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Dilution of Position Calculation for MS Location Estimation in Wireless Communication Systems

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Abstract: *Geometric dilution of precision (GDOP) represents the geometric effect on the relationship between measurement error and positioning determination error. When the measurement variances are equal in each other, GDOP could be the most appropriate selection criterion of location measurement units. GDOP expression has simpler form if all the measurements with the same variance. For time of arrival (TOA) schemes, the maximum volume method of GDOP calculation does not guarantee the optimal selection of the four measurement units. The conventional method for calculating GDOP is to use matrix inversion to all subsets. GDOP was originally used as a criterion for selecting the right 3D geometric configuration of satellites in global positioning systems (GPS). In this paper, we employ GDOP using the matrix inversion method to select appropriate base stations (BSs) in cellular communication systems. The proposed BS selection criterion performs better than the random subsets of four or five BSs chosen from all seven BSs. After BS selection, the proposed distance-weighted method and threshold method for TOA schemes yields superior mobile station (MS) location estimation accuracy. For time of difference arrival (TDOA) schemes, the proposed BS selection criterion provides better MS location estimation. From simulation results, the performances of MS location strongly depend on the relative position of the MS and BSs.*

Keywords: *Geometric dilution of precision (GDOP), Time of arrival (TOA), Time of difference arrival (TDOA), Non-line-of-sight (NLOS).*

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I. Introduction

GDOP is defined under the assumption of equal pseudo-range error variance [1]. GDOP can be approximately inversely proportional to the volume of the tetrahedron formed by four satellites [2-3]. The popular schemes for estimating the location of mobile station (MS) in wireless communication systems include angle of arrival (AOA) [10], time of arrival (TOA) [11], and time difference of arrival (TDOA) [12] techniques. The AOA scheme utilizes an antenna array and a directive antenna to estimate the direction of arrival signal. TOA location scheme measures the propagation time for a radio wave to travel between the MS and a BS. The TDOA scheme measures the time difference between the radio signals.

For both TOA and TDOA schemes, we employ the subset with the minimum GDOP to determine the MS location estimation in cellular communications system. The matrix inversion method is used to calculate GDOP value and only a subset with minimum GDOP is selected for location process. In our simulations, we only consider the subsets of four or five measurements. By using the BS selection criterion, the results imply that an improvement in MS location accuracy is very obvious. Simulation results show that the proposed BS selection criterion always gives the better MS location accuracy comparing with the random subsets of four or five BSs. It is enough for selecting four BSs for the compromise between completeness of data and simplification of computation. With the proposed BS selection criterion, the proposed distance-weighted method and threshold method provide much better MS location estimation for TOA schemes.

Mobile Location Methods for TOA schemes

With the proposed BS selection criterion, MS location can be estimated by the Taylor series algorithm (TSA) [4-5], linear lines of position algorithm (LLOP) [6], distance-weighted method and threshold method which we have proposed in [7].

Taylor Series Algorithm (TSA)

Several methods have been presented to solve the nonlinear problem. TSA is the most useful in linearizing the non-linear equations. By measuring the propagation times of the signals traveling between the MS and various BSs, the distances between the MS and BSs can be obtained. Let t_i denote the propagation time from the MS to BS i. The distances between BS i and the MS can be expressed as

$$r_i = \sqrt{(x - X_i)^2 + (y - Y_i)^2}\ ,\ \ i = 1, 2, \ldots 7 \tag{1}$$

where (x, y) and (X_i, Y_i) are the locations of the MS and BS i, respectively. If (x, y) is the true position and (x_v, y_v) is the initial estimated position, let $x = x_v + \delta_x$, $y = y_v + \delta_y$. The MS location can be estimated by linearizing the TOA equation using Taylor's series expansion and neglecting the higher order terms.

$$r_i \cong r_{vi} + a_{i1}\delta_x + a_{i2}\delta_y \tag{2}$$

where $r_{vi} = \sqrt{(x_v - X_i)^2 + (y_v - Y_i)^2}$, $a_{i1} = \left.\frac{\partial r_i}{\partial x}\right|_{x_v, y_v} = \frac{x - X_i}{r_i}$, $a_{i2} = \left.\frac{\partial r_i}{\partial y}\right|_{x_v, y_v} = \frac{y - Y_i}{r_i}$. δ_x , δ_y are respectively, coordinate offset of x, y.

The linearized TOA equations can be expressed in matrix form as

$$z \cong A\delta \tag{3}$$

where $z = \begin{bmatrix} r_1 - r_{v1} \\ r_2 - r_{v2} \\ \vdots \\ r_i - r_{vi} \end{bmatrix}$, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \vdots \\ a_{i1} & a_{i2} \end{bmatrix}$, $\delta = \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix}$, and $r_i = \sqrt{(x - X_i)^2 + (y - Y_i)^2}$.

The vector variable δ in Eq. (3) can be solved as follows

$$\delta = (A^T A)^{-1} A^T z\ . \tag{4}$$

Calculation starts with an initial guess for the MS location, and update this value iteratively until the magnitude of δ below a given threshold value. This method is recursive and the computational overhead is intensive. It requires a proper initial position guess close to the true solution and convergence is not guaranteed [4-5].

Linear Lines of Position Algorithm (LLOP)

The new geometrical interpretation makes use of linear lines of position (LLOP) to replace the circular LOP for estimating the MS location [6]. The line which passes through the intersections of the two circular LOPs for two TOA measurements can be found by squaring and subtracting the distances obtained by Eq. (1) for $i = 1, 2$ and can be expressed as

$$2(X_1 - X_2)x + 2(Y_1 - Y_2)y = r_2^{\,2} - r_1^{\,2} + (X_1^2 + Y_1^2) - (X_2^2 + Y_2^2)\ . \tag{5}$$

The MS location is identified by

$$Gl = h \tag{6}$$

where $l = \begin{bmatrix} x \\ y \end{bmatrix}$ denotes the MS location,

$$G = \begin{bmatrix} X_1 - X_2 & Y_1 - Y_2 \\ X_1 - X_3 & Y_1 - Y_3 \\ \vdots & \vdots \\ X_1 - X_i & Y_1 - Y_i \end{bmatrix} \text{ and } h = \frac{1}{2}\begin{bmatrix} r_2^2 - r_1^2 + (X_1^2 + Y_1^2) - (X_2^2 + Y_2^2) \\ r_3^2 - r_1^2 + (X_1^2 + Y_1^2) - (X_3^2 + Y_3^2) \\ \vdots \\ r_i^2 - r_1^2 + (X_1^2 + Y_1^2) - (X_i^2 + Y_i^2) \end{bmatrix}.$$

Hence, the solution to Eq. (6) can be obtained by

$$l = (G^T G)^{-1} G^T h \tag{7}$$

Proposed Distance-Weighted Method and Threshold Method

From the viewpoint of geometric approach, TOA value measured at any BS can be used to form a circle, centered at the BS. The MS position is then given by the intersections of the circles from multiple TOA measurements. In order to achieve high accuracy with less effort, distance-weighted method and threshold method which we have proposed in [7] can be applied to determine MS.

Mobile Location Methods for TDOA schemes

For TDOA approach, TSA [4] and least square (LS) method [13] are used to determine the MS location.

Taylor Series Algorithm (TSA)

TDOA measurement is obtained from subtracting two TOA measurements. The range difference between the i th BS and BS1 can be expressed as

$$r_{i1} = r_i - r_1 = \sqrt{(x-X_i)^2 + (y-Y_i)^2} - \sqrt{(x-X_1)^2 + (y-Y_1)^2} \tag{8}$$

Equation (8) into Taylor series and retaining the first two terms produce

$$r_{i1} \cong r_{vi1} + b_{i1}\delta_x + b_{i2}\delta_y \tag{9}$$

Where $r_{vi1} = \sqrt{(x_v-X_i)^2 + (y_v-Y_i)^2} - \sqrt{(x_v-X_1)^2 - (y_v-Y_1)^2}$ $b_{i1} = \frac{\partial r_{i1}}{\partial x} = (\frac{x-X_i}{r_i}) - (\frac{x-X_1}{r_1})$,

$b_{i2} = \frac{\partial r_{i1}}{\partial y} = (\frac{y-Y_i}{r_i}) - (\frac{y-Y_1}{r_1})$. The linearized TDOA equations can be described by

$$\gamma \cong B\delta \tag{10}$$

where $\gamma = \begin{bmatrix} r_{21} - r_{v21} \\ r_{31} - r_{v31} \\ \vdots \\ r_{i1} - r_{vi1} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ \vdots & \vdots \\ b_{i1} & b_{i2} \end{bmatrix}$.

The MS location is obtained through the use of a TSA expansion and the solution to Eq. (8) can be obtained by

$$\delta = (B^T B)^{-1} B^T \gamma . \tag{11}$$

Least Square Method (LS)

By squaring Eq. (1), we can obtain the following equations

$$(x-X_1)^2 + (y-Y_1)^2 = r_1^2 \tag{12}$$

$$(x-X_i)^2 + (y-Y_i)^2 = r_i^2 = (r_{i1} + r_1)^2 ,\ \ i = 2,3,\ldots 7 \tag{13}$$

Subtracting the Eq. (12) from Eq. (13) result in

$$E\delta = F \tag{14}$$

where $E = \begin{bmatrix} X_2 - X_1 & Y_2 - Y_1 \\ X_3 - X_1 & Y_3 - Y_1 \\ \vdots & \vdots \\ X_i - X_1 & Y_i - Y_1 \end{bmatrix}$, $F = \frac{1}{2} \begin{bmatrix} (X_2{}^2 + Y_2{}^2) - (X_1{}^2 + Y_1{}^2) - r_{21}{}^2 - r_{21} \cdot r_1 \\ (X_3{}^2 + Y_3{}^2) - (X_1{}^2 + Y_1{}^2) - r_{31}{}^2 - r_{31} \cdot r_1 \\ \vdots \\ (X_i{}^2 + Y_i{}^2) - (X_1{}^2 + Y_1{}^2) - r_{i1}{}^2 - r_{i1} \cdot r_1 \end{bmatrix}$.

The LS solution for TDOA scheme is [13]

$$\delta = (E^T E)^{-1} E^T F \tag{15}$$

Calculation of GDOP for TOA and TDOA schemes

GDOP was initially developed as a criterion to help select the optimal 3D geometric configuration of satellites in GPS. High GDOP describes a situation in which a relatively small ranging error can result in a large position location error. GDOP is a task of choosing the appropriate measurement units, which results in the better geometric configuration and the more accurate position estimate. In order to improve the positioning accuracy, we should minimize GDOP among the selected measurement units. In the range measurements, the accuracy varies with the error, as well as the relative positions of the MS and BSs. If the measurement errors are uncorrelated and have equal variances, GDOP can be defined as [14]

$$GDOP = \sqrt{trace(H^T H)^{-1}} \ . \qquad (16)$$

The geometry matrix for TOA schemes and TDOA schemes are

$$H = \begin{bmatrix} \partial r_1/\partial x & \partial r_1/\partial y & 1 \\ \partial r_2/\partial x & \partial r_2/\partial y & 1 \\ \vdots & \vdots & \vdots \\ \partial r_i/\partial x & \partial r_i/\partial y & 1 \end{bmatrix}, \text{ and } H = \begin{bmatrix} \partial r_{i1}/\partial x & \partial r_{i1}/\partial y \\ \partial r_{i2}/\partial x & \partial r_{i2}/\partial y \\ \vdots & \vdots \\ \partial r_{ij}/\partial x & \partial r_{ij}/\partial y \end{bmatrix}, \text{ respectively.}$$

Proposed BS selection criterion

To select the most appropriate set of BSs, which will give the minimum positioning error, GDOP effect must be considered in cellular communication systems. When enough measurements are available, the optimal measurements selected with the minimum GDOP can prevent the poor geometry effects, thereby improving the MS location accuracy. The excessive measurement increases the computational load and can not improve the location accuracy. To further reduce the computational overhead and improve location performance, the selection of optimal measurement units is necessary.

In general, the subset with smallest GDOP provides more accurate MS location results. We use a set of four or five BSs selected from among seven to estimate MS location in cellular communication systems, as shown in Fig.1. Those BSs are the ones with the minimum GDOP. The proposed BS selection criterion is as follows: Select n measurements among seven BSs to generate different subset in cellular communication systems, there are divided into $C(7,n)$ measurement subsets. GDOP is computed for all subset of n measurement units and the subset which gives the smallest GDOP is selected. Finally, n measurement units of this subset are used to find out the MS location solution.

Simulation Results

We attempt to improve the performance of the MS location estimate in cellular communication systems. We consider a center hexagonal cell (where the serving BS resides) with six adjacent hexagonal cells of the same size, as shown in Fig. 1. Each cell has a radius of 1 km and the MS location is uniformly distributed in the center cell [8]. The serving BS, that is, BS1, is located at (0, 0). Different methods based on GDOP to select the best subset of four or five BSs to estimate the MS location. The dominant error for wireless location systems is usually due to the NLOS propagation effect. The NLOS propagation model is based on the circular disk of scatterers model (CDSM) [9]. The measured ranges are the sum of the distances between the BS and the scatterer and between the MS and the scatterer.

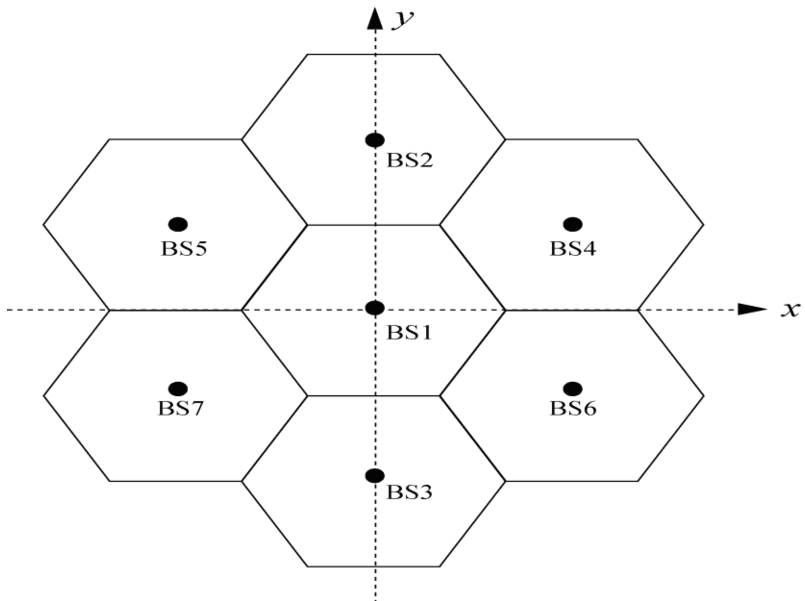

Fig. 1. Seven-cell system layout.

Based on the above BS selection criterion, the most straightforward location method employs the BSs with minimum GDOP to estimate the MS location. To give a comparison of different subsets, Fig. 2 provides the root mean square (RMS) error varies as the radius of CDSM. Four BSs selected randomly with poor geometry perform extremely worse location estimation and the accuracy of mobile location can be strongly affected by the relative geometry between BSs and MS. In order to eliminate the poor geometry effects, the selecting BSs with minimum GDOP criterion can be used and optimal geometric configuration with four measurements are obtained.

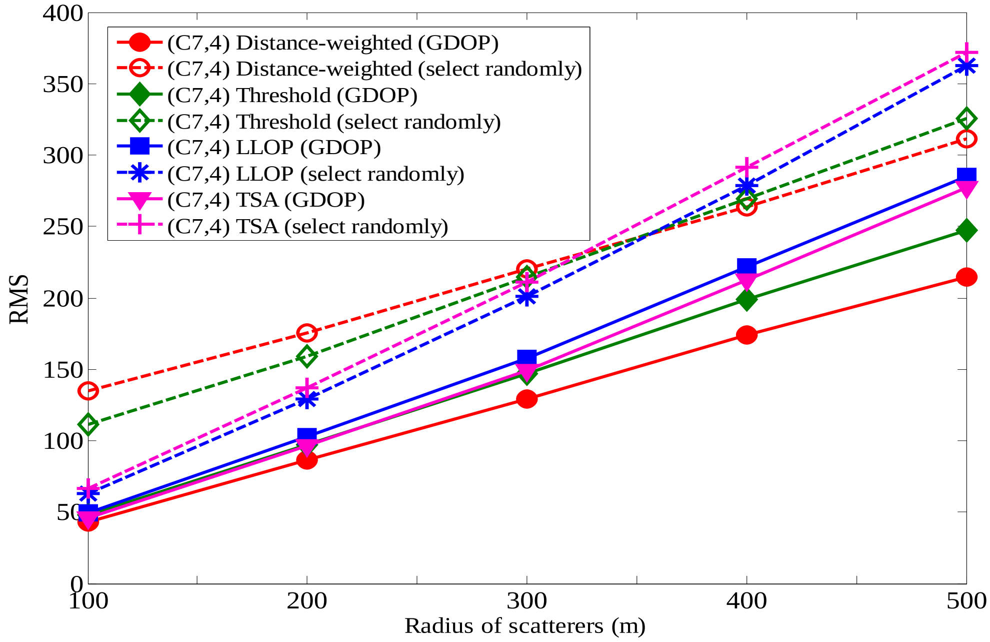

Fig. 2. Average location error versus the disc radius of CDSM.

Figure 3 was performed to examine how the proposed BS selection criterion compares with the subset selecting five BSs randomly when the radii of CDSM are varied. The subset with minimum GDOP always provides much better location estimation than the other subsets with five BSs taken from seven BSs randomly regardless of the different methods.

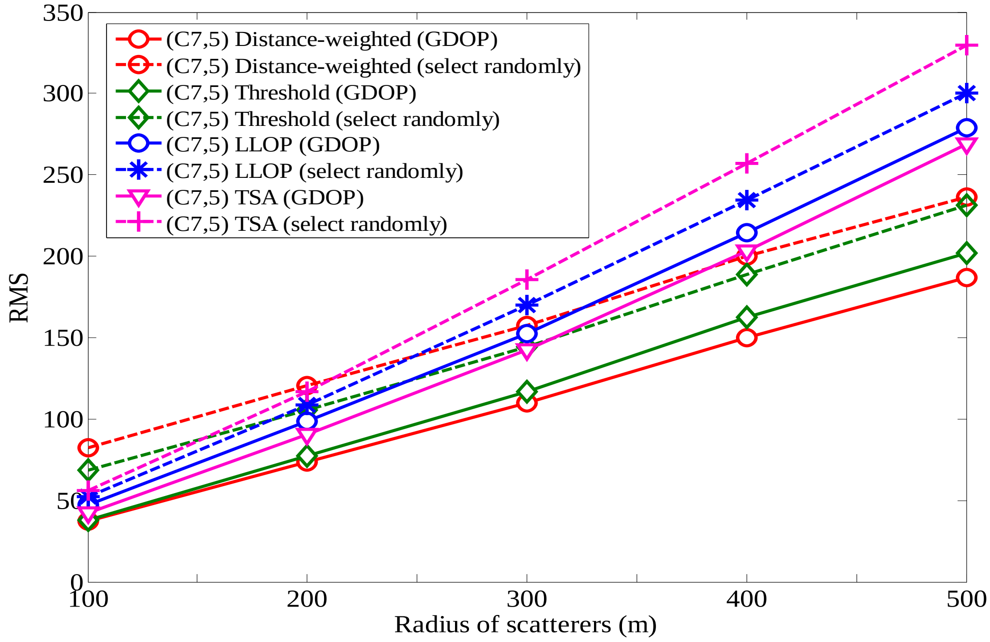

Fig. 3. Comparison of RMS errors when NLOS errors are modeled as CDSM.

Figure 4 compares the results of the subset with minimum GDOP and using all seven BSs method. The radius of the scatters of CDSM is assumed to be 100 m. The larger the number of the selected BSs, the more accurate the positioning is. By using LLOP, distance-weighted method and threshold method, the positioning precision of using all seven BSs slightly overmatched that of the minimum GDOP subset with five BSs. For TSA, the optimal subset with five BSs gives the equal level of performance as using all seven BSs method. From simulation results, it is enough to select four BSs according to the best geometry to obtain the huge decrease of the positioning error.

In TDOA schemes, the NLOS propagation model is based on the uniformly distributed noise model [5], in which the TOA measurement error is assumed to be uniformly distributed over $(0, U_i)$, for $i = 1, 2, \ldots 7$ where U_i is the upper bound. The improvement in MS location estimation using the proposed BS selection criterion can be seen in Fig. 5. Four BSs are randomly selected with relatively poor accuracy and the geometric configuration between BSs and MS is critical affecting the positioning accuracy seriously. It can be seen that LS performs better than TSA method.

Figure 6 shows how the average location error is affected by the upper bound of NLOS errors. The superior performance for the proposed BSs criterion has been demonstrated when comparing the RMS error. Five randomly selected BSs with poor geometry yield bad location estimation and the proposed BSs criterion provides precise location estimation even in severe NLOS conditions.

The upper bound of NLOS error is chosen as follows: $U_i = 300$ m. Figure 7 shows the CDF of the average location error of the minimum GDOP subset and using all seven BSs method. If more BSs are involved in the subset, these methods will give better location performance improvement. The minimum GDOP subset with five BSs and using all seven BSs method provide a comparable level of accuracy in MS location estimation.

Summary

In order to eliminate the poor geometry influence and improve the positioning accuracy, the minimum GDOP subset can be used to estimate the location of MS in cellular communication networks. In our simulations, only four or five BSs with best geometry among seven BSs are chosen to determine MS location. It can be seen that the subset with minimum GDOP for predicting MS location provides high degree of accuracy.

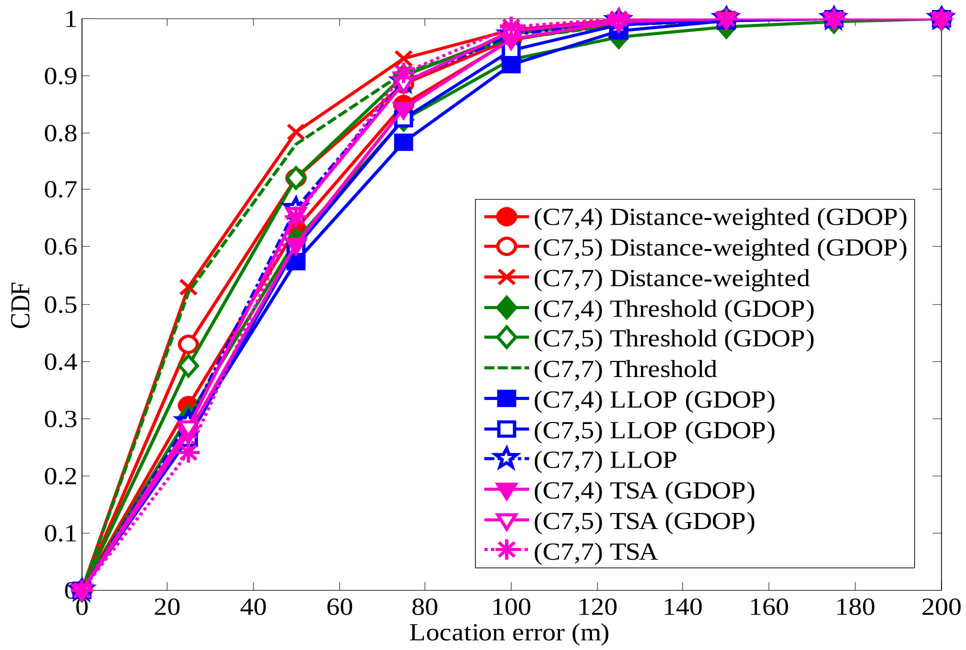

Fig. 4. Comparison of location error CDFs using all seven BSs and the subset with minimum GDOP.

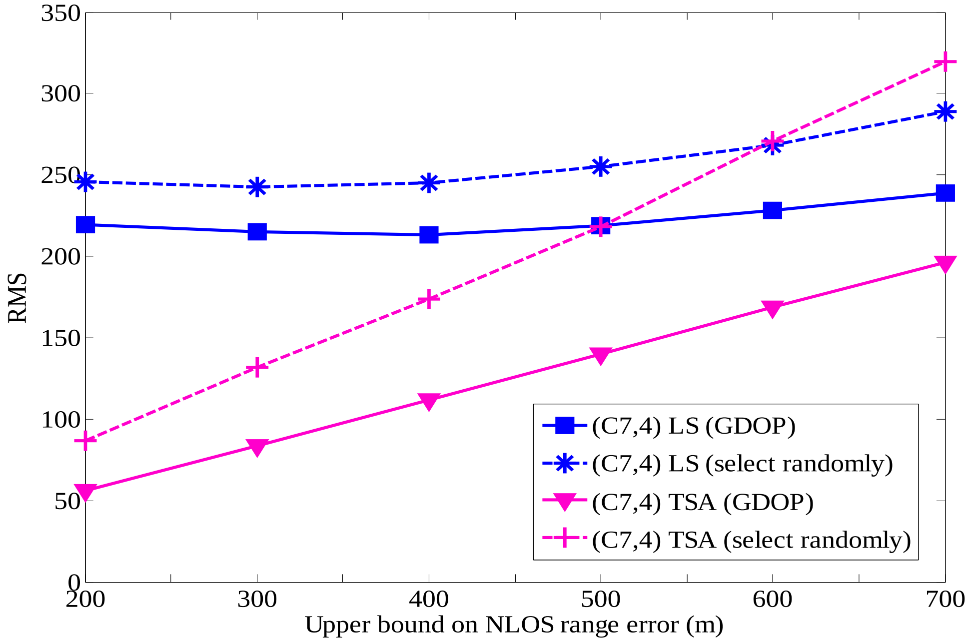

Fig. 5. Average location error versus the upper bound of NLOS errors.

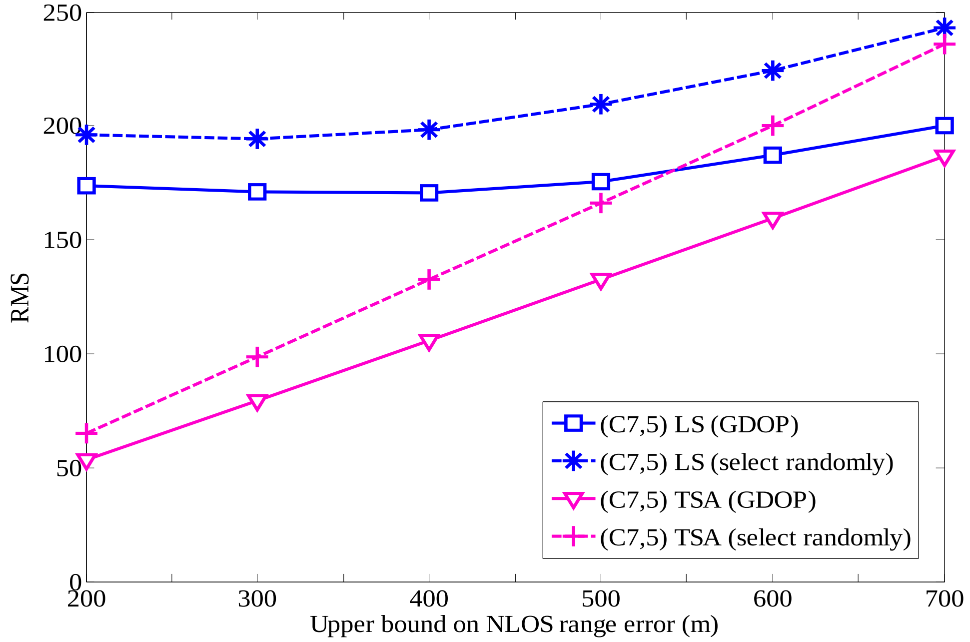

Fig. 6. Performance comparison between the location estimation methods when the upper bound is used to model the NLOS.

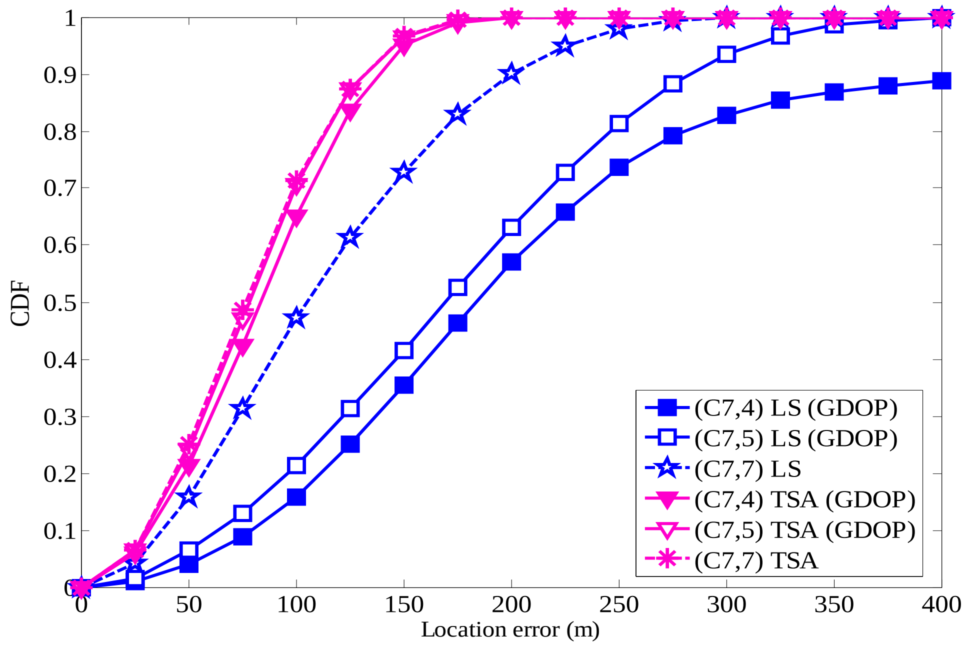

Fig. 7. Comparison of error CDFs when the subset with minimum GDOP and using all seven BSs.

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