Study and Analysis of Fractal Linear Antenna Arrays

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Abstract: A fractal is a recursively generated object having a fractional dimension. They have no characteristic size, and are constructed of many copies of themselves at different scales [1]. In this report, we primarily focus on design of fractal linear array antenna for even number of elements and comparison of fractal linear array antenna for even, odd number of elements and also with conventional antenna[6]. These arrays are analyzed by fractal electrodynamics and to simulate results by MATLAB programming.

Keywords: conventional antenna, fractals, fractal linear arrays, fractal electrodynamics, fractional dimension

I. Introduction

The wireless industry is witnessing a volatile emergence today in present era. Today’s antenna systems demand versatility and inconspicuous. Operators are looking for systems that can perform over several frequency bands are reconfigurable as the demands on the system changes. Some applications require antenna size to be as decreased as possible. Fractal construction method plays a prominent role for these needs. Fractals have non-integral dimensions and their space filling capability could be used for decreasing antenna size and their property of being self-similarity in the geometry leads to have antennas which have a large number of resonant frequencies. Fractal antenna arrays also have Multiband performance is at non-harmonic frequencies. Fractal antenna arrays have good Impedance, good SWR (standing wave ratio) performance on a reduced physical area when compared to non fractal Euclidean geometric methodology. Fractal antenna arrays show Compressed Resonant behavior. At higher frequencies the Fractal antenna arrays are naturally broadband. Polarization and phasing of Fractal antenna arrays is possible. In many cases, the use of fractal element antennas can simplify circuit design. Often fractal antenna arrays do not require any matching components to achieve multiband or broadband performance. Rearrangements could be applied to shape of fractal antenna to make it to resonate at various frequencies.

1.1 Fractal’s Definition:
The word fractal derived from the Latin fractus meaning broken, uneven; any of various extremely irregular curves or shape that repeat themselves at any scale on which they are examined [5].

Fractal comes into two major variations:
1. Deterministic fractals
2. Random fractal

1.2. Types of Fractals:
Deterministic fractal: The fractals of this class are visual. In two-dimensional case they are made of a broken line (or of a surface in three-dimensional case) so-called the generator. Each of the segments which forms the broken line is replaced by broken line generator at corresponding scale for a step of algorithm [5].

Random fractal or stochastic fractals: The stochastic fractals have random characteristics, (i.e.) they have no exact dimensions for expansion. Two dimensional stochastic fractals are used for designing surface of sea or relief modeling [5].
Geometric array or deterministic array: Those fractals that are composed of several scaled down and rotated copies of it, They are called Geometric fractal antenna arrays.

There are different types of geometric fractal antenna arrays [1]:
1) Triadic cantor linear antenna array
2) Sierpinski carpet antenna array
3) Sierpinski triangle antenna array

II. Description of method
In this report we analyze cantor linear array for even and odd number of elements and compare results with each other and also with the conventional linear array.

2.1. Cantor fractal linear array
Cantor linear array: A linear array of isotropic elements uniformly spaced a distance d apart along the z axis, is shown in below figure [1]. The array factor to the corresponding to this linear array may be expressed in the form,

\[ A_F (\psi) = I_0 + 2 \sum_{n=1}^{N} I_n \cos (n \psi) \quad \text{for } 2N+1 \]  \hspace{1cm} (1)

\[ A_F (\psi) = 2 \sum_{n=1}^{N} I_n \cos (n - 1/2) \psi \quad \text{for } 2N \]  \hspace{1cm} (2)

Where,
\( \psi = k d \left[ \cos \theta - \cos \theta_0 \right] \) and
\( k = 2\pi/\lambda \).
The array becomes fractal like, when appropriate elements are turned on and off or removed, then it is given by,
\[ I_n = \begin{cases} 1 & \text{if element is turned on} \\ 0 & \text{if element is turned off} \end{cases} \]

2.2) Fractal linear array for odd number of elements:
The array factor for odd number of elements [1]:
\[ F(s, \psi(\theta)) = \prod_{n=1}^{N} g(s, \psi(\theta)) \]

Where
\( s = \) scale (or) expansion factor.
\( G^w = \) the array factor associated subarray.
The array factor of fractal is comes from, array factor of the linear array of “N” odd radiating elements, because here we expand our array with the “3” iterations. (W.K.T)

\[ I_0 + 2 \sum_{n=1}^{N} I_n \cos (n \psi) \quad \text{for } 2N+1 \]  \hspace{1cm} (1)
Then, the fractal behavioral characteristics such as number of elements, amplitudes of the elements should be substituted in equation (1),

\[
G(s, \psi(\theta)) = 2 \cos(\psi(\theta))
\]

The generating fundamental sub array factor, For \( n \) elements,

\[
g(s, \psi(\theta)) = \cos(s^{-p-1}\psi(\theta))
\]

Here the expansion factor is for “3”,

\[
f(3, \psi(\theta))_{NOR} = \prod_{p=1}^{N} \cos(3^{p-1}\psi(\theta))
\]

Directivity of the odd element array:

\[
D_p(u) = 2^p \prod_{p=1}^{p} \cos^2(3^{p-1} \frac{\pi u}{2})
\]

2.3. The array factor for even number of elements

Hence, fractal array produced by following this procedure belong to a special category of thinned arrays. One of the simplest schemes for constructing fractal array follows the recipe for the cantor set.

Table 2.1 element distribution of the fractal linear array for even elements

<table>
<thead>
<tr>
<th>( n )</th>
<th>Element pattern</th>
<th>Active elements</th>
<th>Total elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1010</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1010000010100000</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>101000001010000000000000000000000101000001010000000000000000000000</td>
<td>8</td>
<td>64</td>
</tr>
</tbody>
</table>

The array factor of the four elements generating sub array with the representation 1010 is, From equation (2),

\[
2 \sum_{n=1}^{N} \cos(n - \frac{1}{2}) \psi, \text{ for } 2N
\]

\[
A.F = 2 \cos(\frac{\psi}{2})
\]

Then the array Factor for cantor linear element of even elements,

\[
g(s, \psi(\theta)) = 2 \cos(s^{-p-1}\psi(\theta)\frac{\theta}{2})
\]

\[
f(4, \psi(\theta))_{NOR} = \prod_{p=1}^{N} \cos(4^{p-1}\psi(\theta)\frac{\theta}{2})
\]

The directivity of even element:

\[
D_p(u) = 2^p \prod_{p=1}^{p} \cos^2(4^{p-1} \frac{\pi u}{2})
\]

III. Results

The field patterns of fractal linear antenna array for even number of elements present multiband characteristic features. A multiband behavior is one in which the side-lobe ratio and radiation pattern characteristics are similar at several frequency bands.

The patterns are constructed for various iterations \( p=1, 2, 3, 4 \), and with scaling factor \( s=4 \), and the minimum separate distance between the array elements is \( d=\lambda \), where \( \lambda \) is the fixed design wavelength. It can be concluded that as each time the wavelength is increased, the closest elements collapse into an equivalent array from the radiation point of view. Hence at each longer wavelength the equivalent array effectively loses one iteration. The pattern of array factor with iteration \( p=3 \) is the envelop of the fine structures of the pattern with \( p=4 \), and so is the pattern with \( p=2 \) and that of \( p=3 \). It can be generalized to the circumstance with \( p \) and \( p+1 \).
the fractal were ideal, it would keep exactly the same shape after collapsing, but since it is band limited fractal, the array and its radiation patterns remain similar but not equal, at each frequency.

![Array factor of cantor linear array for even number of elements](image1)

**Fig 3.1** Array factor of cantor linear array for even number of elements

<table>
<thead>
<tr>
<th>Number of iterations p</th>
<th>Number of active elements</th>
<th>Total number of elements</th>
<th>Resolution(rad)</th>
<th>Sidelobe level (dB)</th>
<th>Beam area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>-3</td>
<td>6.3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>16</td>
<td>2</td>
<td>-3</td>
<td>0.78</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>32</td>
<td>0.75</td>
<td>-2.5</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>64</td>
<td>0.25</td>
<td>-2</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 3.1, number of elements, resolution, side lobe level for uniform cantor fractal array of scaling factor $s=4$ (even elements)

![Array factor of cantor linear array for odd number of elements](image2)

<table>
<thead>
<tr>
<th>Number of iteration, p</th>
<th>Number of active elements</th>
<th>Total number of elements</th>
<th>Resolution(rad)</th>
<th>Side lobe level(dB)</th>
<th>Beam area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0.5</td>
<td>-11.06</td>
<td>6.3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>9</td>
<td>0.5</td>
<td>-5.51</td>
<td>0.78</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>27</td>
<td>0.4</td>
<td>-5.25</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>81</td>
<td>0.03</td>
<td>-5.1</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 3.2, number of elements, resolution, side lobe level (sll) for uniform cantor fractal array with scaling factor $s=3$ (odd element cantor array)[2]
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Table 3.3, number of elements, resolution, and side lobe level for conventional linear array [2]

<table>
<thead>
<tr>
<th>Number of elements</th>
<th>Resolution</th>
<th>Side lobe level(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>-11.3</td>
</tr>
<tr>
<td>2</td>
<td>0.19</td>
<td>-12.8</td>
</tr>
<tr>
<td>3</td>
<td>0.06</td>
<td>-13.3</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>13.25</td>
</tr>
</tbody>
</table>

IV. Conclusion

Fractal linear arrays designed by fractal geometry gives improved results when compared to conventional linear array antenna except side lobe level, and when comparison held on fractal linear array of odd, even elements such as number of elements, beam area, side lobe level and resolution, fractal odd element linear array gives good results than even element antenna except beam area. Automatically directivity of odd number cantor linear antenna array is improved because of less beam area and resolution. The outputs are simulated using MATLAB7.6.

References

Journal Papers:

Books: