

Investigation on the Pattern Synthesis of Subarray Weights for Low EMI Applications

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Abstract: In modern radar applications, it is frequently required to produce sum and difference patterns sequentially. The sum pattern amplitude coefficients are obtained by using Dolph-Chebyshev synthesis method where as the difference pattern excitation coefficients will be optimized in this present work. For this purpose optimal group weights will be introduced to the different array elements to obtain any type of beam depending on the application. Optimization of excitation to the array elements is the main objective so in this process a subarray configuration is adopted. However, Differential Evolution Algorithm is applied for optimization method. The proposed method is reliable and accurate. It is superior to other methods in terms of convergence speed and robustness. Numerical and simulation results are presented.

Key Words: Antenna Arrays, Sum and Difference, Pattern Synthesis, Subarray weighting configuration, Ultra Sidelobe level suppression, Differential Evolution Method.

I. Introduction

The concepts of half-power beamwidth and peak directivity of a linear antenna array pattern are introduced and it is applied to the case of pattern synthesis. For this purpose a well known technique called Dolph-Chebyshev synthesis method [1] is widely used. Namara [2] proposed the excitation matching method based on an expansion in terms of Zolotarev polynomials where each possible grouping, the corresponding subarray coefficients are iteratively computed.

The design of monopulse radar systems [3]-[4] requires the synthesis of both the sum and the difference pattern, which satisfy some specifications such as narrow beamwidth, low sidelobe level (SLL) and high directivity. In order to properly solve the optimal compromise problem in monopulse radar tracking array antennas, several techniques [5]-[6] based on sub arraying have been proposed to reduce the design complexity of the feed network. Shindman et al. [7] developed the techniques for designing the minimum power sidelobes for a main lobe array factor or difference pattern array factor. Although several methods [8]-[10] for implementing monopulse antennas have been proposed, interest has been shown to the methods that use proper feed networks for the design of a single feed for both sum and difference patterns instead to two independent feeds.

Differential Evolution Algorithm (DEA) is most powerful global optimizer and has been successfully applied to array pattern synthesis and also in other areas of the applied electromagnetic problems. In addition to the above, proper amplitude excitation weights of the elements are used to control the beam width and sidelobe level [11]-[12].

The sum pattern amplitude coefficients are obtained by using Dolph-Chebyshev synthesis method where as the difference pattern excitation coefficients will be optimized in this present work. For this purpose optimal group weights will be introduced to the different array elements to obtain any type of beam depending on the application. Optimization of excitation to the array elements is the main objective so in this process a subarray configuration is used to reduce the design complexity of feeding network.

The excitation coefficients for the sum pattern are calculated from the Dolph-Chebyshev method with fixed SLL = -40dB. This problem has already studied and final difference pattern was optimized by Differential Evolution Strategy (DES).

II. Differential Evolution Algorithm

2.1. Initialization:

Differential Evolution is very simple to understand and easy to implement, which begins with a population of D dimensional vectors and denotes by N_s shown in the following equation (1)

$$X_{i,G}, i=1, 2, \dots, N_s \quad (1)$$

Where the index ‘i’ denotes the population and ‘G’ denotes the generation to which the population belongs. DE strategy depends on three main operators that are mutation, crossover and selection. Schematic representation of DE strategy is shown in the figure 1.

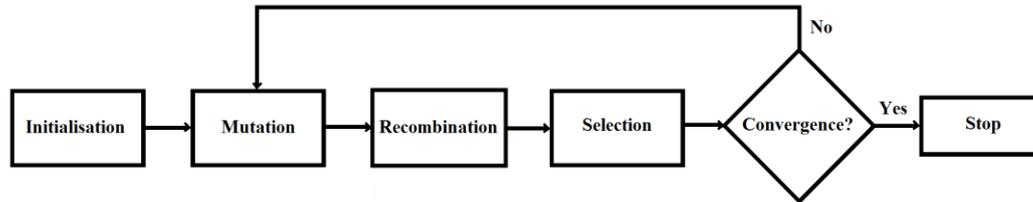


Figure 1: Schematic Representation of DE Strategy.

2.2. Mutation:

The mutation process at each generation begins by randomly selecting three individual variables that are $X_{r_1}, X_{r_2}, X_{r_3}$ in the population set of N_s elements.

For each target vector $X_{i,G}$, a mutant vector is generated according to the following equation

$$V_{i,G+1} = X_{i,G} + F \cdot (X_{r_2,G} - X_{r_3,G}) \tag{2}$$

Where the indexes $r_1, r_2, r_3 \in \{1, 2, 3 \dots N_s\}$ are randomly selected such that $r_1 \neq r_2 \neq r_3 \neq i$, F is a real and constant factor $\in [0, 2]$ which controls the strengthening of the discrepancy variation $(X_{r_2,G} - X_{r_3,G})$.

2.3. Crossover:

Once mutation part is completed, in order to increase the range of the perturbed parameter vectors crossover is introduced. In this stage the parent vector is mixed with the mutated vector to produce a trail vector $U_{j,G+1}$. The trail vector is described as follows.

$$U_{i,G+1} = (U_{1i,G+1}, U_{2i,G+1}, \dots, U_{Di,G+1}) \tag{3}$$

$$U_{ji,G+1} = \begin{cases} V_{ji,G+1} & \text{if } (\text{randm } b(j) \leq C_R) \text{ or } j = k \\ X_{ji,G} & \text{if } (\text{randm } b(j) > C_R) \text{ or } j \neq k \end{cases} \tag{4}$$

Where $j \in \{1, 2 \dots D\}$, k is a random parameter index chosen once for each ‘i’.

C_R is the crossover constant $\in [0, 1]$.

randm b(j) is the j^{th} evaluation of a uniform random number generator with outcome $\in [0,1]$. Figure 2 gives an example of crossover mechanism for 7-dimensional vectors.

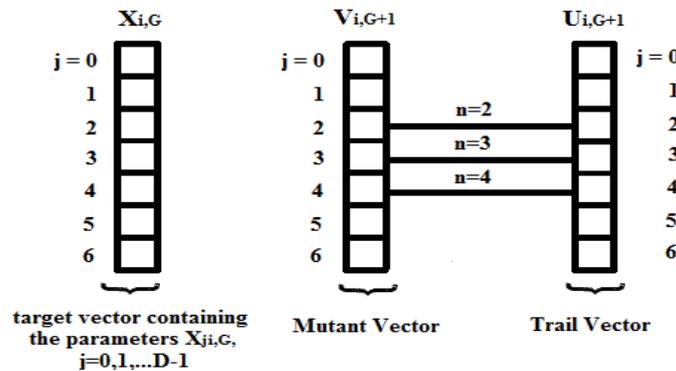


Figure 2: Illustration of the crossover process for D=7 parameters.

2.4. Selection:

Selection takes place with a contest held between the one with best fitness function and the target vector, which are allowed to enter the next generation. The trail vector $U_{i,G+1}$ is compared to the target vector $X_{i,G}$ using the greedy criterion to decide whether it should become a member of generation G+1 or not. Population for the next generation is selected according to the following rule.

$$X_{i,G+1} = \begin{cases} U_{i,G+1} & \text{if } f(U_{i,G+1}) \leq f(X_{i,G}) \\ X_{i,G} & \text{otherwise} \end{cases} \tag{5}$$

Where $j = 1, 2, 3 \dots N_s$.

For this problem, the i^{th} element of the population at the k^{th} iteration is indicated by $V_k(i)$, $(i=1, \dots, N_s)$, which has the following hybrid structure with integer and real variables $V_k(i) = (g_1 \dots g_p, c_1 \dots c_N)$. The key Parameters of the DE algorithm C_R and F should be accurately chosen in order to avoid a premature convergence to local minima or to a slow convergence rate.

III. Mathematical Formulation

3.1 Dolph-Chebyshev Array Design Procedure:

Consider an array of isotropic elements positioned symmetrically along the X-axis. Suppose the distance between any two adjacent elements is 'd', and the array is operated at $\lambda/2$, a symmetric linear array is shown in figure 3.

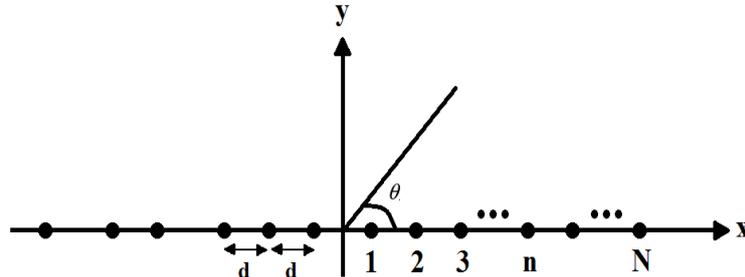


Figure 3: Geometry for N element linear array

Its excitation coefficients are related to Tchebyshev polynomials. The recursion formula for r^{th} Tchebyshev polynomial $T_r(z)$ is given below

$$T_r(z) = 2z T_{r-1}(z) - T_{r-2}(z) \quad (6)$$

Here every polynomial can also be computed using

$$T_r(z) = \cos(r \cos^{-1}(z)) \quad -1 \leq z \leq +1 \quad (7)$$

$$T_r(z) = \cos(r \cosh^{-1}(z)) \quad z < -1, z > +1$$

With this symmetric amplitude excitation the array factor of an array with even or odd number of elements is more than a sum of cosine terms.

To show the approach, consider a linear array of N equally-spaced elements whose array factor $AF(\theta)$.

$$(AF)_N = \sum_{n=1}^M a_n \cos[2(n-1)u] \quad N = 2M \text{ (even)} \quad (8)$$

Where, $u = \frac{\pi d}{\lambda} \cos\theta$

' a_n ' are the amplitude excitation coefficients

' λ ' is wave length in integer times of fundamental frequency,

' θ ' defines the angle at which $AF(\theta)$ is calculated with respect to the broadside direction.

'd' is the inter-element distance,

'M' the number of elements.

3.2 Array synthesis methodology:

The objective of the synthesis is to construct a reduced subarray configuration able to synthesize as better as possible this pattern. To avoid the implementation of several designing network arrays, a subarray configuration is adopted. The sum Pattern of array factor $AF_s(\theta)$ is obtained starting by a set of excitation coefficients, a_n^s , ($n = -N, \dots, -1, 1, \dots, N$) which are assumed to be symmetric i.e., $a_{-n}^s = a_n^s$, ($n = 1, \dots, N$) and are fixed.

In this case, the array space factor is given by

$$AF_s(\theta) = \sum_{n=1}^N a_n^s \cos\left[\frac{1}{2}(2n-1)kd \cos\theta\right] \quad (9)$$

The 'N' number of elements for the array is grouped into 'P' subarrays in order to construct the difference pattern. Each subarray has a weighting coefficient g_{c_n} , $p = 1, \dots, P$, and in order to create a difference pattern, the group of the antennas must be optimized.

In particular, if $c_n = p$, then the n^{th} element is to be connected to the p^{th} subarray. If $c_n = 0$, then the element is not considered in the process of synthesis.

The excitation coefficients of the difference pattern can be obtained by multiplying each coefficient of the sum pattern to the coefficient of the corresponding subarray group weight.

Formally,

$$a_n^d = g_{c_n} \cdot a_n^s, \quad c_n = 1, \dots, p, \dots, P.$$

Here g_{c_n} denotes the Kronecker function, i.e.,

$$\begin{cases} g_{c_n} = 1 & \text{if } c_n = p \\ g_{c_n} = 0 & \text{elsewhere} \end{cases} \quad (10)$$

Where g_p is the group weight of the p^{th} subarray.

A typical configuration for a subarrayed linear array structure with N isotropic elements for sum pattern excitation levels of a_n^s and difference pattern excitation levels of a_n^d are shown in figure 4.

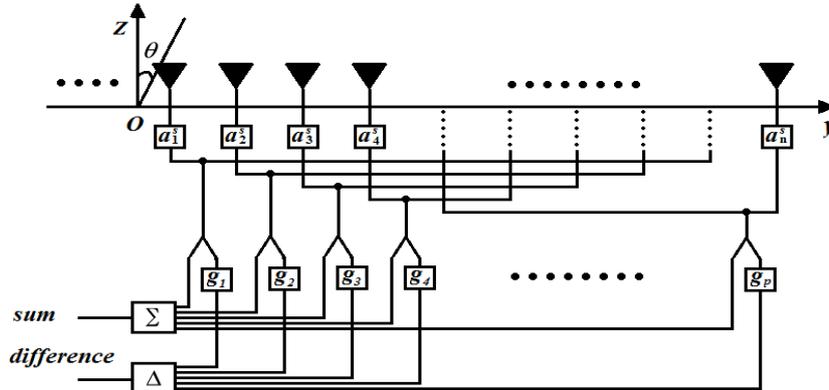


Figure 4: Geometry for N element linear array

Due to the problem of symmetry, only one half of the array is considered in the synthesis problem. In particular, the elements of the array are grouped in to ‘P’ subarrays. Since the excitations of the difference pattern must be antisymmetric, i.e. $a_n^d = -a_n^d$, ($n=1, \dots, N$) to ensure a deep null at the broad side.

In this case, the array space factor is given by

$$AF_d(\theta) = \sum_{n=1}^N a_n^d \sin \left[\frac{1}{2} (2n - 1) k d \cos \theta \right] \quad (11)$$

Where ‘k’ is a wave number of the propagation medium equals to $2\pi/\lambda$

a_n^s Amplitude sum pattern excitation coefficients,

a_n^d Amplitude difference pattern excitation coefficients, and

‘ θ ’ defines the angle at which ‘F’ is calculated with respect to a direction orthogonal to the array.

The sum patterns are obtained for SLL= -40dB and the difference patterns are achieved using proposed method.

This problem can now be deal with an optimization, where the clustering into subarrays ‘ c_n ’ and the subarray group weights ‘ g_p ’ are the optimization variables in order to optimize the difference pattern with maximum directivity. Specifications such as Sidelobe levels and beamwidth for the difference pattern can be optimized with a proper selection of the element grouping and subarray weights. Only half of the array is needed to be considered, since both the patterns are symmetric. The optimized radiation patterns are obtained with fixed SLL’s for various numbers of subarrays.

3.3 Objective function:

To this end, the optimized objective function used for calculating the fitness function associated with the symmetrical linear array can be formulated as follows

$$\text{Fitness} = \text{PSLL}_0 - \text{SLL}_d \quad (12)$$

$$\text{Where } \text{PSLL}_0 = \text{Max}_{\theta \in \text{CS}} \left[20 \log_{10} \left| \frac{E(\theta)}{E_{\text{max}}(\theta)} \right| \right] \quad (13)$$

SLL_d = Desired sidelobe level

$E_{\text{max}}(\theta)$ = Peak value of the Main beam

‘ θ ’ = Steering angle from the broad side of the array, $-90 \leq \theta \leq 90$

‘S’ = Space spanned by the angle excluding the mainlobe.

IV. Numerical Simulation Results And Discussions

In order to validate the effectiveness of Differential Evolution Method we first examine a linear array with 8 subarrays of 40 elements and 60 elements that are spaced $\lambda/2$ distance apart. The excitations for the sum pattern a_n^s are calculated from the dolph-chebyshev method using equation (8) with SLL=-40dB.

In order to construct the difference pattern, the N number of elements for the array is grouped into P subarrays. The excitations for the difference pattern a_n^d are determined by using differential evolution with the equation (9). Figure (5) reports the behavior of the cost function versus the number of iterations.

First investigate a linear array of 40 and 60 elements that are spaced $\lambda/2$ distance apart with 8 sub arrays for sum SLL=-40dB. The optimized radiation patterns of difference array space factor for 8 subarrays are obtained by using the equation (11) and the results are shown in figure (6) and figure (10). As can be seen from these

figures, the resulting first SLL's for difference pattern is reduced to FSL_L= -54.53dB and FSL_L= -54.91dB for 40 and 60 elements.

The radiation patterns of 40, 60 elements for 10, 6, 4, 2 subarrays are computed that are reported in figures (7), (8) and figures (11), (12) and also the corresponding subarray configurations and its group weights are listed in tables (1), (2), (3) and (4).

Finally, for completeness, table (5) presented that 10 subarrays of difference pattern of reduced FSL_L=-54.97dB and table shows that the number of elements increases for increased subarrays, the difference patterns of FSL_L's may be reduced. So finally DES has a more robust exploration ability to reach the optimal point in the search space.

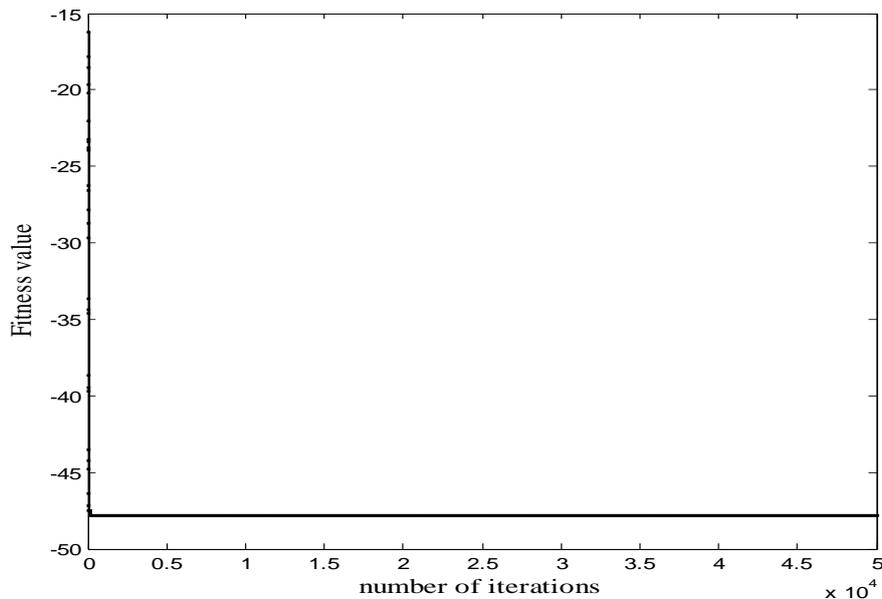


Figure: 5 Behavior of fitness function of 40 elements with SLL = -40dB.

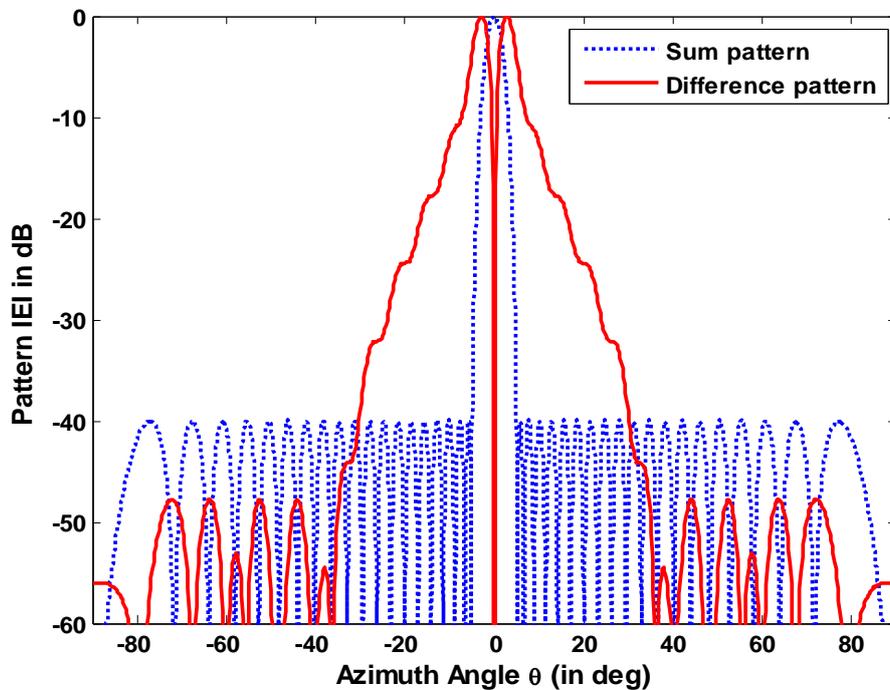


Figure: 6 Sum pattern for a 40 elements Dolph-Chebyshev array space factor with SLL=-40dB and the corresponding optimized difference pattern for P=8 subarrays

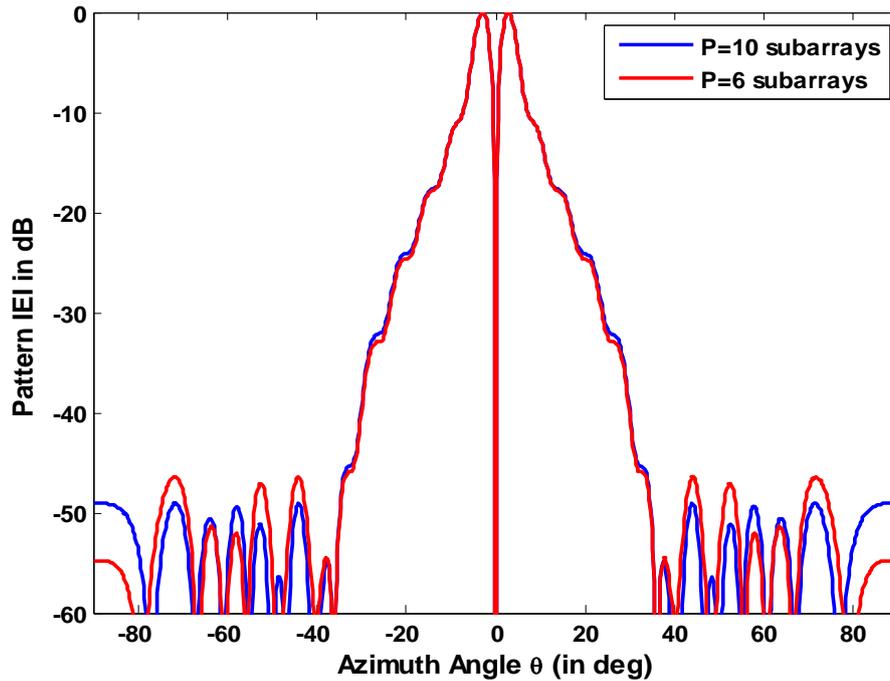


Figure: 7 optimized difference patterns for various values of P obtained by DE when the sum pattern is predetermined from the Dolph-chebyshev synthesis.

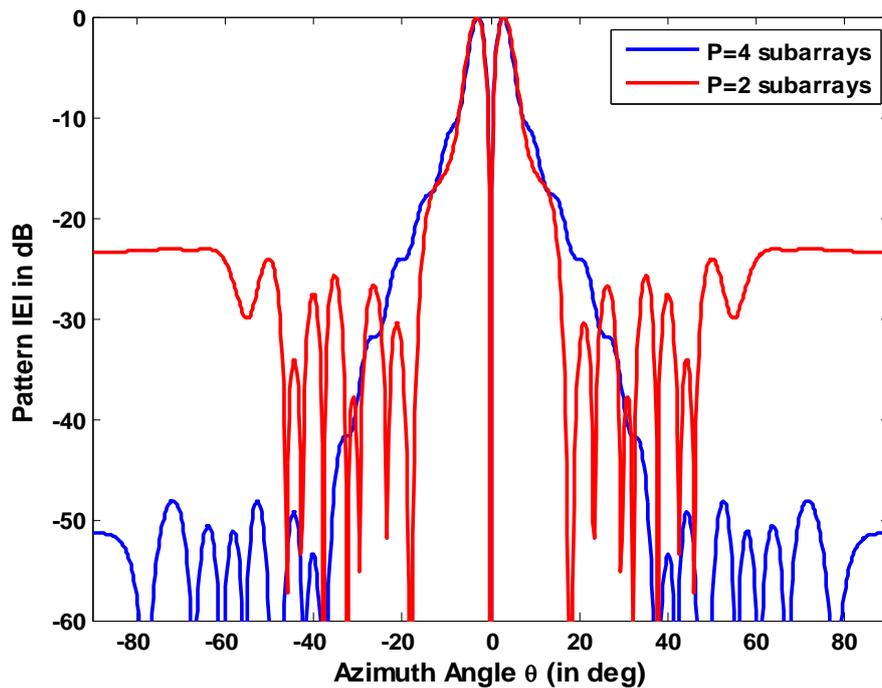


Figure: 8 optimized difference patterns for various values of P obtained by DE when the sum pattern is predetermined from the Dolph-chebyshev synthesis.

Table: 1: Subarray configurations of 40 element with SLL = -40dB

Subarray number	Optimal clustering in to subarrays and here showing only half of the array
P=10	[10,2,1,1,7,5,8,1,1,5,1,1,1,1,1,1,1,1,1,2,10]
P=8	[3,5,8,8,8,1,1,2,1,8,1,8,1,2,1,1,8,1,5,3]
P=6	[4,3,2,1,6,6,6,1,5,6,1,6,6,2,1,2,1,1,3,4]
P=4	[2,3,1,4,4,4,4,4,1,1,1,1,4,4,1,1,4,1,3,2]
P=2	[2,2,1,1,1,1,1,1,1,1,1,1,1,1,1,1,2,2,1,2,2,2]

Table: 2 Subarray weights for the arrays described in table 1

	P=10	P=8	P=6	P=4	P=2
G1	1.0000	0	1.0000	0.9957	0.8359
G2	0.8056	0.9955	1.0000	0.3213	0.3281
G3	1.0000	0.3120	0.7958	0.8105	
G4	0	1.0000	0.3084	1.0000	
G5	1.0000	0.8028	0		
G6	1.0000	1.0000	1.0000		
G7	0.9885	0			
G8	0	0.9969			
G9	0				
G10	0.3164				

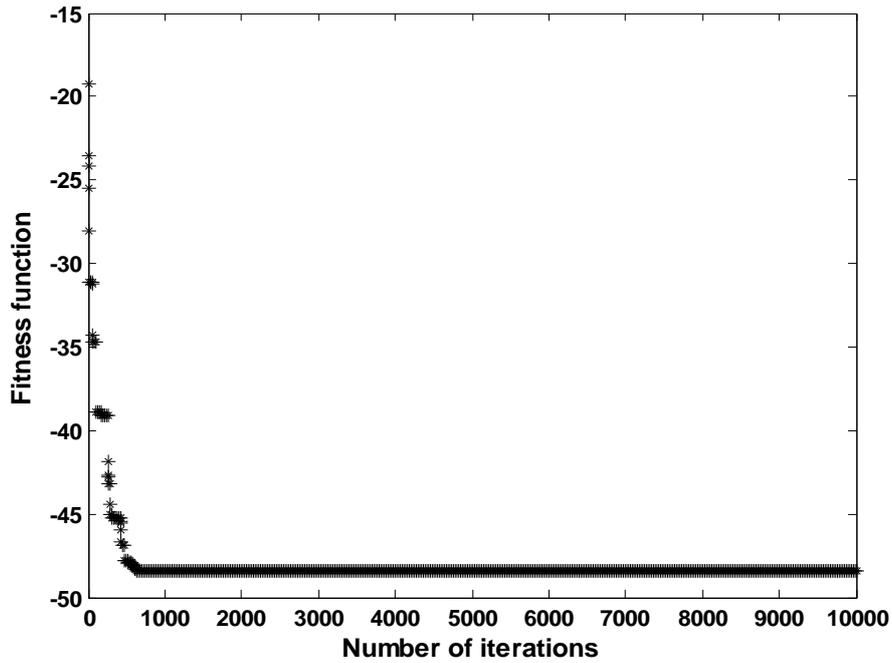


Figure: 9 Behavior of fitness function of 60 elements with SLL = -40dB.

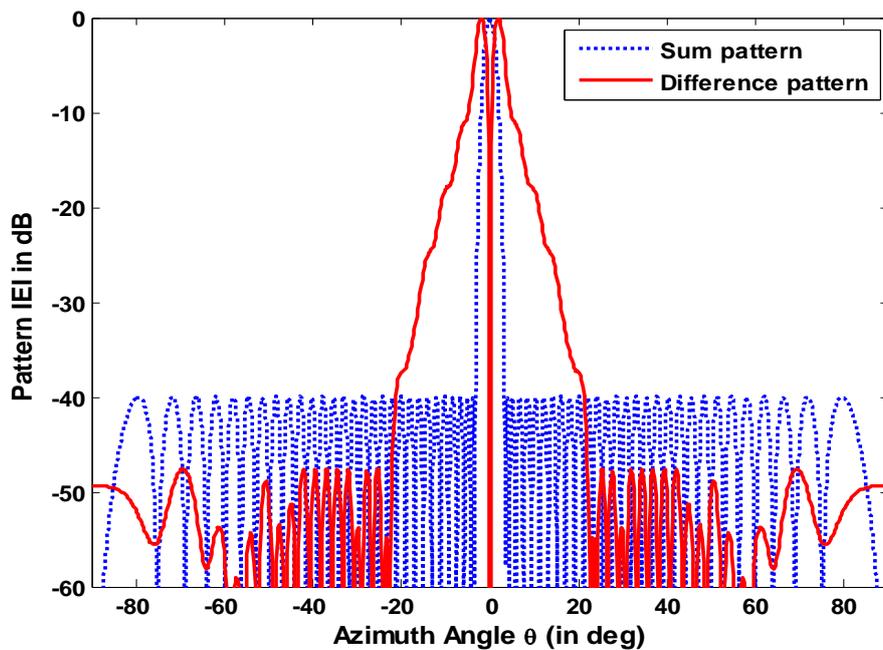


Figure: 10 Sum pattern for a 60 elements Dolph-chebyshev array space factor with SLL=-40dB and the corresponding optimized difference pattern for P=8 subarrays

Table: 5 Optimized difference pattern FSLL's obtained by DE with various values of P sub arrays for SLL = -40dB

Indices P	Number of elements 40	Number of elements 60
P=10	- 54.79 dB	- 54.97 dB
P=8	- 54.53 dB	- 54.91 dB
P=6	-54.43dB	- 51.04 dB
P=4	-41.45dB	- 54.67 dB
P=2	-30.46dB	-35.16 dB

V. Conclusion

In the present context of growing need in the field of electromagnetic problems a relatively simple and straight forward optimization method Differential Evolution Algorithm has been demonstrated. By employing the above, jamming and EMI problems can be reduces effectively. The design of monopulse antennas for which a newly proposed process based on subarray configuration has been reported and discussed with the help of latest results. The method has been checked with several array configurations and successfully synthesizes the sum and difference pattern in a linear array antenna with sufficiently ultra sidelobe level. Hence, the optimal solutions found by DE are equivalently as good as those obtained in 60 element array. Numerical simulation results of several array synthesis problems shows that the DES performs much better than PSO, Ant Colony Optimization and most of the other methods.

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