

## Implementation of MIMO Radar for Multiple Target Detection

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**Abstract:** This paper focuses on the early detection problem of multiple moving targets in statistical MIMO radar systems using TBD techniques. At first, assuming prior knowledge of the number of targets, a binary generalized likelihood ratio test (GLRT) is derived, which shows that the optimal implementation of the GLRT requires multidimensional joint search. To reduce the implementation complexity, a suboptimum multitarget TBD algorithm using successive-target cancellation and polar Hough transform (STC-PHT) is proposed. In addition to low complexity, the new proposed algorithm doesn't need the prior information of the number of targets, and can avoid the implementation of multi-hypothesis test when the number of targets is unknown.

**Keywords:** Antenna, Detect, Processor, Noise, Target, Threshold, Track, Velocity.

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### I. Introduction

Target tracking has been playing a prominent role in today's world. In the past decades tracking has been in a wide area of applications in both military and commercial systems. Tracking involves estimating unknown dynamic quantities of the target of interest on the basis of observed data from sensors. Inspired by recent advances in Multiple-Input Multiple-Output (MIMO) communications, statistical MIMO radar concept is developed. MIMO radars employ multiple transmit waveforms and jointly process signals received from multiple receive antennas. Statistical MIMO radar overcomes target RCS fluctuations by averaging over many decorrelated channels between transmit and receive antennas. Based on the processing chain of measurement to yield final track outputs, tracking methods can be classified as Detect Before Track (DBT) and Track Before Detect (TBD) algorithm. In DBT radar tracking methods consider thresholded measurements, called plots, as an input. When the energy reflected by a target is too low, no plot can be constructed and this target will therefore be declared lost. In high noise and clutter, target tracking techniques, such as the Kalman filter, probabilistic data association (PDA) and multiple hypotheses testing (MHT), declare detections at each measurement time. The detections are then used to estimate the target trajectory. A limitation of these methods is that much of the information contained in the measurements is discarded due to the application of a detection threshold. This is akin to the information loss when hard rather than soft decisions are made in communication system receivers [1].

Track before detect (TBD) is a technique for target detection and tracking that is useful when the signal-to-noise ratio (SNR) is low. Unlike the above approaches, in TBD detections are not declared at each frame. Instead, a number of frames of data are processed, after which the estimated target track is returned when the detection is declared.

The literature on TBD is limited. In the early 1980s, unthresholded CCD images were used to track satellites against background noise. In [2], 3-D matched filtering is applied to moving target detection using optical images, while in [3] a recursive moving-target indication algorithm is used. These methods assume that the target velocity is known. Their performance degrades in the presence of a velocity mismatch or a target maneuver.

The DP approach originally proposed in [5] avoids problems with velocity mismatch and can handle slowly maneuvering targets. The analysis in [6] adequately describes the detection performance of the DP algorithm in [5] but does not address the tracking performance of the algorithm. In [4], the DP algorithm of [5] is modified and extended to improve efficiency, enhance performance in non-Gaussian noise and allow velocity transitions.

While the above studies rely on the critical hypothesis that only a single target may be present in the coverage area, the aim of this work is to investigate TBD procedures in multi-target environments. Also, the number K of prospective targets may be either known or unknown. This is indeed a problem of primary relevance in both military and civil real-world applications.

The contribution of this study is manifold and can be summarized as follows.

1) First, the TBD problem assuming that the number of prospective targets is a priori known at the receiver is solved. In this case, a binary GLRT is derived. Optimal implementation of the GLRT involves a joint (across targets) constrained maximization which can be solved either via dynamic programming with a nonlinear complexity in the number K of targets [8] or via an equivalent minimum network flow optimization with a nonlinear complexity in the number M of integrated frames.

2) Next, the more challenging problem of simultaneously tracking and detecting multiple targets is done using Successive Target Cancellation - Polar Hough Transformation (STC\_PHT). For notation, bold lower and upper case roman letters designate column vectors and matrices, respectively. Superscript 'H' denotes the complex conjugate transpose.

**A. Problem Definition**

The problem of jointly tracking and detecting multiple targets, each approaching the radar along its own trajectory is considered. A generalized GLRT is derived and using Successive Target Cancellation –Polar Hough Transformation number of targets and tracks are found out.

**II. Mathematical Model**

The statistical MIMO radar system under consideration consists of  $N$  omnidirectional antennas for transmitting and  $M$  uniform linear arrays with  $d$  elements for receiving. At each illumination (i.e., during each CPI), each transmit antenna emits a burst signal with  $L$  pulses, and the  $N$  burst signals.

The transmitted signals are orthogonal to each other and are used to generate  $MN$  observation channels. The total number of illuminations is  $Q$ . At the  $q$ th illumination, all the vectors observed by the  $MN$  channels are stacked into a long  $v$ -dimensional column vector  $Z_q$ .

$$z_q = \sum_{k=1}^K H_{q,k} \rho_{q,k} + w_q = H_q \rho_q + w_q \quad (1)$$

Where  $=MNd$ ,  $K$  is the number of targets,

$H_q = [H_{q,1} \dots \dots \dots H_{q,k}]$ , is space time mode matrix and  $\rho_q = [\rho_{q,1}^H \dots \dots \dots \rho_{q,k}^H]^H$ , complex amplitude vector of the  $k$ th target at the  $q$ th illumination, respectively. The  $H_{q,k}$  is a block diagonal matrix which is shown as follows

$$H_{q,k} = \begin{bmatrix} h_{q,k,11} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & h_{q,k,MN} \end{bmatrix} \quad (2)$$

Where  $h_{q,k,mn}$  is the space-time steering vector of the  $k$ th target at the  $q$ th illumination and the  $m$ th observation Channels. The noise  $w_q$  is a complex Gaussian random vector with zero mean and covariance matrix  $R_q$ .

Then the GLRT for (2) is as follows

$$\text{Max}\{(\sum_{q=1}^Q z_q^H S_q^{-1} H_q (H_q^H S_q^{-1} H_q)^{-1} H_q^H S_q^{-1} z_q)\} \quad (3)$$

Where the  $z_1 \dots z_q$  are mutually statistically independent due to time diversity. The  $S_q$  is the maximum likelihood estimation of  $R_q$ , and it is assumed that required training data samples are well selected.  $P_q$  is a set of  $K$  targets positions at the  $q$ th illumination, and each point in  $P_q$  belongs to a two-dimensional measurement space  $(\tau, \theta)$ , where  $\tau$  is the delay (i.e., the range), and  $\theta$  is the azimuth.  $\phi(\cdot)$  is a transition function defining the set of physically admissible target movements between consecutive illuminations.

Assuming that targets are mutually spaced at least one range resolution cell apart at each illumination and each observation channel, as a consequence, the  $H_{q,k}^H S_q^{-1} H_{q,k} = 0$  holds, and the GLRT (3.3) is simplified as follows,

$$\text{Max}\langle \sum_{q=1}^Q \sum_{k=1}^K T_{q,k} \rangle. \quad (4)$$

$$T_{q,k} = \sum_{q=1}^Q \sum_{k=1}^K z_{q,k}^H S_q^{-1} H_{q,k} (H_q^H S_q^{-1} H_q)^{-1} H_{q,k}^H S_q^{-1} z_{q,k}$$

Where  $z_{q,k}$  is the observation vector corresponding to the  $k$ th target at the  $q$ th illumination.

Accordingly, the GLRT can be reduced to the following two steps:

Step 1: According to (3.5), the merit function (i.e., test statistic)  $T_{q,i}$  is evaluated using the observation vector  $z_{q,i}$  in the  $i$ th range-azimuth resolution cell at the  $q$ th illumination. Here, according to the range and azimuth resolutions of the statistical MIMO radar system, the two-dimensional measurement space  $(\tau, \theta)$  is discretized to be  $N_\tau N_\theta$  range azimuth resolution cells, and the state of the  $i$ th data point at the  $q$ th illumination is denoted by the vector  $(T_{q,i}, \tau_{q,i}, \theta_{q,i})$ .

$i=1 \dots \dots N_\tau N_\theta$  As a consequence, all the data points' obtained at the  $q$ th illumination compose the  $q$ th data-set.

Step 2: According to the transition function, all of the  $K$  candidate trajectories throughout the  $Q$  data-sets are found out, and simultaneously the merit functions of data points of these trajectories are accumulated in order to make a decision.

**A. Polar Hough Transformation:**

This algorithm proposed for signal processing in the paper, is the improved version of the algorithm for target detection and track initiation with the Hough transform, proposed by Carlson, Evans and Wilson in [7]. In order to keep the constant false alarm rate in conditions of randomly arriving impulse interference, in the range-azimuth-time space, signal detection is carried out by a CFAR detector instead the detector with a fixed detection threshold. The CFAR processor is a detector, which maintains a constant false alarm probability in the process of target detection. In such a detector, target detection is declared if the signal sample exceeds a preliminary determined threshold.

After radar scan, each of the radars forms the local polar data space  $(r_n, a_n)$ . Where  $r_n$  is  $[0, r_n^{max}]$  and  $a_n$  is  $[0, 360^\circ]$  target range and azimuth, respectively of the  $N^{th}$  radar. All co-ordinate systems are oriented to the "North", and the earth curvature is neglected. The first stage is data association of the N-th radar co-ordinate systems to the Global Co-ordinate System (GCS), as shown in Fig. 1.

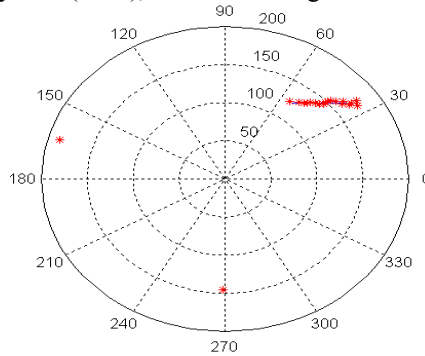


Fig. 1: Local polar data space

The second stage is a Polar Hough Transform (PHT), (fig.3). The PHT maps points (targets) from everyone associated local observation space (associated data map) into curves in the Hough parameter space, termed as the (r-q) plane, by

$$\rho = r_n \cdot \cos(a_n - \theta) \quad 0 < (a_n - \theta) \leq \pi \quad (5)$$

Where the parameter  $\rho$  represents the smallest distance between the line trajectory and the origin of the global polar co-ordinate system, while  $\theta$  is the angle of the vector from the origin to this closest point.

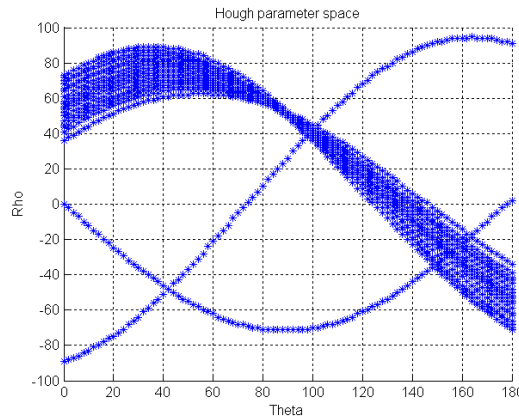


Fig. 2: Local Hough parameter space

If a line trajectory exists in the global (r-a) space, by means of polar Hough transform it is represented as a point of intersection of sinusoids defined by PHT.

The parameters  $\rho$  and  $a_n$  present the linear trajectory in the Hough parameter space and can be transformed back to the global data space showing the current distance to the target. If the number of binary integrations (BI) of data in the Hough parameter space (of intersections in any of the cells in the parameter space) exceeds the detection threshold, both target and linear trajectory detections are indicated, use the Polar Hough Transform for detection of targets located on one straight-line trajectory at the same time (Fig.3).

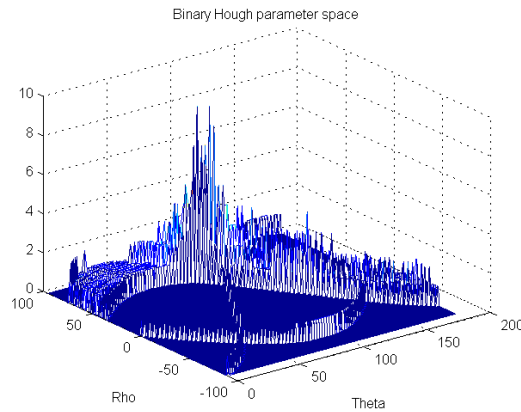


Fig. 3: Binary Houghparameter space

When the difference between the accuracy characteristics is significant it is necessary to adjust the dimensions of the Hough parameter space – to increase or decrease the size of accumulator cell. This leads to resolution decreasing when detection closely located of crossing trajectories.

The Polar Hough Transform is only efficient if a high number of radar measurements fall in the right range-azimuth area which correspondence of one accumulator cell in Hough space, so that the binary integration can be easily detected amid the background noise. This means that the accumulator cell must not be too small, or else some radar measure will fall in the neighboring Hough cells, thus reducing the visibility of the main cell. Also, much of the efficiency of the Polar Hough Transform is dependent on the quality of the input data: the edges must be detected well for the PHT to be efficient. Use of the PHT on noisy background (presence of false alarm) is a very delicate matter and generally, is necessary more radar measurement, since it has the nice effect of attenuating the noise through summation.

### III. The STC-PHT-Based Multiple Target TBD Algorithm

#### A. The Procedures of this Algorithm

Step 1: A primary threshold  $Z1$  is set and any data points with merit function exceeding this threshold are mapped into Hough parameter space by PHT.

Step 2: In the accumulator cell with maximum accumulated value, the transition function is applied to find out all of feasible trajectories in this cell. Then, the accumulated merit function for each feasible trajectory is evaluated and their maximum is compared with the secondary threshold  $Z2$  to declare detection.

Step 3: Firstly, if a target is declared, the number of targets is increased by 1, the corresponding trajectory is recorded, and all the data points which belong to this target should be deleted from all the accumulator cells. Then, whether a target is declared or not, the accumulated value of this accumulator cell is defined as 0. Accordingly, a new Hough parameter space is set up.

Step 4: Repeating the step 2 and 3 throughout the whole Hough parameter space, the estimates of targets' number and trajectories are obtained finally.

It is noted that a key step in this algorithm is evaluation of the primary threshold  $Z1$  and the secondary threshold  $Z2$  which is discussed in section III.

#### B. The Expression of False-Alarm Probability PFA

1) Calculation of the primary threshold  $Z1$

Primary threshold value is calculated by using probability of false alarm as follows

$$p_{fa} = 1 - E_b \left\{ \Psi \left( \frac{bZ_1}{(G + bZ_1)}, MN, G - v1 \right) \right\} \quad (6)$$

Where  $b$  is complex beta-distributed statistic with parameters  $G - v + MN + 1$  and  $v - MN E_b \{ \cdot \}$  denotes the expectation operator with respect to  $b$ ,  $\Psi(\alpha, \beta, \gamma)$  denotes probability distribution function of beta-distribution with parameters  $\alpha$  and  $\beta$  evaluated at  $\gamma$ ,  $G$  is the number of training samples for covariance matrix estimation.

2) Calculation of secondary threshold  $Z_2$

The merit function is distributed as a complex  $F$  distribution conditioned on  $b$ , so it is difficult to solve  $Z_2$  based on the exact distribution of the accumulated merit function. However, if assuming  $G$  tends to  $\infty$  the solving process of  $Z_2$  will be simplified remarkably, and the resulting value can be used for crude approximation of true value. In this case, the equation for solving  $Z_2$  is as follows

$$PY_0 = \int_{Z_2(n_2)}^{\infty} (1/\Gamma(n_2 MN)) y^{n_2 MN - 1} e^{-y} dy \quad (7)$$

The Expression of Detection Probability PD for the  $j$ th valid accumulator cell, the expression of PD is

$$P_D = \sum_{n_2=n_0}^{n_3(j)} [C_{n_3(j)}^{n_2} p_d^{n_2} (1 - p_d)^{n_3(j) - n_2} * PY_1] \quad (8)$$

Where  $PY_1$  denotes the probability that the accumulated merit function of potential target trajectory with length  $n_2$  exceeds  $Z_2$ , and  $p_d$  is the detection probability corresponding to  $Z_1$  and rewritten as follows,

$$p_d = E_{\zeta} \{1 - E_b\} e^{-b\zeta} \sum_{g=0}^{\infty} \Psi((bZ_1/(G + bZ_1)), MN, G - v + 1) \quad (9)$$

Where the random variable  $\zeta$  is the sum of SNRs of all the channels after clutter rejection, and the SNR of each channel is distributed as exponential distribution with mean  $\varepsilon$ . Specially, when  $MN$  observation channels are mutually statistically independent and  $G$  tends to  $\infty$  the equation (3.10) can be simplified as follows

$$P_d = \int_{Z_1/(\varepsilon+1)}^{\infty} (1/\Gamma(MN)) y^{MN - 1} e^{-y} dy \quad (10)$$

$$PY_1 = \int_{Z_2(n_2)/(\varepsilon+1)}^{\infty} (1/\Gamma(n_2 MN)) y^{n_2 MN - 1} e^{-y} dy \quad (11)$$

But for the general cases that target RCSs aren't statistically independent between channels and between illuminations, the performance analysis can be performed only by Monte-Carlo simulations of (9).

PFA denotes the false alarm probability for the entire Hough parameter space.  $n_0$  denotes the minimum length (i.e., the minimum number of data points) which a feasible trajectory must possess. The valid accumulator cell is defined as the cell whose value exceeds  $n_0$  in the accessible Hough space, and the total number of valid accumulator cells is denoted by  $N_1$

For the  $j$ th valid accumulator cells,  $1 \leq j \leq N_1$  the maximum number of feasible data points at each illumination is approximately identical and denoted by  $n_1(j)$  which can be obtained by using the accessible Hough space and transition function. Similarly, the maximum number of illuminations for the  $j$ th valid accumulator cell can also be obtained and denoted by  $n_3(j)$ , naturally,  $n_0 < n_3(j) < Q$

The probability that the accumulated merit function of false trajectory with length  $n_2$  exceeds  $Z_2$  is set to be identical and denoted by  $PY_0$ , which means that  $Z_2$  is a function of  $n_2$ . Further, the false-alarm probability for each valid accumulator cell is also set to be identical. As a consequence, for the  $j$ th valid accumulator cells, the equation

$$1 - \sqrt[N_1]{1 - P_{FA}} = PY_0 * \sum_{n_2=n_0}^{n_3(j)} [C_{n_3(j)}^{n_2} p_1^{n_2} p_2^{n_3(j) - n_2}] \quad (12)$$

Where the  $C_{n_3(j)}^{n_2}$  denotes combinatorial factor,  $p_1 = n_1(j) Pfa$ ,  $p_2 = (1 - Pfa)^{n_1(j)}$  and  $Pfa$  is the false-alarm probability corresponding to  $Z_1$ . Next, the equations for solving  $Z_1$  and  $Z_2$  will be derived based on the statistical characteristics of merit function.

#### IV. Numerical Simulation

We consider a simulation scenario with following design parameter,  $N=M=3$ ,  $d=8$ ,  $L=16$ ,  $Q=25$ ,  $PFA = 10^{-6}$ ,  $n_0 = 6$ , and  $K=2$ . For the sake of simplicity, white Gaussian noise background is assumed, and the effect of range change on  $\varepsilon$  is omitted.

##### A. Results obtained so far

Fig.4 is a gray-scale plot of the merit function in polar coordinates plane after primary thresholding, where the two true trajectories of targets are marked with the sign "o".

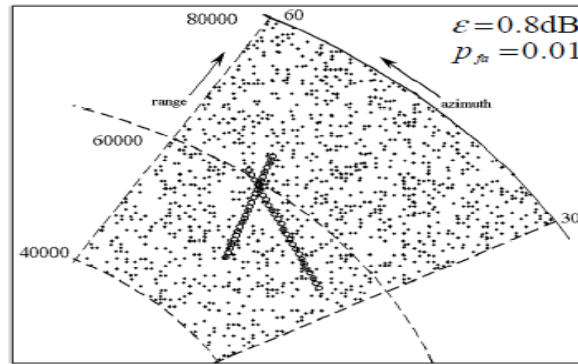


Fig.4 Gray-scale plot of the merit function after primary thresholding

Fig. 5 shows polar plot of the merit function after Primary thresholding.

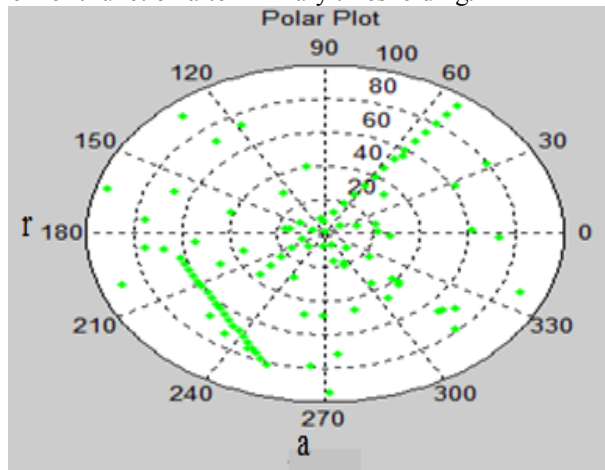


Fig. 5 Polar plot of the Merit function after Primary thresholding

The Polar Hough Transformation (PHT) maps points (targets) from the observation space (polar data map) into curves in the polar Hough parameter space. PHT transforms the polar data of the target ( $r, a$ ) into  $\rho$  and  $\theta$ . Each PHT point corresponds to sinusoid curve and is used for construction of targets Accumulator cells. Histogram is used to track the maximum accumulated value in each cell. The Fig.6 shows the PHT transformation and histogram.

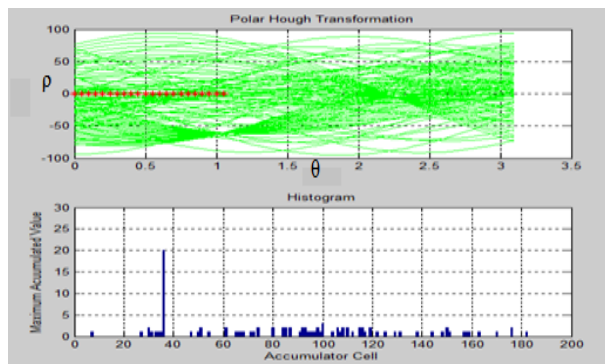


Fig. 6 PHT transformation and Histogram for  $\theta = 1$  (in radians)

Each maximum accumulated value of the Accumulator cell is compared with  $n_2$  if it greater then with secondary threshold value ( $Z_2 = 2.16977$ ) is compared. If it exceeds  $Z_2$  then all PHT points corresponding top and  $\theta$  value are marked as true targets points.

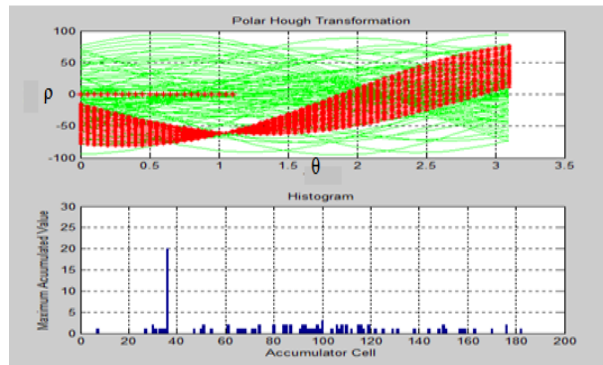


Fig. 7: PHT transformation and Histogram for theta = 1.2 (in radians)

Fig. 7 shows the  $\rho$  and  $\theta$  values which exceed  $Z_2$  and corresponding histogram. True target points are represented by red in colour. Fig. 8 represents the Probability of detection of the system for different values of the pfa as function of signal strength. As the signal strength increases the data points exceeding the threshold values will be more. Hence more data points are available of the processing of the signal. Therefore probability of detection of the overall system increases.

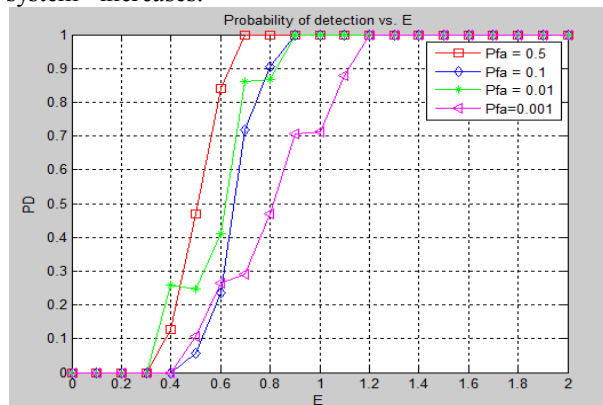


Fig. 8: Probability of detection of the system as function of signal strength.

From the Graph it can be concluded as, when the signal strength is greater than 0.9 values, probability of detection of the system tend towards to unity.

### V. Conclusion

Successive Target cancellation using Polar Hough Transformation based multi target TBD algorithm is designed. The designed MIMO radar was built for the Swerling type 1 target, by considering Alamouti's space time codes over the AWGN noise environment. Merit function of joint search process for target detection is split into many disjoint statistical decision processes, and each of which corresponds to one valid accumulator cell in Polar Hough parameter space. Following conclusions can be drawn

1. This new algorithm can sub optimally splits the joint maximization of the merit function into many disjoint statistical decision processes. This will reduce the implementation complexity.
2. Simulation results analysis has shown this new algorithm can effectively improve the detection performance of statistical MIMO radar at low SNR.

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