

Reed-Solomon Code Performance for GMSK Modulation over AWGN Channel

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Abstract: The main objective of this paper is to find the performance of Reed-Solomon code for Gaussian Minimum Shift Keying (GMSK) modulation over Additive White Gaussian Noise (AWGN) channel. The simulation model provides error detection and correction using Reed-Solomon (RS) Codes. The Original message is encoded and decoded using RS Codes. MATLAB simulation is selected as the investigating tool. The performance of proposed GMSK and RS code combination simulation is compared with uncoded system with the constraint that the transmission bandwidth is constant. We also performed the simulations for different code rates and different block lengths with fixed number of error. The results are presented by a plot between the bit error rates (BER) and signal to noise ratio using Monte Carlo simulation. The results show that for a given bandwidth, it is beneficial to use a smaller code rate but only to a certain value otherwise the system performance deteriorates. The BER performance also improves by decreasing code rate and by taking large block lengths or by increasing redundancy.

Index terms: AWGN, BER (Bit Error Rate), GMSK, Matlab, RS Codes, Partial Band Noise jamming (PBNJ), Galois field.

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I. Introduction

To achieve consistent and reliable data from the information source to the destination is one of the main issues in communication systems. The main objective of any communication system is transmission of data with the minimum error rate no matter whether it is digital or analog. The use of channel codes or Forward Error Correcting Codes in digital communication systems is an integral part of ensuring reliable communication [1] even in the presence of jamming. Jamming is an active attack with a purpose to prevent devices from exchanging information by interfering with their communication. There are various ways to counter jamming effect like making use of highly directional antennas, using Forward Error Correcting Codes & Spread Spectrum Communication. In case of Partial Band Noise jamming (PBNJ) jammer evenly distributes noise power over some frequency bandwidth which is a subset of total bandwidth. So as to mitigate this PBNJ effect one Forward Error Correcting Code is required which is more suitable to work against burst noise. Although there are various codes which work efficiently good for random errors but for burst noise or burst error Reed Solomon Code is the best.

Reed Solomon are non binary byte organised codes which is widely used in wireless communication, compact disc players and computers memories. Reed Solomon Codes are effective for deep fade channel and are considered as a structured sequence that is most widely used in burst error control.

The main objective of this paper is to evaluate the performance of Reed-Solomon codes in error correction control system in terms of bit error rate (BER). In proposed communication system the signal is transmitted using GMSK modulation technique in the presence of Additive White Gaussian Noise (AWGN). In GMSK, which is a subclass of continuous phase modulation the digital data stream is first shaped with Gaussian filter before being applied to MSK modulator. By using Gaussian filter, sideband power gets reduced which in turn yields excellent performance in the presence of interchannel interference (ICI). Compressing the bandwidth although avoids ICI but it causes an expansion in time domain which results in intersymbol interference (ISI). In order to remove ISI effect equalizers are required at the receiver end.

GMSK modulation method, first proposed by K. Murota and K. Hirade [4], is a widely used modulation scheme of cellular systems due to its compact Power Spectral Density and excellent error performance. Although the performance of GMSK has been analyzed by several researchers, coding for GMSK has received little attention [5].

This paper focuses on GMSK and Reed-Solomon (RS) coding. Error control codes insert redundancy into the transmitted data stream so that the receiver can correct errors that occur during transmission. Therefore, the bit interval of the coded bits is selected shorter in order to keep the information transmission rate constant. A

shorter bit interval results in a larger transmission bandwidth. To remain the bandwidth of the coded system same as that of the uncoded system, the modulator used in the coded system must adopt a smaller value of B_b [3].

This paper is organized as follows. The next section i.e. section 2 gives an overview of the system including a description of GMSK modulation and RS codes. Section 3 gives our approach to the bandwidth allocation problem. Simulation results for GMSK modulation with different Reed-Solomon coding rates for maintaining same bandwidth are presented in Section 4, and the conclusions are given in Section 5.

II. System Overview

RS/GMSK system model and Reed-Solomon coding system shown in Figure 1. The performance of various combinations of GMSK and RS codes is evaluated with the constraint that the total system bandwidth is constant. The bandwidth of GMSK can be easily controlled by the parameter B_b . The uncoded system is also evaluated to serve as a benchmark.

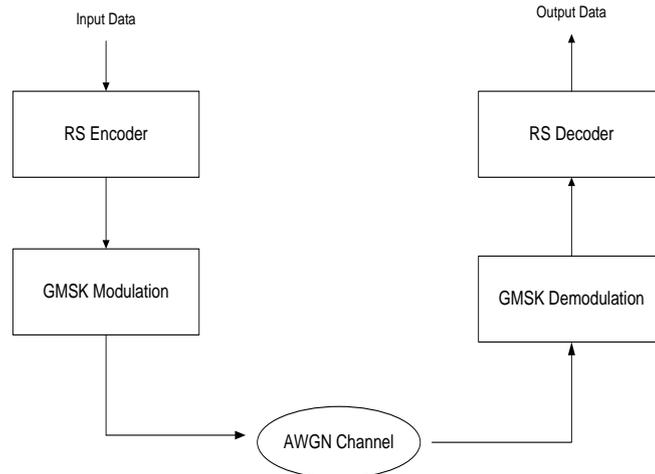


Fig. 1. RS/GMSK Model

A. GMSK Modulation

GMSK, as its name suggests, is based on MSK and was developed to improve the spectral properties of MSK by using a premodulation Gaussian filter. The filter impulse response is expressed as:

$$h(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{t^2}{2\sigma^2}\right) \dots\dots\dots(1)$$

$$\text{where } \sigma = \frac{\sqrt{\ln 2}}{2\pi B_b T}$$

The Gaussian filter is characterized by its $B_b T$ product (B_b is the -3dB bandwidth of the Gaussian prefilter and T is the symbol period.) The lower the $B_b T$ product, the narrower the modulation bandwidth. In this paper, we use $B_b T = 1.0$ and $B_b T = 0.5$ for the uncoded system. For transmission in an AWGN channel, the bit error rate of GMSK is given by

$$P = \frac{1}{h^2} \operatorname{erfc}\left(\sqrt{\frac{d_{\min}^2 E}{2N_0}}\right) \dots\dots\dots(2)$$

Where d_{\min} is the normalized minimum Euclidean distance between the signal representing “0” and the signal representing “1”, E is the energy per transmitted bit and $N_0/2$ is the power spectral density of the AWGN.

B. Reed-Solomon codes

Reed-Solomon codes are block-based error correcting codes with a wide range of applications in digital communications and storage. It is vulnerable to the random errors but strong to burst errors. Hence, it has good performance in fading channel which have more burst errors. In coding theory Reed-Solomon (RS) codes are cyclic error correcting codes invented by Irving S.Reed and Gustave Solomon [6]. They described a systematic way of building codes that could detect and correct multiple random symbol errors. By adding t check symbols to the data, an RS code can detect any combination of up to t erroneous symbols, and correct up to $\lfloor t/2 \rfloor$ symbols. As an erasure code, it can correct up to t known erasures, or it can detect and correct combinations of errors and erasures. Reed-Solomon codes are used to correct errors in many systems including:

- Storage devices (including tape, Compact Disk, DVD, barcodes, etc)
- Wireless or mobile communications (including cellular telephones, microwave links, etc)

- Satellite communications
- Digital television / DVB
- High-speed modems such as ADSL, xDSL, etc.

Block coding schemes involve dividing the input data into k-bit blocks and then mapping each k-bit block into an n-bit block called a codeword, where $n > k$ in the encoding process. (n-k) check bits are added to each k-bit block. The ratio $r = k/n$ is called the code rate. The data is partitioned into symbols of m bits, and each symbol is processed as one unit both by the encoder and decoder. RS codes satisfy: $n \leq 2^{m-1}$ and $n - k \geq 2t$, where t is the number of correctable symbol errors. Reed Solomon codes are polynomial codes over certain finite fields particularly useful in Burst error correction. Encoding & Decoding principles of nonbinary RS codes depends on Galois fields (GF). Symbols from extension galois field (2^n) are used in constructing RS codes. $GF(2^n)$ is extension galois field with 2^n elements. Let β be a primitive element in $GF(2^n)$ & $G(Z)$ be the Generator polynomial with roots $(\beta, \beta^2, \beta^3, \dots, \beta^{N-M})$. Then

$$G(Z) = \prod_{i=1}^{N-M} (Z - \beta^i) \dots \dots \dots (3)$$

Let (m_1, m_2, \dots, m_M) be the message symbols where $m_i \in GF(2^n)$ which is defined by a polynomial

$$P(Z) = m_1 + m_2 Z + m_3 Z^2 + \dots + m_M Z^{M-1} \dots \dots \dots (4)$$

& Hence Codeword Polynomial is

$$C(Z) = P(Z)G(Z) \dots \dots \dots (5) \quad \text{If during transmission some additive errors are introduced due to noise which is described by error polynomial } e(Z) = \sum_{j=0}^{n-1} e_j Z^j \dots \dots \dots (6) \quad \text{then received polynomial becomes } R(Z) = C(Z) + e(Z) \dots \dots \dots (7)$$

Various algebraic Decoding methods like Peterson-Gorenstein-Zierler (PGZ), Berlekamp-Massey Algorithm (BMA) and Euclidean method of Sugiyama are used for RS codes which are based on the idea of determining error location and error correction. Decoding algorithm for t error correcting RS codes is based on considering error polynomial $e(Z)$ which is

$$e(Z) = e_{n-1} Z^{n-1} + e_{n-2} Z^{n-2} + \dots + e_1 Z + e_0 \dots \dots \dots (8)$$

here v is total errors that actually occurs & t is error correcting capability of RS codes. Let these errors occur at locations $i_1, i_2, i_3, \dots, i_v$. The error polynomial can then be written as

$$e(Z) = e_{i_1} Z^{i_1} + e_{i_2} Z^{i_2} + \dots + e_{i_v} Z^{i_v} \dots \dots \dots (9)$$

Let The RS decoded symbol error probability, P_E , in terms of the channel symbol error probability, p, can be written as follows [7]:

$$P_e \approx \frac{1}{2^m - 1} \sum_{j=t+1}^{2^m - 1} \binom{2^m - 1}{j} p^j (1 - p)^{2^m - 1 - j} \dots \dots \dots (5)$$

Since RS code parameters have a significant effect on coding performance, RS codes with different rates are tested here e_{i_k} is the magnitude of k^{th} error. For error correction we must know two things error locations & magnitude of these errors. Thus, the unknowns are $i_1, i_2, i_3, \dots, i_v$ & $e_{i_1}, e_{i_2}, \dots, e_{i_v}$, which signify the locations & the magnitudes of the errors respectively. The syndrome can be obtained by evaluating the received polynomial at α

$$S_1 = v(\alpha) = c(\alpha) + e(\alpha) = e(\alpha) = e_{i_1} Z^{i_1} + e_{i_2} Z^{i_2} + \dots + e_{i_v} Z^{i_v} \dots \dots \dots (10)$$

If error magnitudes are defined as $Y_k = e_{i_k}$ for $k=1, 2, \dots, v$ & error locations are $Z_k = \alpha^{i_k}$ for $k=1, 2, \dots, v$, where i_k is the location of k^{th} error & Z_k is the field element associated with this location then Syndrome can be written as $S_j = Y_1 Z_1^j + Y_2 Z_2^j + \dots + Y_v Z_v^j \dots \dots \dots (11)$

We can evaluate the received polynomial at each of the powers of α , thus we have following set of 2t equations with v unknown error locations Z_1, Z_2, \dots, Z_v & the v unknown error magnitudes Y_1, Y_2, \dots, Y_v ,

$$S_1 = Y_1 Z_1 + Y_2 Z_2 + \dots + Y_v Z_v \dots \dots \dots (12)$$

$$S_2 = Y_1 Z_1^2 + Y_2 Z_2^2 + \dots + Y_v Z_v^2 \dots \dots \dots (13)$$

$$\vdots$$

$$S_{2t} = Y_1 Z_1^{2t} + Y_2 Z_2^{2t} + \dots + Y_v Z_v^{2t} \dots \dots \dots (14)$$

If the error locator polynomial is defined as

$$A(z) = A_v z^v + A_{v-1} z^{v-1} + \dots + A_1 z + 1 \dots \dots \dots (15)$$

Then zeros of this polynomial are the inverse error locations Z_k^{-1} for $k=1, 2, \dots, v$ i.e.

$$A(z) = (1 - zZ_1)(1 - zZ_2) \dots (1 - zZ_v) \dots \dots \dots (16)$$

So, if we know the coefficients of the error locator polynomial $A(z)$, we can obtain the error locations Z_1, Z_2, \dots, Z_v . Since error locations are now known these form a set of 2t linear equations. These can be solved to obtain error magnitudes.

III. PERFORMANCE EVALUATION

In order to determine what combination of coding rate and B_b for the coded system results in the same bandwidth as the uncoded system, we must choose the measure of bandwidth. In this paper, we have used the percent power containment bandwidth, denoted by B_x and defined as the bandwidth which contains $x\%$ of the signal power. B_{90} , B_{99} and $B_{99.9}$ are plotted in Figure 2.

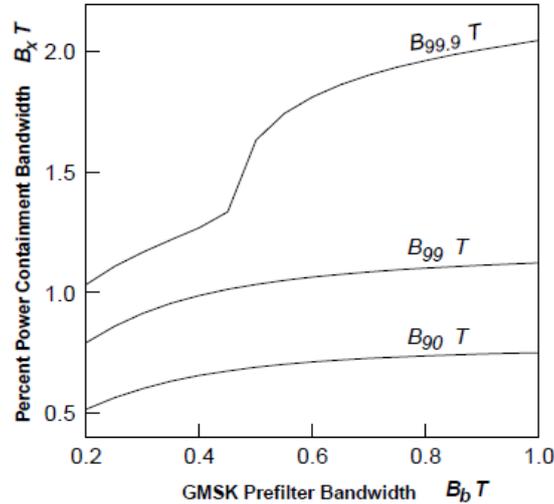


Fig. 2. Percent power containment bandwidths for GMSK

$B_{99.9}$ for GMSK, which is the bandwidth that contains 99:9% of the signal power is used in the simulations. In Equation (2) for the uncoded system, the value of E is E_b , which is the energy per transmitted information bit. For the coded system, the value of E is set to be rE_b , since the energy for the coded bits is spread among the more numerous coded bits. This allows a fair comparison to be made between the uncoded and coded systems. It is complicated to compute the bit error probability p_b by using Equation (2) and (3) because RS codes are nonbinary codes, so we use MATLAB. The simulation model is shown in Figure 3. In the simulations the following parameters are used.

- Input data 100000 symbols
- RS codeword length: 31,63, 127

SIMULATION PARAMETERS

Item	Value
Channel Model	AWGN
Modulation	GMSK
Channel Coding	Reed Solomon
Codeword Length	127,63,31
Data Rate	1Mbps
Frequency	850 MHz

$B_b T$ AND RS CODE PARAMETER COMBINATIONS THAT RESULT IN SYSTEMS WITH EQUAL BANDWIDTHS

n=31			
Uncoded	Coded		
	k=25	k=23	k=21
0.5	0.45	0.35	0.25
1.0	0.51	0.48	0.45

n=63			
Uncoded	Coded		
	k=57	k=53	k=49
0.5	0.47	0.46	0.40
1.0	0.64	0.54	0.49

n=127			
Uncoded	Coded		
	k=107	k=103	k=99
0.5	0.47	0.45	0.40
1.0	0.54	0.51	0.49

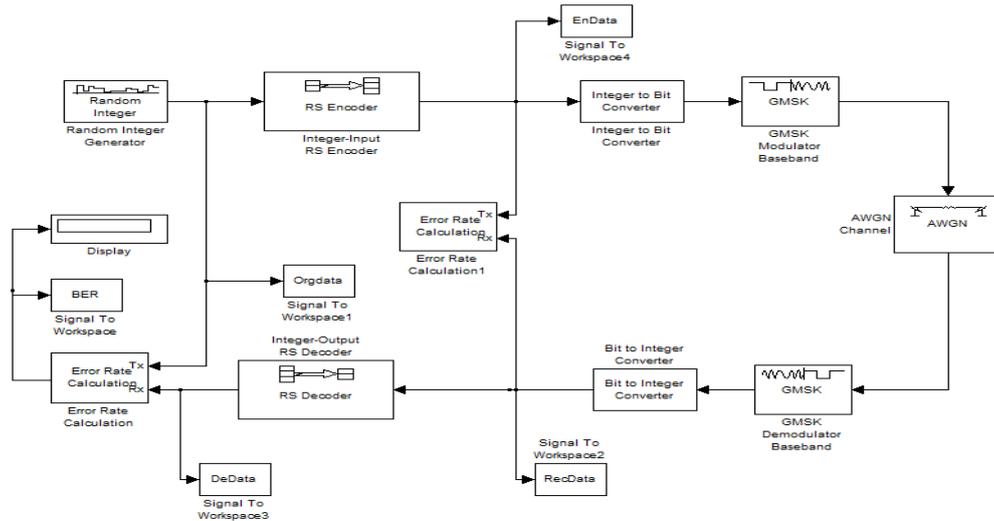


Fig. 3. Proposed Simulation Model

III. RESULT & ANALYSIS

In this section, the parameters used in the simulation model are discussed. Simulation results are also presented. Proposed Simulation is used to evaluate the BER (Bit Error Rates) for different systems. The code parameters used in this simulation are RS(127, k) and RS(63, k). To ensure the bandwidths of the coded and uncoded systems remain the same, first of all we have to calculate the value of B_bT . Let us see how to calculate the value of B_bT , if we set the value of $B_bT = X$ for the uncoded system, then $B_{99,9}T = Y$ from Figure 2. When the RS coding rate is r , the coded $B_{99,9}T = Z$ is calculated from $Z = Y \times r$. The corresponding value of B_bT is found from Figure 2. The parameters used in the simulations are shown in Table I and Table II. The simulation results are shown in Figures 4–10. From Figure 4, it is clear that as the code rate decreases or we can say that as the value of B_bT decreases, it results in decreasing the bit error probability. When the value of B_bT reaches to 0.4 for RS(63,49) codeword probability of BER reaches a minimum value. but, as the code rate decreases further, probability of BER increases instead. The reason is that the code rate r decreases but the bit error rate of GMSK p increases, so the net result of decreasing r depends on how much p increases in response to a decrease in r . Due to decrease in r when p increases more, P_b becomes worse. Similar result can also be found out from Figure 5, 6 and 7 for different system parameters. From Fig 4–10, we can also show the SNR required to achieve $P_b = 10^{-4}$ when the value of $B_bT = 0.5$ and $B_bT = 1.0$ for the uncoded system. A smaller value of B_bT means a smaller code rate. All the figure shows, as the code rate is decreased, the performance improves until a minimum is reached and then it gets worse. The optimal combination of coding and modulation for a given total transmission bandwidth can be found from these figures. The (63, 49) RS shows the maximum gain (6dB) achieved over the uncoded system ($B_bT = 0.5$) (see figure 4 and 6). For $n=127$, the (127,107) RS has the largest gain at (5.8dB) (see figure 5 and 7).

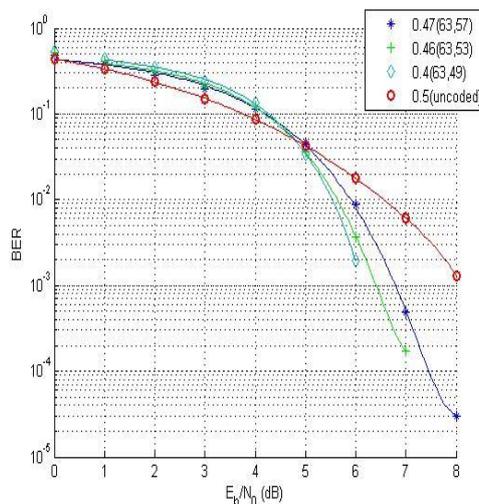


Fig. 4. Performance for GMSK-RS(63:k) code combinations with the same bandwidth as an uncoded system with $B_bT = 0.5$

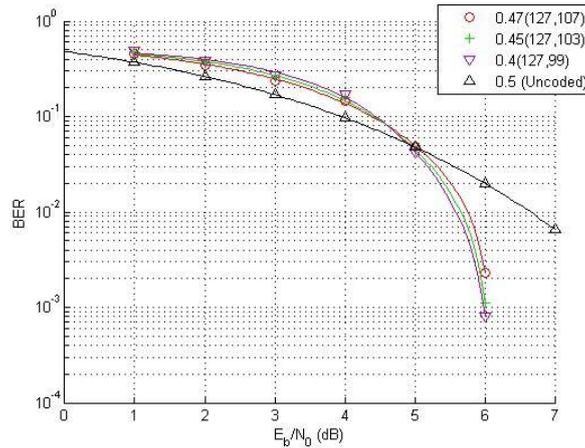


Fig. 5. Performance for GMSK-RS(127;k) code combinations with the same bandwidth as an uncoded system with $B_bT = 0.5$

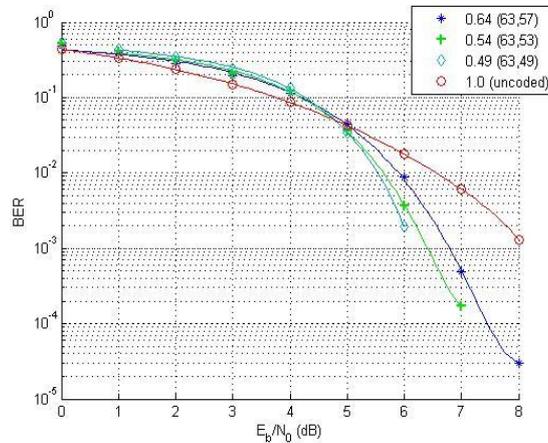


Fig. 6. Performance for GMSK-RS(63;k) code combinations with the same bandwidth as an uncoded system with $B_bT = 1.0$

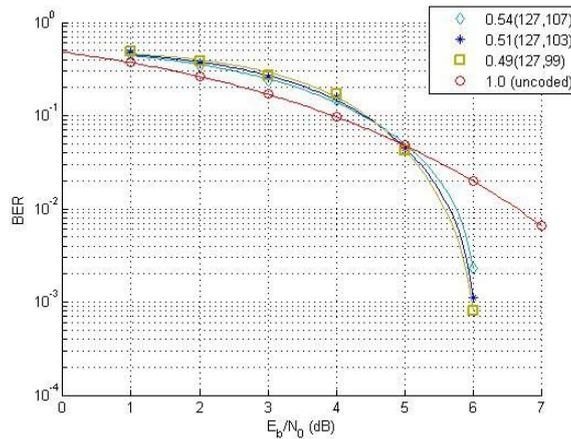


Fig. 7. Performance for GMSK-RS(127;k) code combinations with the same bandwidth as an uncoded system with $B_bT = 1.0$

Next we performed the simulations for RS codes for different code rates and different block length with fixed number of error correction code. We can see from the figure 8, as the block length decreases the BER also decreases and the system performance also improves. But the further decrease in block length causes increase in BER (block length (15, 9) So here the best result comes out with RS (31, 25) with $m= 5$ i.e. number of bits per symbol is 5. We can now say that for RS codes the BER performance improves with the decreasing code rate but by taking large block lengths. The RS code, which is well suited for correction of burst errors, shows a poor BER performance for lower SNR values, because of the random errors introduced by the AWGN.

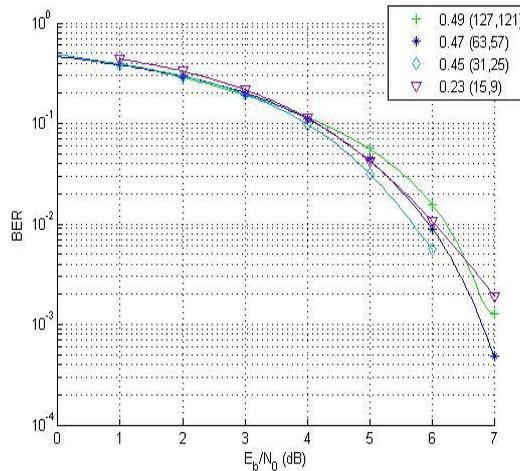


Fig. 8. The BER performance comparison of RS codes for different code rates and different block length but Fix No. of error correction capabilities

To do this experimentsto be run we use the following RS codes set; RS (127,123), RS (127,119), RS (127,111) and RS (127, 95) respectivelyfor simulation results. The above RS code sets have identical codeword symbols (n) whereas the number of datasymbols (k) decreases, which results in decrease in code ratefrom $r = 0.98$ to 0.87 which means increasing in redundancy from 4 to 32 symbols. We have selected the field size 128, the codeword and data symbols can be measured as 7-bit symbols [8]. From the performance curve given below(see fig 9&10) , it isobserved that as the redundancy increases the error-correcting Reed-Solomon code become more efficient i.e., errorperformance improves as the redundancy increases (lower code rate) as shown in fig. 9.we can also see the similar result from fig. 10.for 63 code word symbols.

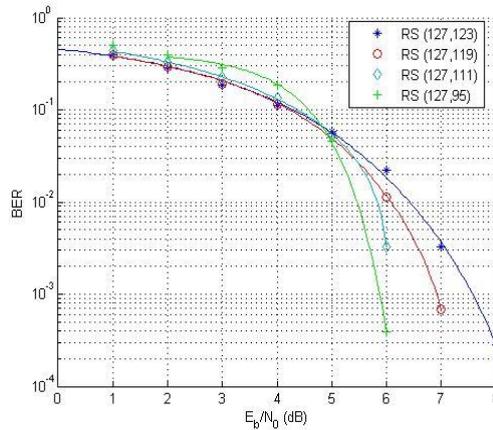


Fig. 9. The BER performance ofReed-Solomon code with increase in Redundancy on codeword 127 having symbol length 7 bits.

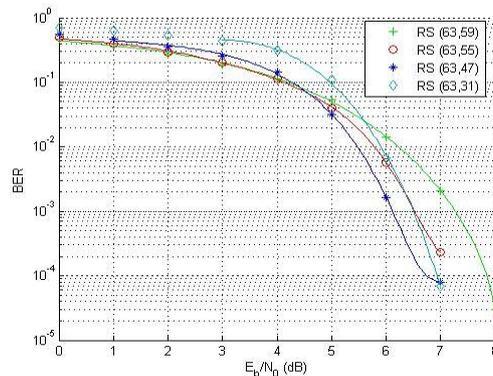


Fig. 10.The BER performance of Reed-Solomon code with increase in Redundancy oncodeword 63 having symbol length 6 bits.

IV. Conclusion

In this paper, we examined the performance of communication system over GMSK modulations which employ RS channel coding. Under a constant bandwidth constraint, we optimized the combination of coding and modulation. The Proposed system results show that for same bandwidth as uncoded system, a coded system bit error rate probability performance can be improved for a given bandwidth by taking a smaller code rate. However, when the code rate further decreases, the system performance decreases. For this reason the optimal code rate was found to be a function of the total system bandwidth. We also performed the simulations for different code rates and different block lengths with fixed number of error correction capabilities and result shows that the BER performance can also be improved by decreasing code rate but for large block lengths. In the proposed technique MATLAB simulation is selected as the investigating tool. The results are presented by a plot between the bit error rates (BER) and signal to noise ratio using Monte Carlo simulation.

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