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Comparative Study of Monotonic and Non- Monotonic Phase LTI systems using Improved Analytical PID controller

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Abstract : *A comparative study of monotonic and non monotonic phase LTI systems using analytical design of PID controller, by means of gain cross over frequency and phase margin specifications, is presented in the paper. The proposed methodology ensures minimum phase margin inside desired bandwidth giving accurate performance to step response for closed loop system. Concept of phase margin needs to be redefined when uncontrolled process presents minimum phase inside bandwidth. For comparative study, bode stability criterion, nyquist stability criterion and unit step response is used.*
Keywords: *Improved analytical controller , linear system compensation, monotonic and non-monotonic phase LTI system.*

I. Introduction

A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a feedback control system. In case of feedback control system for monotonically decreasing phase inside bandwidth, phase margin can be defined as distance between open loop phase at gain crossover frequency and stability limit of $-180°$. After using bode stability criterion, if phase margin is positive then closed loop system is stable [1]

In case of classical frequency response it can be considered that phase margin can be defined on gain cross-over frequency. Whereas this assumption is not valid in general practice, because even for a minimum phase system with a left half-plane zero located near the dominant- poles, its frequency response will present a non-monotonic phase.

This abrupt technique shows an improved frequency response design technique for monotonic or non-monotonic phase (inside the bandwidth) dynamical systems based on gain crossover frequency and phase margin specifications, which is an expansion of the technique shown by Phillips and Harbor [2], by using this feature comparative study of monotonic and non-monotonic phase LTI system is possible. To achieve this feature concept of phase margin can be redefined when the system frequency response shows a non-monotonically decreasing phase inside the bandwidth. The study of phase margin is necessary due to two reasons such as it can be considered as good robustness indicator and which allows bode stability criterion for monotonic and non-monotonic minimum phase systems.

The detailed organization is as follows: section 2 presents problem formulation, some remarks about non-monotonically decreasing phase can be addressed, for that purpose considered the DC-DC buck regulator. Section 3 explains detailed study of PID controller design. Section 4 is concerned about simulation and results where comparative study of monotonic and non-monotonic systems is also explained. Section 5 has conclusion.

II. Problem Formulation

Continuous LTI dynamical system is considered which is given by differential equation,

$$\ddot{y}(t) + a_1\,\dot{y}(t)a_0 y(t) = b_1\dot{u}(t) + b_0\,u(t) \qquad (1)$$

Here LC low pass filter is to be considered which is commonly used in different type of power electronics. Hence assume synchronous buck regulator having model no. IRU 3038 used as voltage tracking application such as DC-DC converter and DDR memory [3]. Buck regulator is taken into considerations which are combination of power stage given by LC low pass filter and pulse width modulation based controller. The transfer function of buck converter is output(regulated voltage) to input(PWM input voltage) is,

$$G(s) = \frac{V_{IN}}{V_{OSC}}\,\frac{(1+sR_C\,C)}{LC_S^2 + s\left(R_C C + \frac{L}{R}\right)+1} \qquad (2)$$

Where ,L is the output inductance, C is the output capacitance, R is the load resistance, R_C is the output capacitor intrinsic resistance, V_{OSC} is the PWM oscillator reference voltage and V_{IN} is the power stage input voltage. Here assume typical application for buck regulator [3], R_C=40ohm, gives transfer function,

$$G(s) = \frac{4(1+1.2\times10^{-5}s)}{3\times10^{-9}\,s^2 + 3.6\times10^{-5}\,s+1} \qquad (3)$$

Where the un-damped frequency is approximately 18 Krad/s and the damping ratio is approximately 0.33. Using this transfer function for buck regulator we can find out bode plot shown in fig.1. Here a valley in phase curve close to un-damped frequency is to be seen. From this we can conclude that left half-plane zero due to the output capacitor intrinsic resistance is not located far enough from the poles. Hence this leads to non monotonic phase compensation because bandwidth is located after zero frequency. Due to this transfer function is in minimum phase and use of bode plot is not possible. Hence we have to study the improved technique of design of PID controller[4].

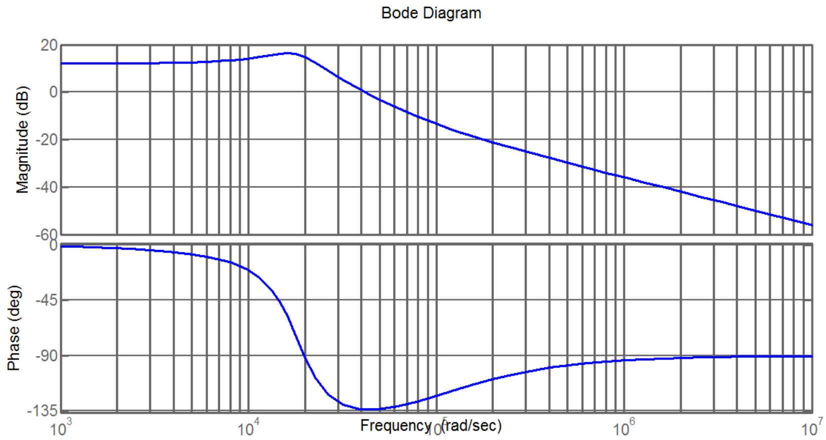

Fig.2 Bode plot for Buck regulator

III. Design Of PID Controller

he PID controller is the most common form of feedback. It was an essential element of early governors and it became the standard tool when process control emerged in the 1940s. In control system today, more than 95% of the control loops are of PID type, most loops are actually PI control. PID controllers are today found in all areas where control is used. The controllers come in many different forms[5]. PID control is an important ingredient of a distributed control system. The controllers are also embedded in many special-purpose control systems. For improving PID control there are various methods such as empiric method by Ziegler and Nichols and analytical compensation based on pole-placement and frequency response method[6] [7] due to this popularity of PID controller is tremendously increased.

Consider equation for PID controller,

$$K(s) = K_P\left(1+\frac{1}{T_1 s}T_D s\right) \qquad (5)$$

Where K_P is the proportional gain, K_I is the integral action time or reset time, and T_D is known as the derivative action time or rate time [1], [3].

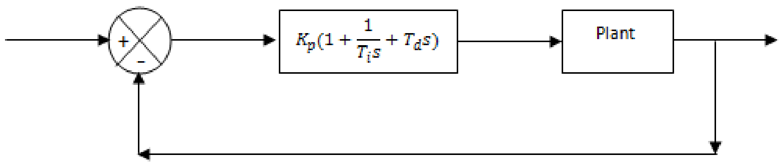

Fig.1 Design of PID control of plant

For designing of PID controller one tuning method is used i.e. Ziegler Nichols tuning method. This process of selecting the controller parameters to meet given performance specifications is known as controller tuning. Ziegler and Nichols suggested rules for tuning PID controllers based on experimental step responses or based on the value of that results in marginal stability when only proportional control action is used. Ziegler–Nichols rules, are useful when mathematical models of plants are not known. However, the resulting system may exhibit a large maximum overshoot in the step response, which is unacceptable. In such case we need series of fine tunings until an acceptable result is obtained. In fact, the Ziegler–Nichols tuning rules give an educated guess for the parameter values and provide a starting point for fine tuning, rather than giving the final settings in a single shot.[6]-[7]. It should be noted that worst case frequency is exactly the gain crossover frequency i.e., it defines a monotonic phase system, then the set of (4) becomes the well-known compensating equations shown by Phillips and Harbor [2].

$$K_P = \frac{1}{|G(j\omega_n)|\sqrt{1+\left(\omega_u T_D - \frac{1}{\omega_u T_I}\right)}} \qquad (5a)$$

$$\omega_u T_D - \frac{1}{\omega_u T_I} = \tan\varphi \qquad (5b)$$

$$\varphi \triangleq -180° + \emptyset_m - < G(j\omega_m) \qquad (5c)$$

One major drawback of PID controller is that, it has less equations than variables therefore it is very important to characterize equations having T_I and T_D values. By rewriting eq (5) in terms of K_P and T_D gives,

$$K_P = \frac{1}{|G(j\omega_n)|\sqrt{1+\Omega^2\left[\tan\varphi + \frac{1-\Omega^{-2}}{\omega_m T_I}\right]^2}} \qquad (6a)$$

$$T_D = \frac{\tan\varphi}{\omega_m} + \frac{1}{\omega_m^2 T_I} \qquad (6b)$$

By using above equations, variations in PI and PD are occurred having a single unique solution each. In general practice, the PID controller should not be usually used to compensate non-monotonic phase dynamical systems since these systems, by their own natural behaviour, already present left half-plane zeros in the open-loop transfer function, yielding a phase-lead effect very similar to the derivative term present in the standard form in (6). Therefore, the next subsections will present the equations and some comments on the PI and PD variations.

3.1 PD controller

Assume T_I equal to infinity in eq 5(a,b) we obtain,

$$K_P = \frac{1}{|G(j\omega_n)|\sqrt{1+\Omega^2 \tan\varphi^2}} \qquad (7a)$$

$$T_D = \frac{\tan\varphi}{\omega_m} \qquad (7b)$$

or $$K_D = K_P T_D = \frac{\omega_u^{-1}}{|G(j\omega_n)|\sqrt{1+\Omega^2 \cot\varphi^2}} \qquad (7c)$$

Now, it should be noted that if monotonicity ratio Ω is larger than K_P gain will be smaller and K_D gain will be larger..

3.2 PI controller

Consider T_D equal to zero in eq 5(a,b),

$$K_P = \frac{1}{|G(j\omega_n)|\sqrt{1+\Omega^{-2} \tan\varphi^2}} \qquad (8a)$$

$$T_I = -\frac{\cot\varphi}{\omega_m} \qquad (8b)$$

$$K_I = \frac{K_P}{T_I} = \frac{\omega_u}{|G(j\omega_n)|\sqrt{1+\Omega^2 \cot\varphi^2}} \qquad (8c)$$

Here, if monotonicity ratio Ω is larger then K_P gain will be larger and K_I gain will be smaller. In case of non monotonic phase system as monotonic phase one should lead overvalued K_P gain and to an undervalued K_D gain..Due to this reason compensation of non monotonic phase system as monotonic system is avoided in case of integral action.

IV. Results And Simulation

In this section tools are presented such as fundamental tools and robustness tools which are widely used in literature.

4.1 Fundamental tools

In fundamental tools consider the factors which are very important for analytical design procedure. This procedure gives gain crossover frequency and phase margin for stable and minimum phase using transfer function which can be obtained from bode criterion. Here damping ratio and un-damped frequency is necessary having transfer function of second order system.

Let us consider negative feedback system into a low-pass behaviour in the open-loop frequency response, then define four parameters which can be used in frequency response compensation for dynamical LTI systems with non monotonic phase inside bandwidth such as worst case frequency ω_m, gain cross over phase margin $\emptyset_u$, gain crossover frequency ω_u, worst case phase margin $\emptyset_m$.

The characteristic equation is gain cross over frequency and stability limit of $-180°$ is ω_u . For bode stability criterion if $\emptyset_u$ is positive then system is stable, when system response represents lowest phase inside bandwidth which is smaller than or equal to the gain crossover frequency ω_u whereas worst case phase margin$\emptyset_m$ is distance between open loop phase in worst case frequencyω_m, and stability limit of $-180°$. Solve equation for frequency domain

$|K(j\omega_u)G(j\omega_u|\angle K(j\omega_u)G(j\omega_u) = 1\angle - 180° + \emptyset_u$ (9a)

With $\emptyset_m \leq \emptyset_u$ (9b)

If the worst case phase margin $\emptyset_m$ is positive then gain crossover phase margin$\emptyset_u$ is positive then system becomes stable, then ω_u is always larger than or equal toω_m. Apply bode stability criterion for above equation it gives,

$|K(j\omega_u| = \frac{1}{|G(j\omega_u)|}$ (10a)

$\angle K(j\omega_m) = -180° + \emptyset_m - \angle G(j\omega_m)$ (10b)

With $\omega_m \leq \omega_u$ (10c)

Here to check how system is monotonic then we have to define monotonicity ratio Ω is given by,

$$\Omega \triangleq \frac{\omega_u}{\omega_m} \quad (11)$$

Ω is always greater than or equal to 1. This ratio indicates that if values of Ω is larger then system tends to non monotonic phase. In next section find the parameters which are useful to shape open loop feedback system into low pass behaviour inside bandwidth and step response for closed loop system in case of gain cross over frequency and phase margin.

4.2 Robustness tools

The analytical design procedure includes various characteristics such as stability, performance and robustness. Above section present stability term now consider factors for designing compensator such as performance and robustnesss. For future procedure $\varkappa_\infty$ theory is considered[1]. By using this theory sensitivity function is given by,

$\|S(j\omega)\|_\infty = \max_m |S(j\omega)| \leq M_s$ (12a)

Where

$S(s) = [1 + K(s)G(s)]^{-1}$ (12b)

Which is more acceptable as robustness measure [1],[9]. Here Nyquist plot is used where $\|S(j\omega)\|_\infty$ is inverse of minimum distance from loop transfer function to critical point(-1,0). Therefore if nyquist plot avoids circle having centre point (-1,0) and radius 1 by M_S then it is said that system is robust related to sensitivity function which are explained in [1],[8],[9]. To obtain better performance of robustness tools consider another complimentary sensitivity function,

$\|T(j\omega)\|_\infty = \max_\omega |T(j\omega)| \leq M_T$ (13a)

where

$T(s) = K(s)G(s)\,[1 + K(s)G(s)]^{-1}$ (13b)

Here nyquist plot having centre $\{-M_T^2/(M_T^2 - 1),0\}$ and radius $(M_T/M_T^2 - 1)$ is used. If nyquist plot avoids circle then it is said that system is robust related to complimentary function. For future work use values given by Skogestad and Postlethwaite as M_S =2 and M_T=1.25 and Kristiansson and Lennartson suggest valuesM_S =1.7 and M_T=1.4 [1],[9]. After comparison we have final conclusion that values suggested by Skogestad and Postlethwaite are more accurate.

4.3 Comparison of monotonic and non-monotonic system

It describes the results obtained from proposed methodology and buck regulator which is shown in section 2. The results can be verified by using MATLAB simulink method. In this case it can be found that the un-damped frequency is near about or equal to 18 krad/s and left half-plane zero frequency is near about or equal to 83 krad/s[6]. Assume switching frequency of the converter to be equal to 200 kHz (approximately 1250 krad/s), by using reference it is said that gain cross-over frequency is about fifth times the switching frequency[3]. Therefore the specified gain cross-over frequency is near about 250 krad/s, from this it concludes that system leads to non-monotonic phase where gain-crossover frequency is beyond the left half-plane zero frequency.

Now the desired phase margin is 40, therefore desired specifications follows the closed loop system performance having overshoot equal to 25% and a settling time equal to 40 s. In this case the feedback system must in second order system.

In case of buck regulator non-monotonic phase system is obtained inside desired bandwidth, here PI controller is superior than PID controller[3]. In this case two parameters for PI controller is studied, first can be studied by using technique of Phillips and Harbor[2] and another from non-monotonic technique and finally both results were compared. The values for compensator are shown in Table 1.

Table 1: Comparison of monotonic and non-monotonic compensation

Parameters	Monotonic	Non-Monotonic
K_P	12.19	14.76
T_I[μsec]	5.86	223.05

From Table 1, worst case frequency is 40 krad/s which is close to frequency of buck regulator for non-monotonic compensation. After comparing values for monotonic and non-monotonic, larger K_P and larger T_I (smaller K_I) value is found for non monotonic systems. For the monotonic compensation (dashed line) and for the non-monotonic compensation (solid line), bode plot is presented in fig.3

It should be noticed that from fig.3, bandwidth is same for both compensation, where non-monotonic system gives larger steady-state error for ramp response and it gives phase margin of at least 40 for all frequency inside bandwidth whereas monotonic compensation only guarantees the phase margin required for the gain crossover frequency[4]. It should be remembered that frequencies inside bandwidth only makes control action.

Monotonic compensation does not guarantees the phase margin inside desired bandwidth, since systems are stable according to Routh–Hurwitz criterion[2],[7]. By using Routh–Hurwitz criterion integral action time T_I is 5.86 microsec and range of gain K_P is located in between 0.023 and 2.646. Therefore there is no real and positive gain able to give the non-monotonically compensated system to instability for the integral action time equal to 223.05 s

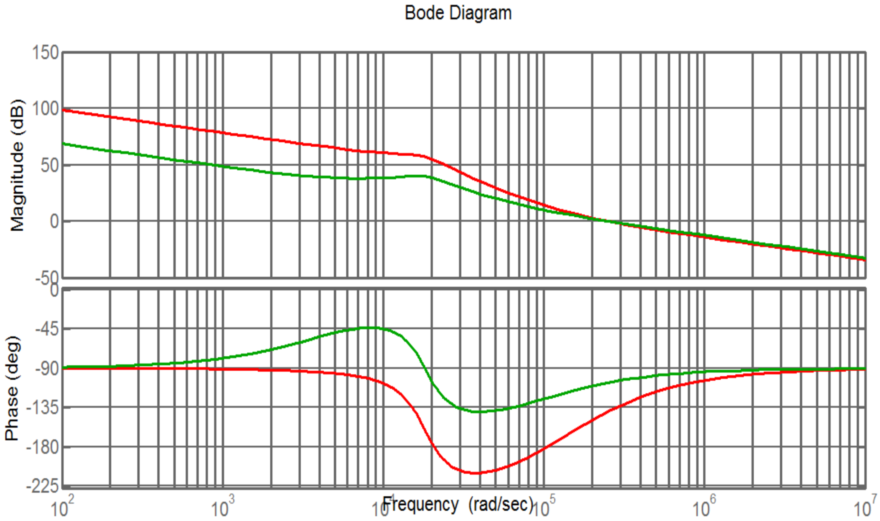

Fig 3. Bode plots for the monotonic compensation (dashed line) and the non-monotonic compensation (solid line)

From future analysis, monotonic compensation gives two openloop gains which cross the stability limit of−180°, hence bode plot[1] is not sufficient for analysis. Therefore to check stability limit of both compensation, we have to use Nyquist plot in frequency response. For the monotonic compensation (dashed line) and for the non-monotonic compensation (solid line) is presented for Nyquist plot in fig.4

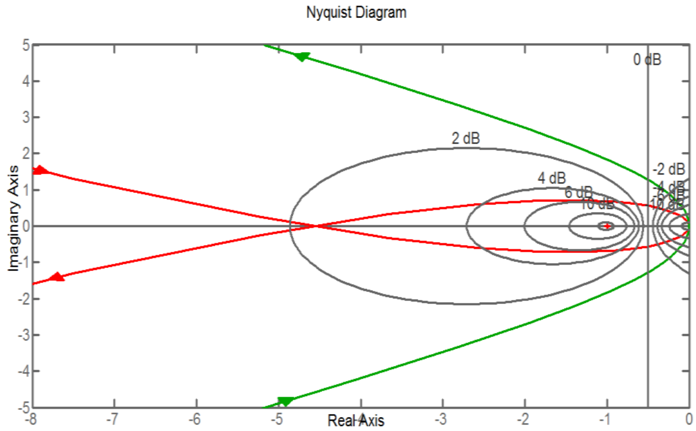

Fig 4. Nyquist plots for the monotonic compensation (dashed line) and the non-monotonic compensation (solid line).

Applying stability criterion for both compensation in Nyquist plot, it should be obtained that monotonic system is not useful for phase margin as it gives one encirclement around the -1 critical point. Whereas it does not happened in case of non-monotonic phase systems. Fig.4 also shows that robustness boundaries (lighter lines) of the sensitivity function (smaller circle) withM_S =2 and complementary sensitivity function (larger circle) withM_T= 1.25[1]. From above fig.4 it is seen that monotonic systems are not robust regarding to complimentary sensitivity function M_T whreas non-monotonic system is robust in case of sensitivity functionM_S . Hence for non-monotonic system Nyquist plot gives accurate shape and results.

To find out accurate performance use of Step response is necessary, unit step response for the monotonic compensation (dashed line) and of the non-monotonic compensation (solid line) is shown in Fig. 4. For unit step response, monotonic phase system use second order system where non-monotonic phase system use first order system. The settling time for the non-monotonically compensated system is approximately 40 s and for monotonically compensated system is approximately 60 s. Whereas overshoot in case of non-monotonic system is smaller than monotonic system, therefore non-monotonic system completes all robustness properties. From above analysis, it should be noticed that non-monotonic phase gives accurate performance measure and its Nyquist plot avoids the circle created by the robustness boundary, which the monotonic compensation does not[1].

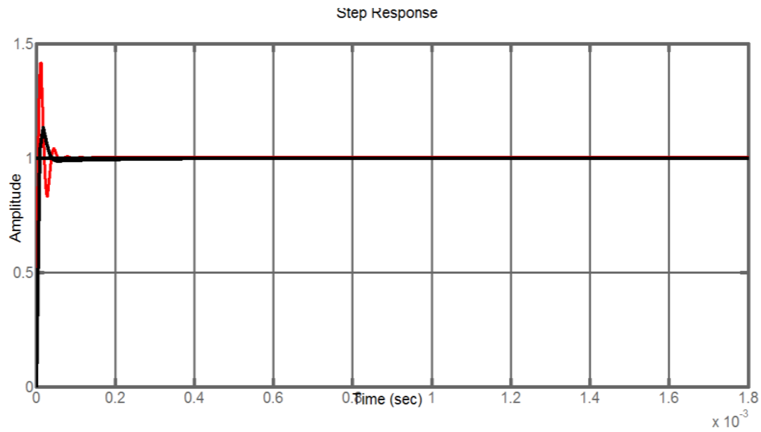

Fig 5. Unit step response of the monotonic compensation (dashed line) and the non-monotonic compensation (solid line).

V. Conclusion

The comparative study of monotonic and non-monotonic phase LTI systems are presented in paper by using improved analytical PID controller design using gain cross-over frequency and phase margin specification. Using this methodology accurate results are obtained. Where minimum phase margin is obtained inside desired bandwidth for closed loop system. Here non-monotonic phase gives more desirable results than monotonic phase results for closed loop system. For comparative study Bode stability criterion, Nyquist criterion and Unit step response are used, whereas nyquist plot are used for stability checking and results obtained by using MATLAB simulink.

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