IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)
e-ISSN: 2278-1676,p-ISSN: 2320-3331, Volume 11, Issue 5 Ver. III (Sep - Oct 2016), PP 85-91
www.iosrjournals.org

The Design of a Phase Gain Damping (PGD) Controller for Feedback Control Systems

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Abstract: *The proportional plus integral plus derivative (PID) controller has been the mainstay of feedback control for decades even though there are slew of other controllers in the books. Simplicity, ease of use, and adequate performance are the lure of the PID. A major drawback of the PID is reliance on trial and error graphical tuning method such as the Ziegler-Nichols, Cohen-Coon, and Fertik. This article introduces the phase, gain, and damping (PGD) controller. By solving a system of nonlinear algebraic equations the designer obtains transfer functions with desired gain, damping, and phase at the closed loop bandwidth of the feedback system. This article also proposes a new method of tuning the PID controller parameters based on the PGD controller coefficients. This approach of using the PGD to tune the PID eliminates the need for cumbersome graphical tuning.*
Keywords: *Phase, Gain, Damping, Controller, PGD, PID*

I. Introduction

A functional block diagram of a feedback control system is shown in Fig. 1. The goal of feedback control is to find a controller $G_c(s)$ which when combined with the given plant $G_p(s)$ gives the feedback system desirable performance as measured by high bandwidth, acceptable damping, and low tracking and steady state errors to specified command inputs. Other performance measures include disturbance suppression as judged by the shape of the sensitivity functions, ample phase margin, 30^0 minimum, [1], and a generous gain margin, typically 12 dB, [2]. Additional requirements such as rise time, settling time, and overshoot are functions of bandwidth and damping.

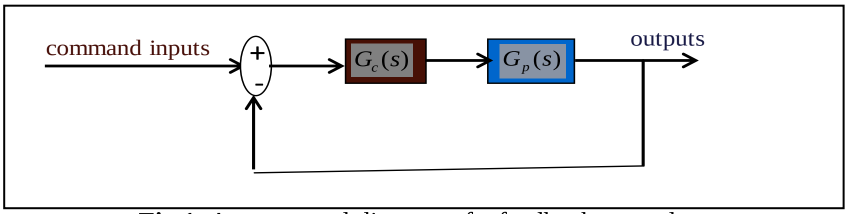

Fig 1. A conceptual diagram of a feedback control system.

Control system engineers have no discretion to alter $G_p(s)$ but what can be done with $G_c(s)$ to enable the system meet control requirements is limited by the designer's control system knowledge. There are many feedback controller design schemes that promise to deliver the aforementioned performance benefits in one form or another. The linear quadratic regulator [3,4,5], linear quadratic Gaussian [6,7], the Luenberger observer [8], pole placement [1,9], and the H-infinity controllers [3,5] are well known. In practice, none of these controllers is as extensively used as the PID, because they have flaws which the latter doesn't. The LQR, LQG, and pole placement are full state feedback controllers; and the observer (also known as estimator) is known to lack robustness [4,7], because the control law is a function of estimated states which may differ significantly from the plant's states. The H-infinity controller is arguably the most prudent choice for multivariable systems but mastery of its design methodology calls for advanced courses in feedback control theory. Consequently, control system designers who are looking for uncomplicated, yet effective controllers, naturally reach for the PID, in spite of its trial and error tuning disadvantage.

The Phase, Gain, and Damping (PGD) controller is introduced in this article. There are relationships between damping, zeros, poles, gain, phase, and the coefficients of a transfer function. These relationships are harnessed to formulate a set of equations whose solution yields transfer functions with specified gain, phase, and damping at the frequency of the closed loop system.

This article is organized as follows. Section 1 is the introduction. Section 2 elucidates the formulation of the PGD controller. The PGD integrator is discussed in Section 3. There is a one-to-one correspondence between the PGD integrator coefficients and the PID controller parameters. This relationship is established in Section 4. Section 5 discusses how the high frequency gain and phase requirements can be relaxed to reduce the number of equations from five to four. Section 6 offers a design example. And Section 7 is the conclusion.

II. The PGD Controller Formulation

This formulation uses the location of the poles and zeros, phase angle, and controller gain to obtain a system of equations that describes the transfer function. A general equation of a monic proper n^{th}-order transfer function has ($2n+1$) coefficients. In this aspect, for a second-order system, the transfer function is

$$g_1(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} \tag{1}$$

Eqn. (1) has five coefficients: $\{b_2, b_1, b_0, a_1, a_0 > 0\}$. The s-plane locations of the zeros of $g_1(s)$ affect the phase margin, steady state error and bandwidth of the feedback system. Further, the zeros of the controller participate in shaping the poles of the closed loop system, which makes the location of the controller zeros critical. Let $\varepsilon_1 \in (-\infty, \infty)$. Eqn. (1) can be written as a ratio of two polynomials $N_1(s)$ (the numerator) and $D_1(s)$ (the denominator). Completing the squares on $N_1(s)$, we have:

$$N_1(s) = (s + \frac{b_1}{2b_2})^2 + \frac{b_0}{b_2} - \frac{b_1^2}{4b_2^2}$$

Suppose $b_0 - \frac{b_1^2}{4b_2} > 0$, then there exists $\varepsilon_1 < 0$ such that

$$f_1 = 4b_2 b_0 - b_1^2 + \varepsilon_1 = 0 \tag{2}$$

Similarly, if $b_0 - \frac{b_1^2}{4b_2} < 0$ then $\varepsilon_1 > 0$ exists such that $f_1 = 0$. That is, the zeros of $g_1(s)$ are complex if $\varepsilon_1 < 0$, else they are real. So the choice of ε_1 enables the designer to make the zeros of $g_1(s)$ real or complex.

The poles of a transfer function determine the magnitude of its damping factor. Let $\varepsilon_2 \in (-\infty, \infty)$. Completing the squares on $D_1(s)$ shows that a relation linking the damping of $g_1(s)$ and its coefficients is given by

$$f_2 = 4a_0 - a_1^2 + \varepsilon_2 = 0 \tag{3}$$

The transfer function $g_1(s)$ has real poles if $\varepsilon_2 \geq 0$, else complex poles. That is, if $\varepsilon_2 < 0$, there exist no coefficients a_1 and a_0 such that $g_1(s)$ has real poles. If real, the poles of $g_1(s)$ become highly disparate as $\varepsilon_2 \to +\infty$, which is consequential because the location of controller poles affect the steady state error of the feedback system.

The closed loop system bandwidth ω_c is a derived requirement, meaning that ω_c is an offspring of some top level requirement given to the control engineer. A transfer function, $g_1(s)$, can be rationalized to separate it into real and imaginary parts so that the arctangent of the ratio of imaginary over real is the phase angle of $g_1(s)$. Let $\theta_{ph}(\omega_c)$ be the phase angle of $g_1(j\omega)$ at the closed loop bandwidth $\omega = \omega_c$. Also, let $k_0 = \tan(\theta_{ph}(\omega_c)) = \bar{I} / \bar{R}$, where

$$\bar{I} = (b_2 a_1 - b_1)\omega_c^3 + (b_1 a_0 - b_0 a_1)\omega_c \tag{4}$$

$$\bar{R} = b_2 \omega_c^4 + (b_1 a_1 - b_2 a_0 - b_0)\omega_c^2 + a_0 b_0 \tag{5}$$

The phase angle $\theta_{ph}(\omega_c) \in [-\pi, \pi]$. Since positive phase angle (phase lead) and stable poles and zeros are desired, $\theta_{ph}(\omega_c)$ is restricted to the open interval $(0, \pi)$. Therefore, $\bar{I} \geq 0$, and $k_0 \bar{R} \geq 0$. Also, there exists $\varepsilon_3, \varepsilon_4 < 0$ such that

$$f_3 = \bar{R} + \varepsilon_3 / k_0 = 0 \quad (6)$$

$$f_4 = \bar{I} + \varepsilon_4 = 0 \quad (7)$$

Let k_s be the controller gain at the closed loop system bandwidth, that is, $k_s = |g_1(j\omega_c)|$. Then the coefficients of $g_1(s)$ and k_s are related as

$$f_5 = (k_s^2 - b_2^2)\omega_c^4 + (k_s^2 a_1^2 - 2k_s^2 a_0 - b_1^2 + 2b_2 b_0)\omega_c^2 + k_s^2 a_0^2 - b_0^2 = 0 \quad (8)$$

The constants in eqns. (6), (7), and (8) are consolidated as follows.

$$\eta_1 = \omega_c^{-2} \qquad \eta_2 = \omega_c^{-4} \qquad \eta_3 = \varepsilon_3 \eta_2 / k_0$$

$$\eta_4 = \varepsilon_4 \omega_c^{-3} \qquad \eta_5 = k_s^2$$

For convenience of access, the equations are collected together thus:

$$f_1 = 4b_2 b_0 - b_1^2 + \varepsilon_1 \quad (9a)$$

$$f_2 = 4a_0 - a_1^2 + \varepsilon_2 \quad (9b)$$

$$f_3 = b_2 + \eta_1(b_1 a_1 - b_2 a_0 - b_0) + \eta_2 a_0 b_0 + \eta_3 \quad (9c)$$

$$f_4 = b_2 a_1 - b_1 + \eta_1(b_1 a_0 - b_0 a_1) + \eta_4 \quad (9d)$$

$$f_5 = \eta_5 - b_2^2 + \eta_1(\eta_5 a_1^2 + 2b_2 b_0 - 2\eta_5 a_0 - b_1^2) + \eta_2(\eta_5 a_0^2 - b_0^2) \quad (9e)$$

The system of eqns. (9) is nonlinear and algebraic and its solution gives the coefficients of $g_1(s)$. Consider the function $F = [f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5]^T$. In terms of F, the system of eqns. (9) is written as

$$F(\Omega, \Gamma) = 0 \quad (10)$$

where, $\Omega = [b_2 \quad b_1 \quad b_0 \quad a_1 \quad a_0]^T$ are outputs, and $\Gamma = [\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3 \quad \varepsilon_4 \quad k_s \quad k_0 \quad \omega_c]^T$ are inputs to be supplied by the designer.

The Jacobian matrix (or matrix of partial derivatives) of eqn. (10) is

$$J = \left[\frac{\partial f_i}{d\Omega} \right]_{i=1,2,3,4,5} \quad (11)$$

In particular,

$\partial f_1 / d\Omega = [4b_0 \quad -2b_1 \quad 4b_2 \quad 0 \quad 0]$; $\qquad \partial f_2 / d\Omega = [0 \quad 0 \quad 0 \quad -2a_1 \quad 4]$;

$\partial f_3 / d\Omega = [1 - \eta_1 a_0 \quad \eta_1 a_1 \quad \eta_2 a_0 - \eta_1 \quad \eta_1 b_1 \quad \eta_2 b_0 - \eta_1 b_2]$.

And $\partial f_4 / d\Omega$ and $\partial f_5 / d\Omega$ are computed similarly.

The steps for a numerical solution of eqn. (10) is as follows.

Specify values for Γ.

Assume an initial value Ω_0 for Ω.

Evaluate $F(\Omega, \Gamma)$ and J.

Evaluate $Error = -J^{-1}F$, and the norm of Error $NE = \|Error\|$.

If Ω_0 is a solution of eqn. (10), then $F(\Omega_0, \Gamma) \cong 0 \Leftrightarrow NE \leq \gamma_0 \geq 0$.

Else, the updated value of Ω is $\Omega_{i+1} = \Omega_i + Error$.

Continue until $NE \leq \gamma_0$ ■

The actual value of γ_0 depends on how exact the solution is desired. Typically, $\gamma_0 \leq 10^{-7}$. The initial condition is chosen arbitrarily and the algorithm converges quickly if Γ satisfies eqn. (10). An in-depth treatment of solutions of nonlinear equations is found in [10].

III. The PGD Integrator

A controller $G_c(s) = g_1(s)$, even when $g_1(s)$ has high DC gain, may not give acceptable steady state error and disturbance suppression. The primary usage of an integrator is to give the feedback system a desired system type to shape the tracking and steady state errors as the designer intends. Assuming a second-order system, the integrator block in the phase, gain, and damping controller is designed as follows. Let

$$g_2(s) = \frac{s^2 + d_1 s + d_0}{s^2 + c_1 s} \tag{12}$$

with the unknowns $\{d_1, d_0, c_1 > 0\}$. Then for $\varepsilon_5 \in (-\infty, \infty)$ and $\varepsilon_6 \in (0, \infty)$, and following the steps used in deriving eqn. (9),

$$f_6 = 4d_0 - d_1^2 + \varepsilon_5 = 0 \tag{13}$$

$$f_7 = -c_1^2 + \varepsilon_6 = 0 \tag{14}$$

$$\Rightarrow c_1 = \varepsilon_6^{0.5} \tag{15}$$

The integrator is active at low frequency. Let $k_L = |g_2(\omega_L)|$, $\omega_L << \omega_c$. In terms of k_L, ω_L, d_0, and d_1, eqn. (8) is re-written as

$$f_8 = (k_L^2 - b_2^2)\omega_L^4 + (k_L^2 a_1^2 - 2k_L^2 a_0 - d_1^2 + 2b_2 d_0)\omega_L^2 + k_L^2 a_0^2 - d_0^2 = 0 \tag{16}$$

It is noted that eqn. (16) is for the general case of eqn. (1). For the special case of eqn. (12), $b_2 = 1$, $a_0 = 0$, and $a_1 = c_1 = \varepsilon_6^{0.5}$. Substituting these values in eqn. (16):

$$f_9 = (k_L^2 - 1)\omega_L^4 + (k_L^2 \varepsilon_6 - d_1^2 + 2d_0)\omega_L^2 - d_0^2 = 0 \tag{17}$$

Let $\tau_1 = \omega_L^{-2}$, $\tau_2 = \omega_L^{-4}$, and $\tau_3 = k_L^2(1 + \omega_L^{-2}\varepsilon_6) - 1$. Then

$$f_{10} = \tau_1(2d_0 - d_1^2) - \tau_2 d_0^2 + \tau_3 = 0 \tag{18}$$

The coefficients d_1 and d_0 are obtained by solving

$$\begin{bmatrix} f_6 & f_{10} \end{bmatrix}^T = H(d_1, d_0, \varepsilon_5, k_L, \omega_L) = 0 \tag{19}$$

The third coefficient c_1 is given by eqn. (15).

IV. Tuning the PID Controller Parameters

The PID is a well known controller and widely used in process and robotics industries and commercial controller hardware (Franklin et al, 2006). Beside graphical parameter tuning, feedback system designers frequently use software auto-tune environment to select PID parameters. The tuning algorithms used in these software are hidden to the users and as a consequence designers may lose sight of the engineering involved in the PID tuning. The PGD controller offers a formulaic approach to choosing the PID controller parameters, thus eliminating the need for graphical tuning. The transfer function of a PID controller is given by

$$g_3(s) = K_p + K_I / s + \frac{K_D s}{s/\tau_D + 1} \tag{20}$$

where, K_P. K_I and K_D are proportional, integral, and derivative gains, respectively, and the pole τ_D is inserted to make the controller proper and hardware implementable. Comparing eqns. (12) with (20), it is deduced that

$$\tau_D = c_1 \tag{21}$$

$$K_I = d_0 / c_1 \tag{22}$$

$$K_P = d_1 / c_1 - d_0 / c_1^2 \tag{23}$$

$$K_D = 1/c_1 + d_0 / c_1^3 - d_1 / c_1^2 \tag{24}$$

Obtaining the PID controller parameters in terms of the PGD coefficients can proceed in one of two ways. In one approach, eqns. (15) and (19) are solved to find the coefficients $\{d_1, d_0, c_1\}$ and then compute $\{\tau_D, K_I, K_P, K_D\}$. Alternatively, a control system designer may wish to tune the PID parameters based on the PID phase angle at a specified frequency ω_L. Following the exposition in sections 2 and 3, the set of equations whose solutions yield $\{d_1, d_0, c_1\}$ are

$$f_a = 4d_0 - d_1^2 + \varepsilon_5 = 0 \tag{25}$$

$$f_b = k_0^{-1}\bar{I} - \bar{R} = 0 \tag{26}$$

$$c_1 = \varepsilon_6^{0.5} \tag{27}$$

where,

$\bar{I} = (c_1 - d_1)\omega_L^3 - d_0 c_1 \omega_L$ and $\bar{R} = \omega_L^4 + (d_1 c_1 - d_0)\omega_L^2$

The PID phase angle $\theta_{ph} \in [-\pi, 0]$. Consider, for instance, the case where the designer wants the PID to have a phase angle of -80^0 at $\omega_L = 10$ rad/s, then $k_0 = \tan(-80\pi/180) = -5.6713$. Let $\varepsilon_6 = 0.1$ and $\varepsilon_5 = 2363$. Inserting these numbers in eqns. (25) to (27) the PGD coefficients are computed as $c_1 = 0.3162$, $d_1 = 49.76$, and $d_0 = 28.38$. Then the PID parameters are obtained using eqns. (21) to (24).

V. Discussions

5.1 Relaxation of the High Frequency Gain Constraint

Equations (9) represent the general case where phase and high frequency gain are regulated and there are five unknowns. The integrator has only three unknowns and that significantly simplifies the system of equations. In both equations (9) and (19), the DC gains of the transfer functions are not restricted so they can assume any values. If the constraint imposed on k_s is relaxed then eqn. (8) becomes irrelevant and the high frequency gain is no longer under the manipulation of the designer. In that case the DC gain, $k_d > 0$, must be fixed, and $b_0 = k_d a_0$. This alternative is useful in obtaining transfer functions with unity DC gain where $k_d = 1$. There are four unknowns in this case: $\{b_2, b_1, a_1, a_0 > 0\}$, and eqn. (6) becomes

$$f_3 = b_2\omega_c^4 + (b_1 a_1 - b_2 a_0 - k_d a_0)\omega_c^2 + k_d a_0^2 + \varepsilon_3 / k_0 = 0$$

Also eqn. (7) becomes

$$f_4 = (b_2 a_1 - b_1)\omega_c^3 + (b_1 a_0 - k_d a_0 a_1)\omega_c + \varepsilon_4 = 0$$

Let,

$\beta_1 = \omega_c^{-2}$ $\qquad$ $\beta_2 = k_d \omega_c^{-4}$

$\beta_3 = k_0^{-1}\varepsilon_3\omega_c^{-4}$ $\qquad$ $\beta_4 = \varepsilon_4\omega_c^{-3}$

The system of equations similar to eqns. (9) is

$$f_a = 4b_2 k_d a_0 - b_1^2 + \varepsilon_1 \tag{28a}$$

$$f_b = 4a_0 - a_1^2 + \varepsilon_2 \tag{28b}$$

$$f_c = b_2 + \beta_1(b_1 a_1 - b_2 a_0 - k_d a_0) + \beta_2 a_0^2 + \beta_3 \tag{28c}$$

$$f_d = b_2 a_1 - b_1 + \beta_1(b_1 - k_d a_1)a_0 + \beta_4 \tag{28d}$$

The weakness of eqns. (28), compared with eqns. (9), is that the high frequency gain is no longer influenced by the designer so it can take on any value.

5.2 Restriction of the Phase Angle

Another simplification of eqns. (9) can be achieved if, with fixed DC gain, $\theta_{ph}(\omega_c)$ is restricted to the open interval $(0, \pi/2)$. In that case the constraint equations $\bar{R} + \varepsilon_3 / k_0 = 0$ and $\bar{I} + \varepsilon_4 = 0$ are replaced by $k_0^{-1}\bar{I} - \bar{R} = 0$, with $\Omega = [b_2 \quad b_1 \quad a_1 \quad a_0]^T$ and $\Gamma = [\varepsilon_1 \quad \varepsilon_2 \quad k_d \quad k_s \quad k_0 \quad \omega_c]^T$.

Let,

$\beta_1 = k_0^{-1}\omega_c^{-1}$ $\qquad$ $\beta_2 = \omega_c^{-2}$ $\qquad$ $\beta_3 = k_0^{-1}\omega_c^{-3}$

$\beta_4 = k_d\omega_c^{-4}$ $\qquad$ $\beta_5 = k_s^2$ and $\beta_6 = (k_s^2 - k_d^2)\omega_c^{-4}$

Hence the system of equations becomes

$$f_a = 4b_2k_da_0 - b_1^2 + \varepsilon_1 \quad (29a)$$

$$f_b = 4a_0 - a_1^2 + \varepsilon_2 \quad (29b)$$

$$f_c = -b_2 + \beta_1(b_2a_1 - b_1) - \beta_2(b_1a_1 - b_2a_0 - b_0) + \beta_3(b_1 - k_da_1)a_0 - \beta_4a_0^2 \quad (29c)$$

$$f_d = -b_2^2 + \beta_2(\beta_5a_1^2 - 2\beta_5a_0 - b_1^2 + 2k_db_2a_0) + \beta_6a_0^2 + \beta_5 \quad (29d)$$

With eqns. (29) the designer can choose the value of the DC gain k_d; the high frequency gain k_s; and make the poles and zeros of $g_1(s)$ complex, real, and highly unequal. So the designer has a tool to obtain a truly tailor-made transfer function.

VI. The PGD Controller Design Example

Let the transfer function of the controller be

$$G_c(s) = K_C g_2 g_1(s)$$

where, $K_C \geq 1$ is a controller gain needed to give the feedback system the required gain margin; and $g_2(s)$ and $g_1(s)$, respectively, are of the forms of eqns. (12) and (1). For this example, $K_C = 1$. The input to be supplied is $\Gamma = [\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3 \quad \varepsilon_4 \quad k_s \quad k_0 \quad \omega_c]^T$. It is required that the controller provides a positive phase angle of 155^0 at the closed loop bandwidth frequency of 355 rad/s. From the given phase angle, $k_0 = -0.4663$; and $\omega_c = 355$. The designer reserves the discretion to choose ε_1, ε_2, ε_3, ε_4, and k_s to satisfy eqn. (10). Suppose the designer wants all the poles and zeros of $g_1(s)$ to be real, then $\varepsilon_1, \varepsilon_2 > 0$. For this problem the following constants are chosen: $\varepsilon_1 = 1.8 \times 10^{10}$ and $\varepsilon_2 = 2249700$. The inequality conditions on eqns. (6) and (7) demand that $\varepsilon_3, \varepsilon_4 < 0$, so for this problem, $\varepsilon_3 = \varepsilon_4 = -5 \times 10^{15}$. Finally, k_s is chosen as $k_s = 14.66$. Inserting these numbers in eqn. (10) and solving numerically yields:

$$g_1(s) = \frac{3220s^2 + 348660s + 8041100}{s^2 + 10739s + 28271000}$$

The designer also has the discretion in specifying the integrator. For this problem the variables were chosen as follows: $\varepsilon_6 = 0.1$, $\varepsilon_5 = 2363.4$, $\omega_L = 10 << \omega_c$, and $k_L = 5$. Solving eqn. (19) the integrator block obtained is

$$g_2(s) = \frac{s^2 + 49.3474s + 17.9407}{s(s + 0.3162)}$$

The transfer function $g_1(s)$ provides a phase lead of 155^0 at the closed-loop system bandwidth of 355 rad/s. Fig. 2 is the Bode plot of the controller $G_c(s)$ which is the product of $g_2(s)$ and $g_1(s)$. The left axis is the magnitude and the phase is represented in the right axis.

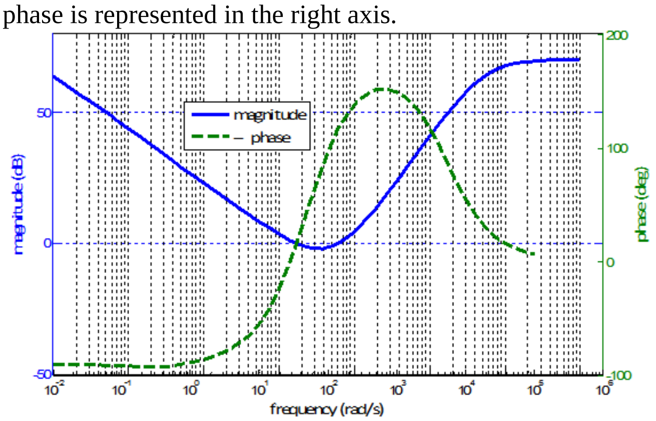

Fig. 2. A Bode plot of the controller with the left axis (solid plot) as magnitude and right axis (dash plot) as phase.

VII. Conclusion

A major shortcoming of the PID controller is its trial and error and graphical tuning methods. In contrast, the PGD controller, introduced in this article, empowers the control engineer to model the controllers precisely. This article also exhibits a formulaic approach to tuning the PID controller parameters. Solutions of sets of nonlinear algebraic equations yield transfer functions with gain, damping, and phase lead at the frequencies intended by the designer. This is important because feedback system stability and other performance requirements are met if the controller produces the right amount of phase, damping, and gain at appropriate frequencies. Other uses of transfer functions with specified phase lead are in tuned resonant circuits and communication networks to provide high quality factor, or correct phase lag introduced into the received signals by the communication channels.

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