Mathematical Modeling of Indirect Field Oriented Controlled Induction Motor

S. O. Bhoyar¹, A. S. Sindekar²

¹(Department of Electrical Engineering, Government College of Engineering, Amravati (M.S.), India)
²(Department of Electrical Engineering, Government College of Engineering, Amravati (M.S.), India)

Corresponding Author: S. O. Bhoyar

Abstract: Induction motors are widely used in industrial applications hence its speed control has been an eminent area of research. The squirrel cage induction motors are cheap, simple and robust in construction, and requires less maintenance. Due to these advantages it has been widely used for fixed speed application in industries. The main purpose is to replace the DC motor by an induction motor and merge the advantages of both the motors together into variable speed brushless motor drive and eliminate the associated problems. However industrial applications require efficient control of IM drives. Control of Induction Motor (IM) is known to be difficult owing to the fact that the mathematical models of IM are highly nonlinear and time variant. The innovation of vector control techniques has solved the problems of controlling the induction motor for precise applications.

This paper describes the concept of vector control technique. There are essentially two general methods of vector control. One called the direct or feedback method, and the other, the indirect or feed forward method. Indirect Field Oriented Control (IFOC) induction motor drives are used in high performance systems in various industrial applications due to their relative simple configuration, as compared to the Direct Torque Controlled (DTC) technique. This work focuses on the indirect field oriented control of induction motor.

Keywords: Induction motor, indirect field oriented control, vector control, space vector pulse width modulation (SVPWM).

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Nomenclature

\( V_{ds}, V_{qs} \): Instantaneous values of direct- and quadrature-axis stator voltage components respectively expressed in the stationary reference frame
\( i_{ds}, i_{qs} \): Instantaneous values of direct- and quadrature-axis stator current components respectively expressed in the stationary reference frame
\( \psi_{ds}, \psi_{qs} \): Instantaneous values of the direct- and quadrature-axis stator flux linkages expressed in the rotor reference frame
\( L_m \): Mutual inductance
\( L_s \): Stator inductance
\( L_r \): Rotor inductance
\( R_s \): Stator resistance
\( R_r \): Rotor resistance
\( p \): Differential operator
\( \omega_r \): Angular rotor speed
\( \omega_s \): Angular speed of the rotor-flux-oriented reference frame
\( \omega_{s1} \): Angular slip frequency

1. Introduction

Nowadays more than 60% of all the electrical energy generated in the world is utilized by cage induction machines. Variable speed or adjustable torque control of electrical motor drives are essential requirements in almost all-modern industrial manufacturing processes. In the past, direct current (DC) motor was largely used in the field of the variable speed applications, where torque and flux are naturally decoupled and controlled independently by the torque producing current and the flux producing current. However, DC motor has disadvantages like maintenance, sparking, difficulty in commutation at high current and voltage so it is limited to low power and low speed. Traditionally variable speed electric machines were based on DC motors, but for the last 50 years, AC drives using induction machines are now finding increasing acceptance in various
industrial applications because of its advantages like, ruggedness, less requirement of maintenance, higher reliability and efficiency, low cost, size and weight compared to the DC motors. There is a demand for high performance electric drives capable of accurately executing torque, speed or position demands. As a result of the important progress in the power electronics and micro-computing, the control of the AC electric machines is known to be a considerable development and a possibility of the real time implantation applications. It is widely recognized that the induction motor is the main actuator for industrial applications. However, the precise induction motor control is not easy to develop as induction motor drive is a complicated non-linear system and rotor variables cannot be measured directly. Also the physical parameters are different for different operating conditions. Also it is difficult to achieve high starting torque like DC motors. Control of induction motors can be done using various techniques. The most common techniques are:

a) Constant voltage/frequency control (V/F)
b) Field orientation control (FOC)
c) Direct torque control (DTC).

The first one is considered as scalar control since it adjusts only magnitude and frequency of the voltage or current with no concern about the instantaneous values of motor quantities. It does not require knowledge of parameters of the motor, and it is an open-loop control. Thus, it is a low cost simple solution for low performance applications such as fans and pumps. The other two methods are in the space vector control category because they utilize both magnitude and angular position of space vectors of motor variables, such as the voltage and flux. Vector control offers a number of benefits including speed control over a wide range, precise speed regulation, fast dynamic response, and operation above base speed. This control strategy can provide the same performance as achieved from a separately excited DC machine, and is proven to be well adapted to all type of electrical drives associated with induction machines. This paper focuses on the field oriented control of induction motor.

Vector control, also called field-oriented control (FOC), is a variable frequency control method in which the stator currents of a three-phase AC electric motor are identified as two orthogonal components that can be visualized with a vector. The field oriented control consists of controlling the stator currents represented by a vector. This control is based on projections that transform a three phase time and speed dependent system into a two coordinate (d and q frame) time invariant system. These transformations and projections lead to a structure similar to that of a DC machine control.

FOC machines need two constants as input references: the torque component (aligned with the q coordinate) and the flux component (aligned with d coordinate). The three-phase voltages, currents and fluxes of AC-motors can be analyzed in terms of complex space vectors. As Field Orientated Control is simply based on projections, the control structure handles instantaneous electrical quantities. This makes the control accurate in every working operation (steady state and transient) and independent of the limited bandwidth mathematical model.

II. Dynamic Modeling of Induction Motor

The induction motor model can be derived in a number of different reference frames. This makes it easier to fix the reference frame to a particular motor quantity and adjust the model accordingly. Most of induction motors are the rotary type with basically a stationary stator and a rotating rotor. The dynamic model of the induction motor is derived by transforming the three phase quantities into two phase direct and quadrature axes quantities. The mathematical model in compact form can be represented in the stationary reference frame as follows:

\[
\begin{bmatrix}
V_i \\
V_q \\
i_{ds} \\
i_{qs}
\end{bmatrix} =
\begin{bmatrix}
R_s + L_s p & 0 & L_m \rho & 0 \\
0 & R_s + L_s p & 0 & L_m \rho \\
L_m \rho & \omega_r L_m & R_r + L_r p & \omega_r L_r \\
-\omega_r L_m & L_m \rho & -\omega_r L_r & R_r + L_r p
\end{bmatrix}
\begin{bmatrix}
ds \\
qs \\
i_{ds} \\
i_{qs}
\end{bmatrix}
\]

The electromagnetic torque of the machine can be obtained by following equation:

\[
T_e = \frac{3}{2} \left( \psi_{ds} i_{qs} - \psi_{qs} i_{ds} \right) \quad \text{...........(1)}
\]

where,

\[
\psi_{ds} = L_s i_{ds} + L_m i_{dr} \quad \text{...........(2)}
\]

and

\[
\psi_{qs} = L_s i_{qs} + L_m i_{qr} \quad \text{...........(3)}
\]
III. Indirect Field Oriented Control

The indirect vector control based on rotor flux linkages is the most popular method in which it involves aligning direct axis and quadrature axis of the stationary frame with the rotor flux frame. Hence unit vectors are generated in a feed forward manner derived as follows:

The rotor fluxes in direct axis and quadrature axis are written as

\[
\frac{d\psi_{dr}}{dt} + \frac{R_r}{L_r} \psi_{dr} - \frac{L_m}{L_r} R_r i_{ds} - (\omega_e - \omega_r) \psi_{qr} = 0 \quad \ldots \ldots \ldots (4)
\]

\[
\frac{d\psi_{qr}}{dt} + \frac{R_r}{L_r} \psi_{qr} - \frac{L_m}{L_r} R_r i_{qs} - (\omega_e - \omega_r) \psi_{dr} = 0 \quad \ldots \ldots \ldots (5)
\]

For decoupling control,

\[
\psi_{qr} = 0 \quad \ldots \ldots \ldots (6)
\]

And hence

\[
\frac{d\psi_{qr}}{dt} = 0 \quad \ldots \ldots \ldots (7)
\]

So the entire flux is directed along d-axis. Substituting equation (6) and (7) in equation (4)&(5) we get,

\[
(\omega_e - \omega_r) = \frac{L_m R_r}{\psi_{dr} L_r} i_{qs}
\]

\[
\omega_{sl} = (\omega_e - \omega_r) \quad \ldots \ldots \ldots (8)
\]

If rotor flux \(\psi_{qr}\) is constant, then \(\psi_{e} = \psi_{dr}\)

Hence slip speed can be calculated from q-axis reference current and rotor reference flux as follows:

\[
\omega_{sl} = \frac{L_m R_r}{\psi_{dr} L_r} i_{qs} \quad \ldots \ldots \ldots (9)
\]

The synchronous reference frame speed is the sum of the angular slip speed and the angular rotor speed which can be given by

\[
\omega_e = \omega_{sl} + \omega_r \quad \ldots \ldots \ldots (10)
\]

The unit vector is generated by integrating the synchronous reference frame speed as

\[
\theta_e = \int \omega_e dt \quad \ldots \ldots \ldots (11)
\]

IV. The Control Scheme

The indirect field oriented control scheme has been explained in fig. 1. The two phase currents \(i_a\) and \(i_c\) are feed to the Clarke’s transformation module. The outputs of the transformation are designated as \(i_{ds}^s\) and \(i_{qs}^s\). These two components of the current are the inputs to the Park transformation that gives the current in the (d, q) rotating reference frame. The \(i_{ds}\) and \(i_{qs}\) components are compared to the references \(i_{ds}^*\) (the flux reference) and \(i_{qs}^*\) (the torque reference). As in synchronous permanent magnet motors, the rotor flux is fixed (determined by the magnets) there is no need to create one. The outputs of the current controllers are \(V_{ds}^*\) and \(V_{qs}^*\); they are applied to the inverse Park transformation. The outputs of this projection are \(V_{ds}^s\) and \(V_{qs}^s\) which are the components of the stator vector voltage in the stationary orthogonal reference frame. These are the inputs to the Space Vector PWM. The Voltage Source Inverter uses Space Vector Pulse Width Modulation technique because of the well-known advantages of better DC bus voltage utilization, reduced harmonic currents and considerable freedom of placement of the space vector in a sector through the choice of switching frequency. The outputs of this block are the signals that drive the inverter. The Space Vector Pulse Width Modulation is explained clearly in further sections. Note that both Park and inverse Park transformations needs the rotor angle, \(\theta_e\) which is calculated using equation (7).
V. Space Vector Pulse Width Modulation (SVPWM) Technique

Space Vector PWM (SVPWM) refers to a special switching sequence of the upper three power switches of a three-phase power inverter. It generates less harmonic distortion in the output voltages and/or currents applied to the phases of an AC motor to provide more efficient use of DC link voltage.

To implement the space vector PWM, the voltage equations in the (a, b, c) reference frame are transformed into the stationary (d, q) reference frame that consists of the horizontal (d) and vertical (q) axes as shown in Fig. 2.

Fig. 2 Voltage Space Vector and its components in (d, q) reference frame

The three control inputs [a, b, c] can map into eight combinations. As a result, six non-zero vectors and two zero vectors are possible. Six nonzero vectors (V1–V6) define the axes of a hexagon as depicted in Fig. 3.

Fig. 3 Basic switching vectors and sectors
The angle between any adjacent two non-zero vectors is 60 degrees. The two zero vectors \( V_0 \) and \( V_7 \) are at the origin and apply zero voltage to the load. The eight vectors are called the basic space vectors and are denoted by \( V_0, V_1, V_2, V_3, V_4, V_5, V_6 \) and \( V_7 \). The same transformation can be applied to the desired output voltage of the inverter to get the desired reference voltage vector \( V_{ref} \) in the d-q plane.

The objective of space vector PWM technique is to approximate the reference voltage vector \( V_{ref} \) using the eight switching patterns. One simple method of approximation is to generate the average output of the inverter over a small period, \( T \) to be equal to \( V_{ref} \) in the same period. The Space Vector PWM can be implemented by the following steps:

**Step 1:** Determine \( V_d, V_q, V_{ref} \) and angle \( \alpha \)

\[
V_d = V_{an} - \frac{1}{2}V_{bn} - \frac{1}{2}V_{cn}
\]

\[
V_q = \frac{\sqrt{3}}{2}V_{bn} - \frac{\sqrt{3}}{2}V_{cn}
\]

\[
|V_{ref}| = \sqrt{V_d^2 + V_q^2}
\]

\[
\alpha = \tan^{-1}\left( \frac{V_q}{V_d} \right)
\]

**Step 2:** Determine time duration \( T_1, T_2 \) and \( T_0 \)

\[
T_1 = T_z \cdot a \cdot \frac{\sin(\frac{\pi}{3} - \alpha)}{\sin(\pi/3)}
\]

\[
T_2 = T_z \cdot a \cdot \frac{\sin(\alpha)}{\sin(\pi/3)}
\]

\[
T_0 = T_z - (T_1 + T_2)
\]

where \( T_z = \frac{1}{f_s} \) and \( a = \frac{|V_{ref}|}{V_{dc}} \)

**Step 3:** Determine the switching time of each transistor (S1 to S6)

**VI. Results**

The simulation of the indirect field oriented control of induction motor drive is modulated in MATLAB. The specification of the induction motor and the control system is given in table 1 and 2.

**Table no 1:** Specifications of the Motor

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power(kW)</td>
<td>150</td>
</tr>
<tr>
<td>Normal line voltage(V)</td>
<td>460</td>
</tr>
<tr>
<td>Rated frequency(Hz)</td>
<td>50</td>
</tr>
<tr>
<td>Stator resistance(mΩ)</td>
<td>14.85</td>
</tr>
<tr>
<td>Stator leakage inductance(mH)</td>
<td>0.3027</td>
</tr>
<tr>
<td>Rotor resistance(mΩ)</td>
<td>9.295</td>
</tr>
<tr>
<td>Rotor leakage inductance(mH)</td>
<td>0.3027</td>
</tr>
<tr>
<td>Mutual inductance(mH)</td>
<td>10.46</td>
</tr>
<tr>
<td>Number of pole pairs</td>
<td>2</td>
</tr>
</tbody>
</table>

**Table no 2:** Specifications Of The Control Scheme

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux controller ( K_p )</td>
<td>100</td>
</tr>
<tr>
<td>Flux controller ( K_i )</td>
<td>30</td>
</tr>
<tr>
<td>Low pass cutoff frequency(Hz)</td>
<td>16</td>
</tr>
<tr>
<td>Sampling time(micro-sec)</td>
<td>20</td>
</tr>
<tr>
<td>Maximum switching frequency(Hz)</td>
<td>20000</td>
</tr>
<tr>
<td>Sampling frequency(Hz)</td>
<td>10000</td>
</tr>
<tr>
<td>DC bus capacitance(mF)</td>
<td>7.5</td>
</tr>
</tbody>
</table>

The set values of speed and torque of the motor in the simulation is maintained at 1500 rpm and 800 Nm. As shown in fig. 4, the speed precisely follows the acceleration ramp till it achieves it set speed of 1500 rpm. The set value of speed is observed to be achieved at 1.7 sec. From fig. (5), (6) and (7), the stable values of electromagnetic torque, the stator current and DC bus voltage respectively are observed at 1.7 sec.
VII. Conclusion

The principles of Indirect Field Oriented control scheme for achieving vector control of a cage induction motor are explained along with detailed block diagrams and equations. It shows that the indirect field oriented control is a good approach for controlling the induction machine without the need of sensors and hence reducing the complexity of the drive system. The induction machine responses to torque changes very quickly and precisely. The simulation results obtained have confirmed the effectiveness of IFOC using SVPWM technique.
The simulation results of the speed and the electromagnetic torque show that the speed regulation of induction machine is achieved in less time. Hence this scheme is suitable for the applications where dynamic response of the motor is required such as the traction vehicles and variable speed industrial processes.

References