}$, $p_{03} = \frac{\beta_0}{a\lambda_1+b\lambda_2+\beta_0}$
$p_{10} = \frac{\alpha}{\alpha_0+\alpha}$, $p_{17} = \frac{\alpha_0}{\alpha_0+\alpha}$, $p_{20} = f^*(a\lambda_1 + b\lambda_2 + \beta_0)$, $p_{24} = \frac{\beta_0}{a\lambda_1+b\lambda_2+\beta_0}\{1 - f^*(a\lambda_1 + b\lambda_2 + \beta_0)\}$,
$p_{25} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\beta_0}\{1 - f^*(a\lambda_1 + b\lambda_2 + \beta_0)\}$,
$p_{26} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\beta_0}\{1 - f^*(a\lambda_1 + b\lambda_2 + \beta_0)\}$, $p_{30} = m^*(0)$,
$p_{42} = m^*(0)$ $p_{52} = p_{61} = f^*(0)$, $p_{70} = r^*(0)$
For $g(t) = \alpha e^{-\alpha t}$, $f(t) = \theta e^{-\theta t}$,, $m(t) = \Upsilon e^{-\Upsilon t}$ and $r(t) = \beta e^{-\beta t}$, we have

$$p_{21.6} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \beta_0}\{1 - f^*(a\lambda_1 + b\lambda_2 + \beta_0)\}$$

$$p_{22.5} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2 + \beta_0}\{1 - f^*(a\lambda_1 + b\lambda_2 + \beta_0)\}$$

But, $f^*(0) = g^*(0) = r^*(0) = m^*(0) = 1$ and $p + q = 1$, $a + b = 1$
It can be easily verified that

$$p_{01} + p_{02} + p_{03} = p_{10} + p_{17} = p_{20} + p_{24} + p_{25} + p_{26} = p_{30} = p_{42} = p_{52} = p_{61} = p_{70} = p_{20} + p_{21.6} + p_{22.5} + p_{24} = 1$$

The mean sojourn times (μ_i) is the state S_i are
$\mu_0 = \frac{1}{a\lambda_1+b\lambda_2+\beta_0}$, $\mu_1 = \frac{1}{\alpha_0}$, $\mu_2 = \frac{1}{a\lambda_1+b\lambda_2+\beta_0+\theta}$, $\mu_3 = \frac{1}{\Upsilon}$, $\mu_4 = \frac{1}{\Upsilon}$, $\mu_5 = \frac{1}{\theta}$, $\mu_6 = \frac{1}{\theta}$, $\mu_7 = \frac{1}{\beta}$,
$\mu_2' = \frac{1}{\theta}$
Also
$m_{01} + m_{02} + m_{03} = \mu_0$, $m_{10} + m_{17} = \mu_1$, $m_{20} + m_{24} + m_{25} + m_{26} = \mu_2$, $m_{30} = \mu_3$, $m_{42} = \mu_4$, $m_{52} = \mu_5$, $m_{61} = \mu_6$, $m_{70} = \mu_7$ and

$$m_{20} + m_{21.6} + m_{22.5} + m_{24} = \mu_2'$$

IV. Reliability And Mean Time To System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$,

$$\phi_0(t) = Q_{01}(t) + Q_{02}(t) \,\&\, \phi_2(t) + Q_{03}(t)$$

$$\phi_2(t) = Q_{20}(t) \,\&\, \phi_0(t) + Q_{24}(t) + Q_{25}(t) + Q_{26}(t) \quad (1)$$

Taking LST of above relations (1) and solving for $\phi_0^{**}(s)$
We have

$$R^*(s) = \frac{1 - \phi_0^{**}(s)}{s}$$

The reliability of the system model can be obtained by taking Laplace inverse transform of the above equation.
The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N_1}{D_1} \quad (2)$$

Where $N_1 = \mu_0 + p_{02}\mu_2$ and $D_1 = 1 - p_{02}p_{20}$ (3)

V. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at an instant't' given that the system entered regenerative state S_i at $t = 0$. The recursive relations for $A_i(t)$ are given as:

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©\, A_2(t) + q_{03}(t)©\, A_3(t)$$

$$A_1(t) = q_{10}(t)©A_0(t) + q_{17}(t)©A_7(t)$$

$$A_2(t) = M_2(t) + q_{20}(t)©A_0(t) + q_{21.6}(t)©\, A_1(t) + q_{22.5}(t)©\, A_2(t) + q_{24}(t)©\, A_4(t)$$

$$A_3(t) = q_{30}(t)©A_0(t)$$

$$A_4(t) = q_{42}(t)©A_2(t)$$

$$A_7(t) = q_{70}(t)©A_0(t) \quad (4)$$

where $M_0(t) = e^{-(a\lambda_1+b\lambda_2+\beta_0)t}$ and $M_2(t) = e^{-(a\lambda_1+b\lambda_2+\beta_0)t}\,\overline{F(t)}$
Taking LT of relations (4) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} s\, A_0^*(s) = \frac{N_2}{D_2} \quad (5)$$

Where $N_2 = (1 - p_{22.5} - p_{24})\mu_0 + p_{02}\mu_2$
$D_2 = (1 - p_{22.5} - p_{24})\mu_0 + [p_{21.6} + p_{01}p_{20}]\mu_1 + p_{02}\mu_2' + p_{03}(1 - p_{22.5} - p_{24})\mu_3 + p_{02}p_{24}\mu_4 + [p_{01}(1 - p_{22.5} - p_{24}) + p_{02}]p_{17}\mu_7$ (6)

VI. Busy Period Of The Server

(a). Due to Hardware Repair

Let $B_i^H(t)$ be the probability that the server is busy in repairing the unit due to hardware failure at an instant 't' given that the system entered state $S_i\ at\ t = 0$. The recursive relations for $B_i^H(t)$ are as follows:

$$B_0^H(t) = q_{01}(t)©B_1^H(t) + q_{02}(t)©B_2^H(t) + q_{03}(t)©B_3^H(t)$$
$$B_1^H(t) = W_1^H(t) + q_{10}(t)©B_0^H(t) + q_{17}(t)©B_7^H(t)$$
$$B_2^H(t) = q_{20}(t)©B_0^H(t) + q_{21.6}(t)©B_1^H(t) + q_{22.5}(t)©B_2^H(t) + q_{24}(t)©B_4^H(t)$$
$$B_3^H(t) = q_{30}(t)©B_0^H(t)$$
$$B_4^H(t) = q_{42}(t)©B_2^H(t)$$
$$B_7^H(t) = q_{70}(t)©B_0^H(t) \qquad (7)$$

where $W_1^H(t) = \overline{G(t)}\,dt$

(b). Due to software Up-gradation

Let $B_i^S(t)$ be the probability that the server is busy due to up-gradation of the software at an instant't' given that the system entered the regenerative state $S_i\ at\ t = 0$. We have the following recursive relations for $B_i^S(t)$:

$$B_0^S(t) = q_{01}(t)©B_1^S(t) + q_{02}(t)©B_2^S(t) + q_{03}(t)©B_3^S(t)$$
$$B_1^S(t) = q_{10}(t)©B_0^S(t) + q_{17}(t)©B_7^S(t)$$
$$B_2^S(t) = W_2^S(t) + q_{20}(t)©B_0^S(t) + q_{21.6}(t)©B_1^S(t) + q_{22.5}(t)©B_2^S(t) + q_{24}(t)©B_4^S(t)$$
$$B_3^S(t) = q_{30}(t)©B_0^S(t)$$
$$B_4^S(t) = q_{42}(t)©B_2^S(t)$$
$$B_7^S(t) = q_{70}(t)©B_0^S(t) \qquad (8)$$

where

$$W_2^S(t) = e^{-(a\lambda_1+b\lambda_2+\beta_0)t}\,\overline{F(t)} + \left(a\lambda_1 e^{-(a\lambda_1+b\lambda_2+\beta_0)t}©1\right)\overline{F(t)} + \left(b\lambda_2 e^{-(a\lambda_1+b\lambda_2+\beta_0)t}©1\right)\overline{F(t)} + \left(\beta_0 e^{-(a\lambda_1+b\lambda_2+\beta_0)t}©1\right)\overline{F(t)}$$

(c). Due to Hardware Preventive Maintenance

Let $B_i^I(t)$ be the probability that the server is busy in preventive maintenance of the unit before hardware failure given that the system entered state $S_i\ at\ t = 0$. We have the following recursive relations for $B_i^I(t)$:

$$B_0^{Pm}(t) = q_{01}(t)©B_1^{Pm}(t) + q_{02}(t)©B_2^{Pm}(t) + q_{03}(t)©B_3^{Pm}(t)$$
$$B_1^{Pm}(t) = q_{10}(t)©B_0^{Pm}(t) + q_{17}(t)©B_7^{Pm}(t)$$
$$B_2^{Pm}(t) = q_{20}(t)©B_0^{Pm}(t) + q_{21.6}(t)©B_1^{Pm}(t) + q_{22.5}(t)©B_2^{Pm}(t) + q_{24}(t)©B_4^{Pm}(t)$$
$$B_3^{Pm}(t) = W_3^{Pm}(t) + q_{30}(t)©B_0^{Pm}(t)$$
$$B_4^{Pm}(t) = W_4^{Pm}(t) + q_{42}(t)©B_2^{Pm}(t)$$
$$B_7^{Pm}(t) = q_{70}(t)©B_0^{Pm}(t) \qquad (9)$$

where $W_3^{Pm}(t) = W_4^{Pm}(t) = \overline{M(t)}\,dt$

(d). Due to Hardware Replacement

Let $B_i^{Rp}(t)$ be the probability that the server is busy in replacement of the unit due to hardware failure given that the system entered state $S_i\ at\ t = 0$. We have the following recursive relations for $B_i^{Rp}(t)$:

$$B_0^{Rp}(t) = q_{01}(t)©B_1^{Rp}(t) + q_{02}(t)©B_2^{Rp}(t) + q_{03}(t)©B_3^{Rp}(t)$$
$$B_1^{Rp}(t) = q_{10}(t)©B_0^{Rp}(t) + q_{17}(t)©B_7^{Rp}(t)$$
$$B_2^{Rp}(t) = q_{20}(t)©B_0^{Rp}(t) + q_{21.6}(t)©B_1^{Rp}(t) + q_{22.5}(t)©B_2^{Rp}(t) + q_{24}(t)©B_4^{Rp}(t)$$
$$B_3^{Rp}(t) = q_{30}(t)©B_0^{Rp}(t)$$
$$B_4^{Rp}(t) = q_{42}(t)©B_2^{Rp}(t)$$
$$B_7^{Rp}(t) = W_7^{Rp}(t) + q_{70}(t)©B_0^{Rp}(t) \qquad (10)$$

Where $W_7^{Rp}(t) = \overline{R(t)}\,dt$

Taking LT of relations (7), (8), (9) and (10), solving for $B_0^{H^*}(t),\ B_0^{S^*}(t),\ B_0^{Pm^*}(t)\ and\ B_0^{Rp^*}(t)$. The time for which server is busy due to repairs, up-gradations, replacements and preventive maintenance respectively are given by

$$B_0^H(t) = \lim_{s\to 0} s\,B_0^{H^*}(t) = \frac{N_3^H}{D_2} \qquad (11)$$

$$B_0^S(t) = \lim_{s\to 0} s\,B_0^{S^*}(t) = \frac{N_3^S}{D_2} \qquad (12)$$

$B_0^{Pm}(t) = \lim_{s\to 0} s\, B_0^{Pm^*}(t) = \frac{N_3^{Pm}}{D_2}$ (13)

$B_0^{Rp}(t) = \lim_{s\to 0} s\, B_0^{Rp^*}(t) = \frac{N_3^{Rp}}{D_2}$ (14)

where

$N_3^H = [p_{01}(1-p_{22.5}-p_{24}) + p_{02}p_{21.6}]\mu_1$, $N_3^S = p_{02}\mu_2$,

$N_3^{Pm} = p_{03}(1-p_{22.5}-p_{24})\mu_3 + p_{02}p_{24}\mu_4$, $N_3^{Rp} = [p_{01}(1-p_{22.5}-p_{24}) + p_{02}p_{21.6}]p_{17}\mu_7$

$and\ D_2\ is\ already\ mentioned.$ (15)

VII. Expected Number Of Hardware Repairs

Let $NHR_i(t)$ be the expected number of hardware repairs by the server in (0, t] given that the system entered the regenerative state $S_i\ at\ t=0$. The recursive relations for $NHR_i(t)$ are given as:

$$NHR_0(t) = Q_{01}(t) \,\&\, NHR_1(t) + Q_{02}(t) \,\&\, NHR_2(t) + Q_{03}(t) \,\&\, NHR_3(t)$$

$$NHR_1(t) = Q_{10}(t) \,\&\, [1 + NHR_0(t)] + Q_{17}(t) \,\&\, NHR_7(t)$$

$$NHR_2(t) = Q_{20}(t) \,\&\, NHR_0(t) + Q_{21.6}(t) \,\&\, NHR_1(t) + Q_{22.5}(t) \,\&\, NHR_2(t) + Q_{24}(t) \,\&\, NHR_4(t)$$

$$NHR_3(t) = Q_{30}(t) \,\&\, NHR_0(t)$$

$$NHR_4(t) = Q_{42}(t) \,\&\, NHR_2(t)$$

$$NHR_7(t) = Q_{70}(t) \,\&\, NHR_0(t) \quad (16)$$

Taking LST of relations (16) and solving for $NHR_0^{**}(s)$. The expected number of hardware repair is given by

$NHR_0 = \lim_{s\to 0} sNHR_0^{**}(s) = \frac{N_4}{D_2}$ (17)

Where $N_4 = p_{10}[p_{01}(1-p_{22.5}-p_{24}) - p_{02}p_{21.6}]$ $and\ D_2\ is\ already\ mentioned.$ (18)

VIII. Expected Number Of Software Up-Gradations

Let $NSU_i(t)$ be the expected number of software up-gradations in (0, t] given that the system entered the regenerative state $S_i\ at\ t=0$. The recursive relations for $NSU_i(t)$ are given as follows:

$$NSU_0(t) = Q_{01}(t) \,\&\, NSU_1(t) + Q_{02}(t) \,\&\, NSU_2(t) + Q_{03}(t) \,\&\, NSU_3(t)$$

$$NSU_1(t) = Q_{10}(t) \,\&\, NSU_0(t) + Q_{17}(t) \,\&\, NSU_7(t)$$

$$NSU_2(t) = Q_{20}(t) \,\&\, [1 + NSU_0(t)] + Q_{21.6}(t) \,\&\, [1 + NSU_1(t)] + Q_{22.5}(t) \,\&\, [1 + NSU_2(t)] + Q_{24}(t) \,\&\, NSU_4(t)$$

$$NSU_3(t) = Q_{30}(t) \,\&\, NSU_0(t)$$

$$NSU_4(t) = Q_{42}(t) \,\&\, NSU_2(t)$$

$$NSU_7(t) = Q_{70}(t) \,\&\, NSU_0(t) \quad (19)$$

Taking LST of relations (19) and solving for $NSU_0^{**}(s)$. The expected numbers of software up-gradation are given by

$NSU_0(\infty) = \lim_{s\to 0} sNSU_0^{**}(s) = \frac{N_5}{D_2}$ (20)

Where $N_5 = p_{02}\,(p_{20} + p_{21.6} + p_{22.5})\, and\ D_2\ is\ already\ mentioned$ (21)

IX. Expected Number Of Hardware Preventive Maintenance

Let $NHI_i(t)$ be the expected number of hardware preventive maintenance by the server in (0, t] given that the system entered the regenerative state $S_i\ at\ t=0$. The recursive relations for $NHPm_i(t)$ are given as:

$$NHPm_0(t) = Q_{01}(t) \,\&\, NHPm_1(t) + Q_{02}(t) \,\&\, NHPm_2(t) + Q_{03}(t) \,\&\, NHPm_3(t)$$

$$NHPm_1(t) = Q_{10}(t) \,\&\, NHPm_0(t) + Q_{17}(t) \,\&\, NHPm_7(t)$$

$$NHPm_2(t) = Q_{20}(t) \,\&\, NHPm_0(t) + Q_{21.6}(t) \,\&\, NHPm_1(t) + Q_{22.5}(t) \,\&\, NHPm_2(t) + Q_{24}(t) \,\&\, NHPm_4(t)$$

$$NHPm_3(t) = Q_{30}(t) \,\&\, [1 + NHPm_0(t)]$$

$$NHPm_4(t) = Q_{42}(t) \,\&\, [1 + NHPm_2(t)]$$

$$NHPm_7(t) = Q_{70}(t) \,\&\, NHPm_0(t) \quad (22)$$

Taking LST of relations (22) and solving for $NHPm_0^{**}(s)$. The expected number of hardware preventive maintenance is given by

$$NHI_0 = \lim_{s\to 0} sNHPm_0^{**}(s) = \frac{N_6}{D_2} \quad (23)$$

Where $N_6 = p_{02}p_{24} + p_{03}(1 - p_{22.5} - p_{24})$ *and* D_2 *is already mentioned.* (24)

X. Expected Number Of Hardware Replacement

Let $NHRp_i(t)$ be the expected number of hardware replacement by the server in (0, t] given that the system entered the regenerative state S_i at $t = 0$. The recursive relations for $NHRp_i(t)$ are given as:

$$NHRp_0(t) = Q_{01}(t) \,\&\, NHRp_1(t) + Q_{02}(t) \,\&\, NHRp_2(t) + Q_{03}(t) \,\&\, NHRp_3(t)$$

$$NHRp_1(t) = Q_{10}(t) \,\&\, NHRp_0(t) + Q_{17}(t) \,\&\, NHRp_7(t)$$

$$NHRp_2(t) = Q_{20}(t) \,\&\, NHRp_0(t) + Q_{21.6}(t) \,\&\, NHRp_1(t) + Q_{22.5}(t) \,\&\, NHRp_2(t) + Q_{24}(t) \,\&\, NHRp_4(t)$$

$$NHRp_3(t) = Q_{30}(t) \,\&\, NHRp_0(t)$$

$$NHRp_4(t) = Q_{42}(t) \,\&\, NHRp_2(t)$$

$$NHRp_7(t) = Q_{70}(t) \,\&\, [1 + NHRp_0(t)] \quad (25)$$

Taking LST of relations (25) and solving for $NHRp_0^{**}(s)$. The expected number of hardware replacement is given by

$$NHRp_0 = \lim_{s\to 0} sNHRp_0^{**}(s) = \frac{N_7}{D_2} \quad (26)$$

Where $N_7 = p_{01}p_{17}(1 - p_{22.5} - p_{24}) + p_{02}p_{17}p_{21.6}$ *and* D_2 *is already mentioned.* (27)

XI. Cost-Benefit Analysis

The profit incurred to the system model in steady state can be obtained as:

$$P = K_0A_0 - K_1B_0^H - K_2B_0^S - K_5B_0^{Pm} - K_7B_0^{Rp} - K_3NHR_0 - K_4NSU_0 - K_6NHPm_0 - K_8NHRp_0 \quad (28)$$

Where

$K_0 = Revenue\ per\ unit\ up - time\ of\ the\ system$

$K_1 = Cost\ per\ unit\ time\ for\ which\ server\ is\ busy\ due\ to\ hardware\ repair$

$K_2 = Cost\ per\ unit\ time\ for\ which\ server\ is\ busy\ due\ to\ software\ up - gradation$

$K_3 = Cost\ per\ unit\ repair\ of\ the\ failed\ hardware$

$K_4 = Cost\ per\ unit\ up - gradation\ of\ the\ failed\ software$

$K_5 = Cost\ per\ unit\ time\ for\ which\ server\ is\ busy\ due\ to\ hardware\ preventive\ maintenance$

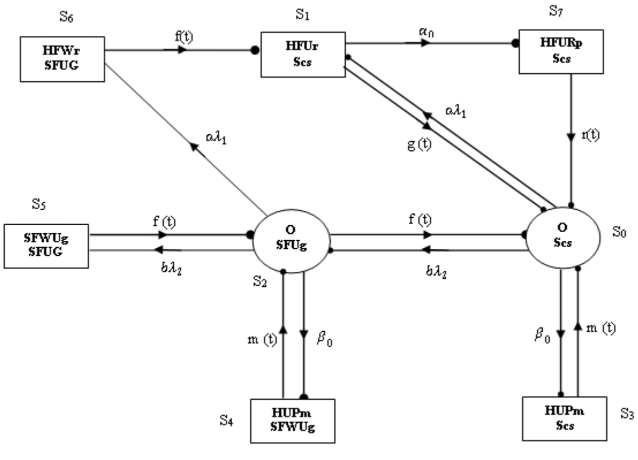
$K_6 = Cost\ per\ unit\ replacement\ of\ the\ failed\ hardwarepreventive\ maintenance$

$K_7 = Cost\ per\ unit\ time\ for\ which\ server\ is\ busy\ due\ to\ hardware\ replacement$

$K_8 = Cost\ per\ unit\ inspection\ of\ the\ failed\ hardwarereplacement$

and $A_0, B_0^H, B_0^S, B_0^{Rp}, B_0^I, NHR_0, NSU_0, NHRp_0, NHI_0$ *are already defined.*

Figure 1: State Transition Diagram



○ Up-State □ Failed State • Regenerative Point

XII. Conclusion

From figures 2 to 4, the behaviour of some important performance measures such as MTSF, availability and profit with respect to hardware failure rate (λ_1) has been observed for arbitrary values of various parameters including $K_0 = 15000$, $K_1 1000$, $K_2 = 700$, $K_3 = 1500$, $K_4 = 1200, K_5 = 300, K_6 = 600, K_7 = 800, K_8 = 1400$ with a=0.6 & b=0.4 as shown respectively. It is found that these measures go on decreasing with the increase of hardware failure rate, software failure rates and hardware undergoes for preventive maintenance. But, their values increase with the increase of hardware repair rate (α), up-gradation rate (θ), preventive maintenance rate (ϒ) and replacement rate (β). On the other hand, if the values of a and b are interchanged i.e. a=0.4 and b=0.6, than MTSF, availability and profit of the system highly increase as compared to other parameters.

XIII. Graphical Presentation Of Reliability Measures

Figure 2: MTSF Vs Hardware Failure Rate (λ1)

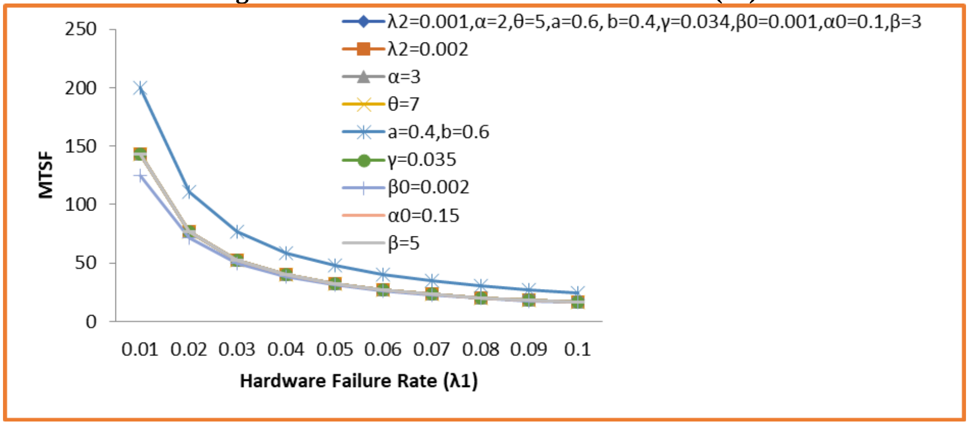


Figure 3: Availability Vs Hardware Failure Rate (λ1)

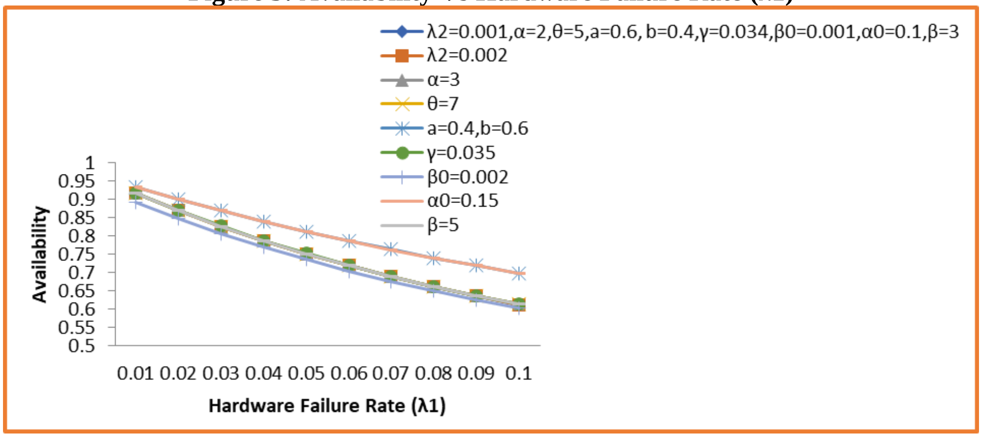
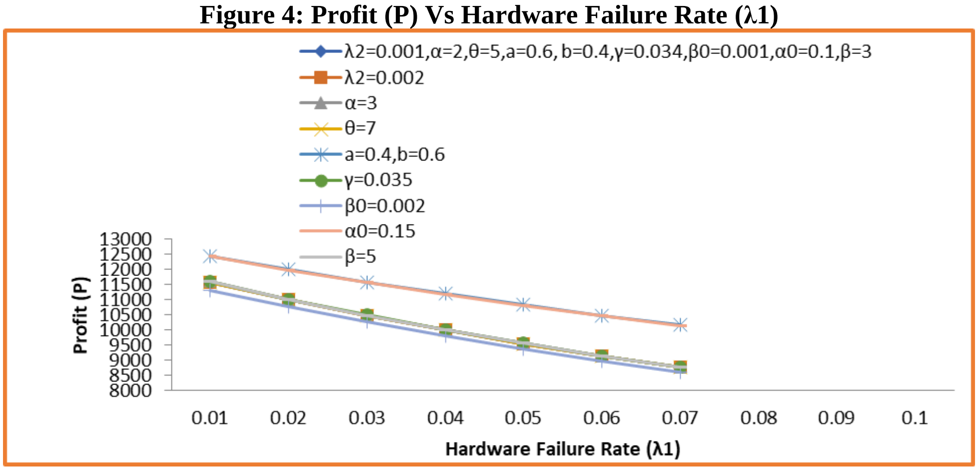


Figure 4: Profit (P) Vs Hardware Failure Rate (λ1)

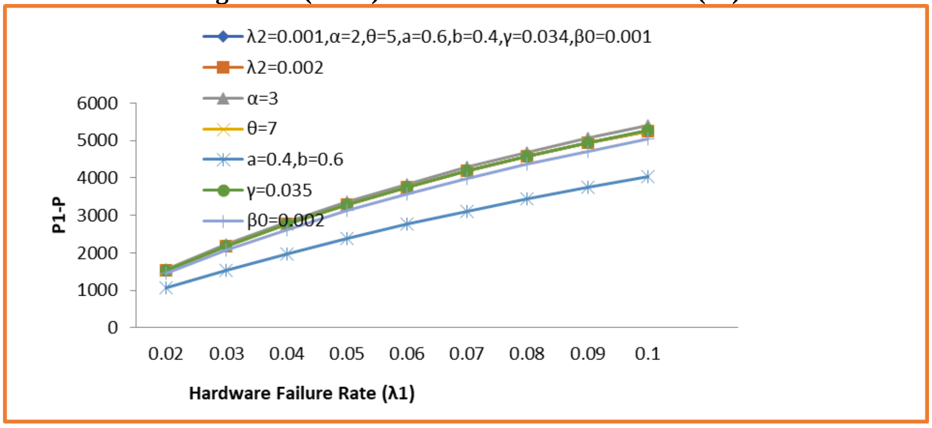


XIV. Comparative Study Of Profits Of The System Models

The profit of the present computer system model has been compared with that of the model Munday et al. (2019) as shown in Figure 5. It is observed that the present model is less profitable as compared to that model. Thus, in a computer system with software redundancy in cold standby, the idea of priority to preventive maintenance over software up-gradation and maximum repair time of hardware component is not helpful in increasing the profit of the system model.

XV. Graphical Presentation Of Profit Difference (P1 – P)

Figure 5: (P1–P) Vs Hardware Failure Rate (λ1)



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