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Iterative Learning Control-Based Optimal Linear Quadratic Digital Tracker for Sampled-Time Active Magnetic Bearing System

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Abstract: *This paper presents an iterative learning control (ILC) methodology-based optimal linear quadratic digital tracker (LQDT) for the five degree of freedom (five-DOF) active magnetic bearing (AMB) system as a multi-input multi-output sampled-date system. The combination of ILC and an observer is given out the perfect tracking responses and steady-state. The paper is organized as: (i) System-state observer Kalman filter identification (OKID) is used to construct control system. (ii).*

Keywords: *Active magnetic bearing (AMB), iterative learning control (ILC), linear quadratic digital tracker (LQDT), five degree of freedom (five-DOF), observer Kalman filter identification (OKID).*

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I. System-state observer Kalman filter identification (OKID) is used to construct control system

This part is derived as, apply the OKID method to construct the control loop. And after that, ILC is applied to achieve desired goal.

A. Observer Kalman identification algorithm

Basic observer equation

In order to apply the observer Kalman filter identification method, the system state is required to perform as

$$x(k+1) = Ax(k) + Bu(k)\,, \qquad (7.1a)$$

$$y(k) = Cx(k) + Du(k)\,, \qquad (7.1b)$$

where $x(k) \in \Re^{n\times1}$, $y(k) \in \Re^{m\times1}$ and $u(k) \in \Re^{r\times1}$ are state, output, and control input vectors, respectively, and $A \in \Re^{n\times n}$, $B \in \Re^{n\times r}$, $C \in \Re^{m\times n}$ and $D \in \Re^{m\times r}$ are system, input, output and direct transmission term system matrices, respectively.

- **Zero initial condition**

Assuming zero initial condition $x(0) = 0,$ for $k = 0, 1, 2, \cdots, l-1$

$$x(0) = 0\,,$$

$$y(0) = Du(0)\,,$$

$$x(1) = Bu(0)\,,$$

$$y(1) = CBu(0) + Du(1)\,,$$

$$x(2) = ABu(0) + Bu(1)\,,$$

$$y(2) = CABu(0) + CBu(1) + Du(2)\,,$$

$$\vdots$$

$$x(l-1) = \sum_{i=1}^{l-1} A^{i-1}Bu(l-1-i),$$

$$y(l-1) = \sum_{i=1}^{l-1} CA^{i-1}Bu(l-1-i) + Du(l-1)$$

$y(k-1)$ for $k = 1, 2, 3, \cdots, l$
can be grouped in a matrix form to yield
$y = YU,$
where
$y = [y(0) \quad y(1) \quad y(2) \quad \cdots \quad y(l-1)],$
$Y = [D \quad CB \quad CAB \quad \cdots \quad CA^{l-2}B],$
and

$$U = \begin{bmatrix} u(0) & u(1) & u(2) & \cdots & u(l-1) \\ 0 & u(0) & u(1) & \cdots & u(l-2) \\ 0 & 0 & u(0) & \cdots & u(l-3) \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & u(0) \end{bmatrix},$$

$y \in \Re^{m \times l}$, $Y \in \Re^{m \times rl}$, $U \in \Re^{rl \times l}$,

m is the number of outputs, l is the number of data samples and r is the number of inputs. Equation $y = YU$ is a matrix representation of the relationship between the input and output time histories. Matrix Y contains all the Markov parameters. Matrix U is a block upper-triangular input matrix. When the states of system are inaccessible, an observer is usually applied to estimate the states from the information of input and output. Therefore, add and subtract the term $Gy(k)$, the observer of system can be rewritten as

$$x(k+1) = Ax(k) + Bu(k) + Gy(k) - Gy(k)$$
$$= (A + GC)x(k) + (B + GD)u(k) - Gy(k)$$
$$= \bar{A}x(k) + \bar{B}v(k) , \qquad (6.2)$$

where
$\bar{A} = A + GC, \bar{B} = [B + GD, -G],$
and $v(k) = \begin{bmatrix} u(k) \\ y(k) \end{bmatrix}$

and G is an $n \times m$ arbitrary matrix that can be used to make the desired stable matrix $\bar{A}$. In fact, system (6) is an observer equation if the state $x(k)$ is considered as an observer state vector. Therefore, the Markov parameters of system (1) will be referred to as the observer Markov parameters.
From system (6), it is easy to show that

$$x(k+1) = \bar{A}x(k) + \bar{B}v(k),$$
$$x(k+2) = \bar{A}x(k+1) + \bar{B}v(k+1)$$
$$= \bar{A}^2 x(k) + \bar{A}\bar{B}v(k) + \bar{B}v(k+1),$$
$$\vdots$$
$$x(k+p) = \bar{A}x(k+p-1) + \bar{B}v(k+p-1)$$
$$= \bar{A}^p x(k) + \bar{A}^{p-1}\bar{B}v(k) + \bar{A}^{p-2}\bar{B}v(k+1) +$$
$$\cdots + \bar{B}v(k+p-1)$$

and

$$y(k+p) = Cx(k+p) + Du(k+p)$$
$$= C\bar{A}^p x(k) + C\bar{A}^{p-1}\bar{B}v(k) + C\bar{A}^{p-2}\bar{B}v(k+1) +$$
$$\cdots + C\bar{B}v(k+p-1) + Du(k+p)$$

for $k = 0, \cdots, l-1-p$, one has

$\bar{y} = C\bar{A}^p x + \bar{Y}\bar{V}$,

where

$\bar{y} = [y(p) \quad y(p+1) \quad \cdots \quad y(l-1)] \in \Re^{m\times(l-p)}$,

$x = [x(0) \quad x(1) \quad \cdots \quad x(l-p-1)] \in \Re^{n\times(l-p)}$,

$\bar{Y} = [D \quad C\bar{B} \quad C\bar{A}\bar{B} \quad \cdots \quad C\bar{A}^{(p-1)}\bar{B}] \in \Re^{m\times[r+(r+m)p]}$

and

$$\bar{V} = \begin{bmatrix} u(p) & u(p+1) & \cdots & u(l-1) \\ v(p-1) & v(p) & \cdots & v(l-2) \\ v(p-2) & v(p-1) & \cdots & v(l-3) \\ \vdots & \vdots & \ddots & \vdots \\ v(0) & v(1) & \cdots & v(l-p-1) \end{bmatrix}.$$

$\in \Re^{[r+(r+m)p]\times(l-p)}$

where $\bar{A}^p$ is sufficiently small and $mp \ge n$, so can be approximated by neglecting the first term $C\bar{A}^p x$, such that

$\bar{y} = \bar{Y}\bar{V}$

$\bar{y}\bar{V}^T = \bar{Y}\bar{V}\bar{V}^T$

$\bar{y}\bar{V}^T(\bar{V}\bar{V}^T)^{-1} = \bar{Y}$

- **Nonzero initial condition**

Consider the discrete multivariable linear system described by

$x(k+1) = Ax(k) + Bu(k)$ (7.3a)

$y(k) = Cx(k) + Du(k)$ (7.3b)

Add and subtract the term $Gy(k)$ to the right-hand side of the state equation, then Eq. (8) can be rewritten by

$x(k+1) = Ax(k) + Bu(k) + Gy(k) - Gy(k)$

$= (A+GC)x(k) + (B+GD)u(k) - Gy(k)$

$\Rightarrow x(k+1) = \bar{A}x(k) + \bar{B}v(k)$ (7.4)

For nonzero initial conditions, the above equation is easy to show that

$x(k+1) = \bar{A}x(k) + \bar{B}v(k)$

$x(k+2) = \bar{A}x(k+1) + \bar{B}v(k+1)$

$= \bar{A}^2 x(k) + \bar{A}\bar{B}v(k) + \bar{B}v(k+1)$

$\vdots$

$x(k+p) = \bar{A}x(k+p-1) + \bar{B}v(k+p-1)$

$= \bar{A}^p x(k) + \bar{A}^{p-1}\bar{B}v(k) + \bar{A}^{p-2}\bar{B}v(k+1) + \cdots + \bar{B}v(k+p-1)$ Using the measurement equation yields

$y(k+p) = Cx(k+p) + Du(k+p)$

$= C\bar{A}^p x(k) + C\bar{A}^{p-1}\bar{B}v(k) + C\bar{A}^{p-2}\bar{B}v(k+1) + \cdots$

$+ C\bar{B}v(k+p-1) + Du(k+p)$

$\Rightarrow \bar{y} = C\bar{A}^p x + \bar{Y}\bar{V}$ (7.5)

where the states in x are bounded and $\bar{A}^p$ is sufficiently small the above equation can be approximated by

$\Rightarrow \bar{y} = \bar{Y}\bar{V}$

where

$$\bar{V} = \begin{bmatrix} u(p) & u(p+1) & \cdots & u(l-1) \\ v(p-1) & v(p) & \cdots & v(l-2) \\ v(p-2) & v(p-1) & \cdots & v(l-3) \\ \vdots & \vdots & \vdots & \ddots \\ v(0) & v(1) & \cdots & v(l-p-1) \end{bmatrix}$$
$$\in \Re^{[r+(r+m)p]\times(l-p)}$$

where, all the rows of $\bar{V}$ must be linearly independent. The maximum value of p, which is the upper bound of the order of the deadbeat observer, is the number that maximizes the number $(r+m)p+r \le l-p$ of the independent rows of $\bar{V}$. So, $l \ge (r+m+1)p+r$. On the other hand, the lower bound of p must be chosen such that $p \cdot \max(r,m) \ge n$, where r and m are the numbers of inputs and outputs, respectively, and n is the order of the system. Obviously, p might be smaller than the true order of the system for a multiple-input multiple-output system, while for a single-input single-output system p must be greater than or equal to the true order of the system.

- **Computation of observer Markov parameters**

The observer Markov parameters $\bar{Y}_k = C\bar{A}^{k-1}\bar{B}$ include the system Markov parameters $Y_k = CA^{k-1}B$ and the observer gain Markov parameters $Y_k^o = CA^{k-1}G$. The system Markov parameters and the observer gain Markov parameters are used to combine a Hankel matrix.

\- **System Markov Parameters**

To recover the system Markov parameters in Y from the observer Markov parameters in $\bar{Y}$, partition $\bar{Y}$ such that

$$\bar{Y} = \begin{bmatrix} D & C\bar{B} & C\bar{A}\bar{B} & \cdots & C\bar{A}^{(p-1)}\bar{B} \end{bmatrix} \qquad (7.6)$$
$$\triangleq \begin{bmatrix} \bar{Y}_0 & \bar{Y}_1 & \bar{Y}_2 & \cdots & \bar{Y}_p \end{bmatrix}$$

where

$$\bar{Y}_0 = D,$$
$$\bar{Y}_k = C\bar{A}^{k-1}\bar{B}$$
$$= \begin{bmatrix} C(A+GC)^{k-1}(B+GD) & -C(A+GC)^{k-1}G \end{bmatrix}$$
$$\triangleq \begin{bmatrix} \bar{Y}_k^{(1)} & -\bar{Y}_k^{(2)} \end{bmatrix}$$

for $k = 1, 2, 3, \cdots$.

The system Markov parameters of the system can be reformulated as

$$Y_1 = CB = C(B+GD)-(CG)D = \bar{Y}_1^{(1)} - \bar{Y}_1^{(2)}D.$$
$$\bar{Y}_2^{(1)} = C(A+GC)(B+GD)$$
$$= CAB + CGCB + C(A+GC)GD$$
$$= Y_2 + \bar{Y}_1^{(2)}Y_1 + \bar{Y}_2^{(2)}D$$
$$Y_2 = CAB = \bar{Y}_2^{(1)} - \bar{Y}_1^{(2)}Y_1 - \bar{Y}_2^{(2)}D.$$

$\bar{Y}_3^{(1)} = C(A+GC)^2(B+GD)$

$= C(A^2 + GCA + AGC + GCGC)(B+GD)$

$= CA^2B + CGCAB + C(A+GC)GCB +$

$C(A+GC)^2 GD$

$= Y_3 + \bar{Y}_1^{(2)}Y_2 + \bar{Y}_2^{(2)}Y_1 + \bar{Y}_3^{(2)}D$.

Then, one has

$Y_3 = CA^2B = \bar{Y}_3^{(1)} - \bar{Y}_1^{(2)}Y_2 - \bar{Y}_2^{(2)}Y_1 - \bar{Y}_3^{(2)}D$.

By induction, the general relationship between the system Markov parameters Y_k and the observer Markov parameters $\bar{Y}_k$ is

$Y_0 = \bar{Y}_0 = D$,

$Y_k = \bar{Y}_k^{(1)} - \sum_{i=1}^{k} \bar{Y}_i^{(2)} Y_{k-i}$ for $k = 1, 2, \cdots, p$,

$Y_k = -\sum_{i=1}^{p} \bar{Y}_i^{(2)} Y_{k-i}$ for $k = p+1, \cdots, \infty$.

- **Observer Gain Markov Parameters**

To identify the observer gain G, first recovers the sequence of parameters as follows:

$Y_k^o = CA^{k-1}G$ for $k = 1, 2, 3, \cdots$.

In terms of the observer gain Markov parameters, in fact, the first parameter of equation in the sequence is

$Y_1^o = CG = \bar{Y}_1^{(2)}$.

The next parameter in the sequence is obtained by considering $\bar{Y}_2^{(2)}$

$\bar{Y}_2^{(2)} = C\bar{A}G = (CAG + CGCG)$

$= Y_2^o + \bar{Y}_1^{(2)} Y_1^o$.

Then, one has

$Y_2^o = \bar{Y}_2^{(2)} - \bar{Y}_1^{(2)} Y_1^o$.

Similarly, one gets

$\bar{Y}_3^{(2)} = C\bar{A}^2 G$

$= (CA^2G + CGCAG + C\bar{A}GCG)$.

$= Y_3^o + \bar{Y}_1^{(2)} Y_2^o + \bar{Y}_2^{(2)} Y_1^o$

Then, one has

$Y_3^o = \bar{Y}_3^{(2)} - \bar{Y}_1^{(2)} Y_2^o - \bar{Y}_2^{(2)} Y_1^o$.

The general relationship can be summarized as follows:

$Y_1^o = CG = \bar{Y}_1^{(2)}$,

$Y_k^o = \bar{Y}_k^{(2)} - \sum_{i=1}^{k-1} \bar{Y}_i^{(2)} Y_{k-i}^o$

for $k = 2, 3, \cdots, p$,

$$Y_k^o = -\sum_{i=1}^{p} \bar{Y}_i^{(2)} Y_{k-i}^o$$

for $k = p+1, \cdots, \infty$.

- **Eigensystem realization algorithm**

The Hankel matrix $\hat{H}(k-1)$ from the combined observer Markov parameters is associated with the system and observer as

$$\hat{H}(k-1) = \begin{bmatrix} \Upsilon_k & \Upsilon_{k+1} & \cdots & \Upsilon_{k+\beta-1} \\ \Upsilon_{k+1} & \Upsilon_{k+2} & \cdots & \Upsilon_{k+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ \Upsilon_{k+\alpha-1} & \Upsilon_{k+\alpha} & \cdots & \Upsilon_{k+\alpha+\beta-2} \end{bmatrix},$$

where $\hat{H}(k-1)_{(m\times\alpha)\times[\beta\times(m+r)]}$ $\alpha \geq 0$ and $\beta \geq 0$ are sufficiently large arbitrary integers and $\Upsilon_k = [Y_k \quad Y_k^o] = [CA^{k-1}B \quad CA^{k-1}G]$. Notice that it's required $(m\times\alpha) < [\beta\times(m+r)]$ and $\alpha \geq p$. However, a large α may induce a large numerical computation error. When the combined observer Markov parameters are determined, the eigen-system realization algorithm (ERA) method is used to obtain the desired discrete system realization $[A, B, C, G]$ through singular value decomposition (SVD) of the Hankel matrix.

The ERA processes the factorization of the block data matrix, started for $k = 1$, using the singular value decomposition $\hat{H}(0) = R\sum S^T$, where the columns of matrices R and S are orthonormal and $\sum$ is a rectangular matrix of the form as follows

$$\sum = \begin{bmatrix} \sum_{\tilde{n}} & 0 \\ 0 & 0 \end{bmatrix},$$

where $\sum_{\tilde{n}} = diag[\sigma_1, \sigma_2, \cdots, \sigma_{n_{\min}}, \sigma_{n_{\min}+1}, \cdots, \sigma_{\tilde{n}}]$ contains monotonically non-increasing entries $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{n_{\min}} \geq \sigma_{n_{\min}+1} \geq \cdots \sigma_{\tilde{n}} > 0.$ Here, some singular values $(\sigma_{n_{\min}+1}, \cdots, \sigma_{\tilde{n}})$ are relatively small $(\sigma_{n_{\min}+1} \ll \sigma_{n_{\min}})$ and negligible in the sense that they contain more noise information than system information. In order to construct the low order observer of the system, let's define $\sum n_{\min} = diag[\sigma_1, \sigma_2, \cdots, \sigma_{n_{\min}}]$. In other words, the reduced model of order $n_{\min}$ after deleting singular values $(\sigma_{n_{\min}+1}, \cdots, \sigma_{\tilde{n}})$ is then considered as the robustly controllable and observable part of the realized system with an acceptable closed-loop performance. Simultaneous realizations of the system and observer by the ERA are given as

$$\hat{H}(0) = R\sum S^T = [R\sum^{1/2}][\sum^{1/2} S^T] = \{P\}\{Q\}$$

where

$$P = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{\alpha-1} \end{bmatrix} \text{ and } Q = \begin{bmatrix} B & AB & \cdots & A^{\beta-1}B \end{bmatrix},$$

then

$A = \Sigma_{n_{\min}}^{-1/2} V_{n_{\min}}^{T} \hat{H}(1) S_{n_{\min}} \Sigma_{n_{\min}}^{-1/2}$,

$\begin{bmatrix} B & G \end{bmatrix} =$ First ($r+m$) columns of $\Sigma_{n_{\min}}^{1/2} S_{n_{\min}}^{T}$

$C =$ First m rows of $V_{n_{\min}} \Sigma_{n_{\min}}^{1/2}$

In order to more improve the performance, this paper used the iterative learning control to achieve this goal. This proposed method use $\alpha = 2$, and $\beta = 3$

B. Iterative learning control

This part will leads us go through an optimal tool, herein distances tracking error of AMB system are reduced, and steady-state is improved. The propose tool based on learning by the past active of some tasks in the system. In order to achieve the high performance and keep the advantage of the high gain system for sampled-data AMB system. Consider the discrete-time system minimum phase as

$\dot{x}_{cj}(\mathrm{kT}) = Ax_{dj}(\mathrm{kT}) + Bu_{dj}(\mathrm{kT}) + d(\mathrm{kT})$ (7.7a)

$y_{cj}(\mathrm{kT}) = Cx_{dj}(\mathrm{kT}) + Du_{dj}(\mathrm{kT}) + s(\mathrm{kT})$ (7.7b)

where $x_{cj}(0) = x(0),$ and j is iteration number, $d(\mathrm{kT}),\ s(\mathrm{kT})$ are unknown disturbances. Then can imagine there exists an equivalently artificial system model.

$\dot{x}_a(\mathrm{kT}) = Ax_a(\mathrm{kT}) + Bu_a(\mathrm{kT})$

$y_a(\mathrm{kT}) = Cx_a(\mathrm{kT}) + Du_a(\mathrm{kT}) + s_a(\mathrm{kT})$

$s_a(\mathrm{kT})$ denotes to actual steady-state error signal between the actual output of the system $y_{cj}(\mathrm{t})$ and pre-specified trajectory signal. Where the artificial control can be determined as

$u_a(\mathrm{kT}) = -K_a x_a(\mathrm{kT}) + S_a r(\mathrm{kT}) + C_a(\mathrm{kT}) + C_{u_a} u_a^{*}(\mathrm{kT})$

where

$K_a = \tilde{R}_a^{-1}(B^T P_a + N_a^T),$

$S_a = -\tilde{R}_a^{-1}\left[(C - DK_a)(A - BK_a)^{-1}B - D\right]^T Q_d,$

$C_{ad}(\mathrm{kT}) = -S_a s_a(\mathrm{kT}),$

$C_a^{*} =_a^{-1} (I_m + B^T[(A-B)]^{-1}K_a^T\}R_a,$

$N_a = C^T Q_a D,$

$\tilde{R}_a = R_a + D^T Q_a D,$

Then $s_a(\mathrm{kT})$ is determined as

$s_{aj}(\mathrm{kT}) = \sum_{i=0}^{j} e_{i-1}(\mathrm{kT})$

Where $e_{i-1}(\mathrm{kT}) = y_{c(\mathrm{i}-1)}(\mathrm{kT}) - r(\mathrm{kT}),$

Then apply optimal error compensation ILC leads to

$u_d(\mathrm{kT}) =$

$-K_d x_d(\mathrm{kT}) + S_d r(\mathrm{kT}) + C_d(\mathrm{kT}) + C_{u_d} u_d^{*}(\mathrm{kT})$

$$K_d = \tilde{R}_d^{-1}(B^T P_d + N_d^T),$$

$$S_d = -\tilde{R}_d^{-1}\left[(C - DK_d)(A - BK_d)^{-1}B - D\right]^T Q_d,$$

$$C_d(\mathrm{kT}) = -S_d s_d(\mathrm{kT}),$$

$$C_d^* = -\tilde{R}_d^{-1}(I_m + B^T[(A - B)]^{-1}K_d^T\}R_d,$$

$$N = C^T Q_d D,$$

$$\tilde{R}_d = R_d + D^T Q_d D,$$

An optimal LQDT with pre-specified measurement output and control input trajectories for the discrete-time controllable and observable system with both an input-to-output direct-feedthrough term and known system disturbances is summarized as follows [16].

Consider the controllable and observable linear discrete-time system with an input-to-output direct-feedthrough term and known/estimated system disturbances $d(k)$ and $s(k)$

$$x_d(k+1) = Gx_d(k) + Hu_d(k) + d(k), \quad (7.8a)$$

$$y_d(k) = Cx_d(k) + Du_d(k) + s(k), \quad (7.9b)$$

where $G \in \Re^{n \times n}$, $H \in \Re^{n \times m}$, $C \in \Re^{p \times n}$, and $D \in \Re^{p \times m}$ are state, input, output, and direct-feedthrough matrices, respectively. $x_d(k) \in \Re^n$ is the state vector, $u_d(k) \in \Re^m$ is the control input, and $y_d(k) \in \Re^p$ is the measurable output.The design goal is to determine the optimal control sequence $u_d(0),\ u_d(1),\ u_d(2),\ \cdots,\ u_d(N_f - 1)$ that minimizes the linear quadratic performance index for a finite time process

$$J(x_d, u_d) = \frac{1}{2}\left[y_d(N_f) - r(N_f)\right]^T S\left[y_d(N_f) - r(N_f)\right] +$$
$$\frac{1}{2}\sum_{k=0}^{N_f - 1}\left\{\left[y_d(k) - r(k)\right]^T Q_d\left[y_d(k) - r(k)\right] + \left[u_d(k) - u_d^*(k)\right]^T R_d\left[u_d(k) - u_d^*(k)\right]\right. \quad (14)$$

where Q_d is a $p \times p$ positive definite or positive semi-definite real symmetric matrix, R_d is an $m \times m$ positive definite real symmetric matrix, S is a $p \times p$ positive definite or positive semi-definite real symmetric matrix, $r(k)$ is a pre-specified output trajectory, and $u_d^*(k)$ is a pre-specified input trajectory. The resulting continuous-time state-feedback control law is given by

$$u_d(k) = -K_d x_d(k) + E_d r(k) + C_d(k) + C_u^* u_d^*(k), \quad (7.10)$$

where

$$K_d = \tilde{R}_d^{-1}\bar{P},\ E_d = \tilde{R}_d^{-1}\left\{D^T + H^T\left[I_n - \left(G - HK_d\right)^T\right]^{-1}\left(C - DK_d\right)^T\right\}Q_d,$$

$$C_d(k) = \tilde{R}_d^{-1}\left\{H^T\left[\left(G-HK_d\right)^T - I_n\right]^{-1}\left(C-DK_d\right)^T - D^T\right\}Q_d s(k)$$

$$+\tilde{R}_d^{-1}H^T\left\{\left[\left(G-HK_d\right)^T - I_n\right]^{-1}\left(G-HK_d\right)^T - I_n\right\}Pd(k)$$

$$= -E_d s(k) + Z_d d(k),$$

$$Z_d = \tilde{R}_d^{-1}H^T\left\{\left[\left(G-HK_d\right)^T - I_n\right]^{-1}\left(G-HK_d\right)^T - I_n\right\}P$$

$$C_u^* = \tilde{R}_d^{-1}\left\{H^T\left[\left(G-HK_d\right)^T - I_n\right]^{-1}K_d^T + I_m\right\}R_d,$$

$$\bar{R}_d = R_d + D^T Q_d D,$$

$$N_d = C^T Q_d D,$$

$$\tilde{R}_d = \bar{R}_d + H^T PH,$$

$$\bar{P} = H^T PG + N_d^T,$$

and P satisfies the algebraic Riccati equation

$$P = G^T PG + C^T Q_d C - \bar{P}^T \tilde{R}_d^{-1}\bar{P}$$

$$= G^T PG + C^T Q_d C - \left(H^T PG + N_d^T\right)^T \quad (7.11)$$

$$\left[\bar{R}_d + H^T PH\right]^{-1}\left(H^T PG + N_d^T\right).$$

C. Sampled-data controlled system

Let the corresponding digitally controlled model of (x) be described as

$$\dot{x}_d(t) = Ax_d(t) + Bu_d(t) + d(t) \;,\; x_d(0) = x_0 \quad (7.12a)$$

$$y_d(t) = Cx_d(t) + s(t), \quad (7.12b)$$

where $u_d(t) \in \Re^m$ is piecewise-constant, such that $u_d(\mathrm{t}) = u_d(kT)$, for $kT < t < (k+1)T$, and $T > 0$, is the period of sampling and hold. Let $u_d(\mathrm{t})$ be a discrete-time state-feedback control law of the for

$$u_d(kT) = -K_d\, x_d(kT) + E_d r^*(kT) + C_d(kT) + C_u^* u_d^*(kT),$$
$$kT < t < (k+1)T,$$

where $K_d \in \Re^{m\times n}$ and $E_d \in \Re^{m\times p}$ are the feedback and feed-forward digital gains, respectively, $C_d(kT)$ is the compensatory signal, and $r^*(kT)$ is a piecewise-constant reference input vector to be determined in terms of $r(kT)$ for tracking purpose. The digital reference input vector with tracking purpose is specified as $r^*(k) = r(k+1)$. For the direct-feedthrough term-free case. The viewpoint has been proved in [18-21]. The overall digitally controlled closed-loop system becomes

$$\dot{x}_d(t) = Ax_d(t) + B[-K_d\, x_d(kT) + E_d r(kT+T) + C_d(kT) + C_u^* u_d^*(kT)], \quad (7.13)$$

$x_d(0) = x_0$,

for $kT < t < (k+1)T$, , where the controller is realized using a zero-order-hold, the discrete-time model is described as

$$x_d((k+1)T) = G\, x_d(kT) + Hu_d(kT) + H_d d(kT), \quad (7.14a)$$

$$y_d(kT) = Cx_d(kT) + s(kT) \quad (7.14b)$$

where

$G = e^{AT}$,

$H = (G-I)^{-1} A^{-1} B$, if A^{-1} exists

Or

$H = \sum_{i=0}^{\infty} \frac{T^{i+1}}{(i+1)!} A^i B$, if A^{-1} does not exist, and

$H_d = (G-I)^{-1} A^{-1} I_n$, if A^{-1} exists

Then the control system is constructed as follows

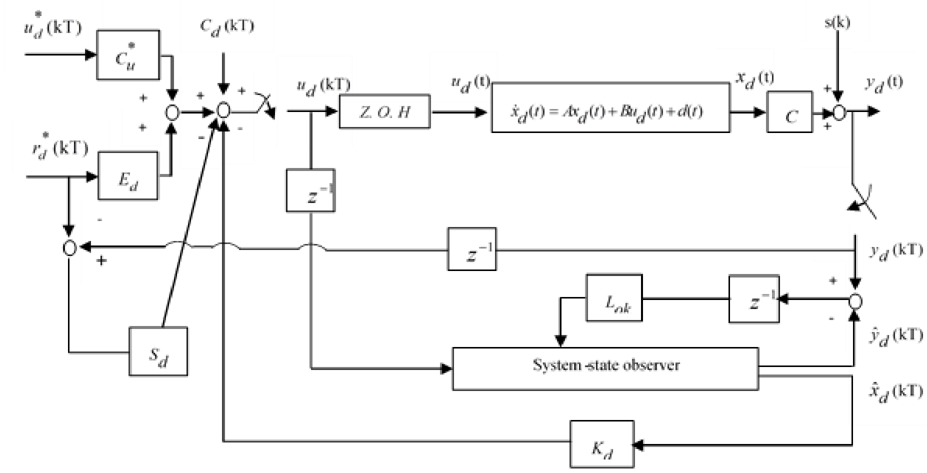
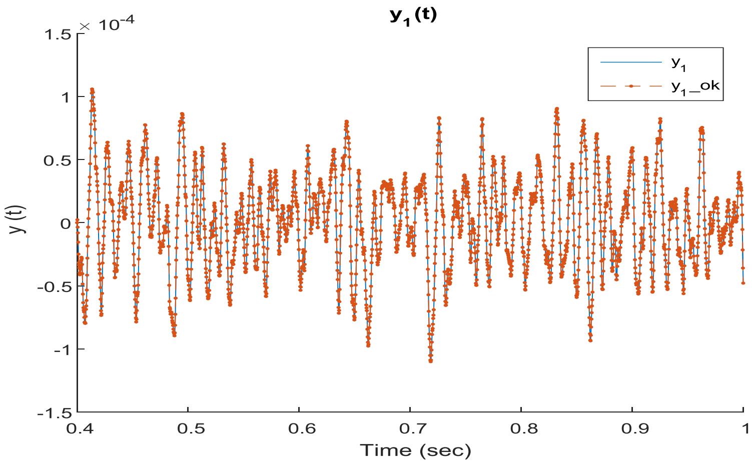
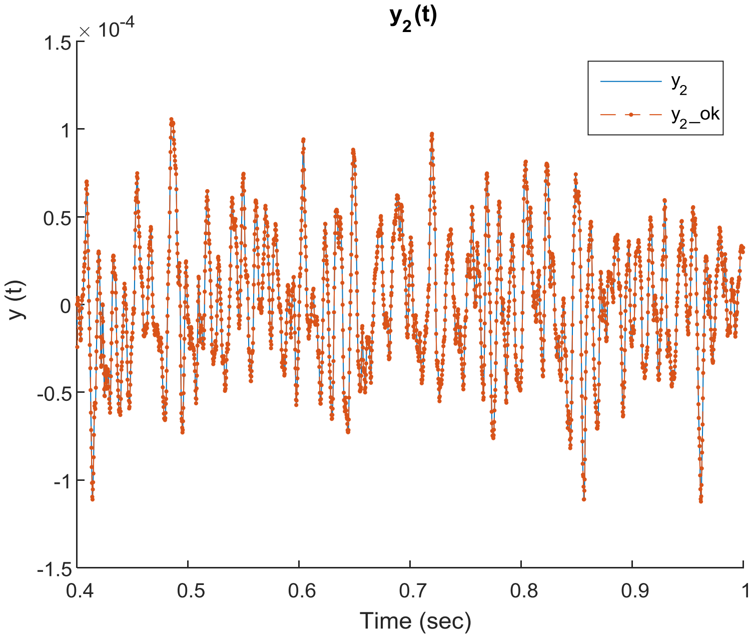


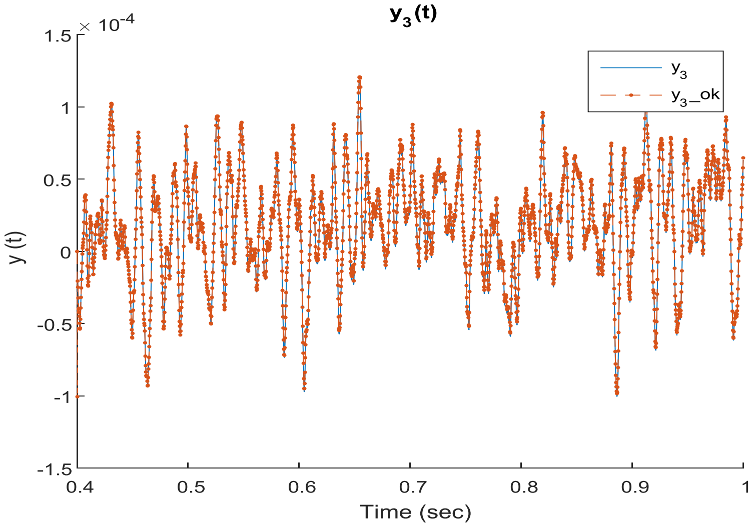
Figure 2.1 Iterative learning control for five DOF AMB system



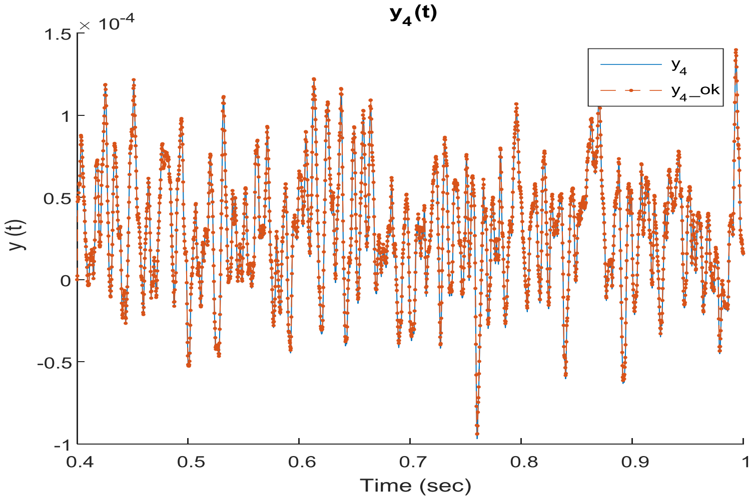
(a)



(b)

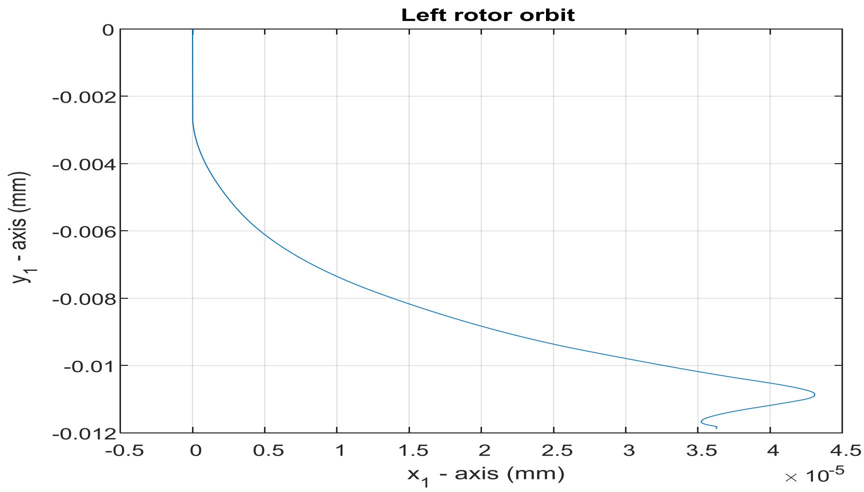


(c)

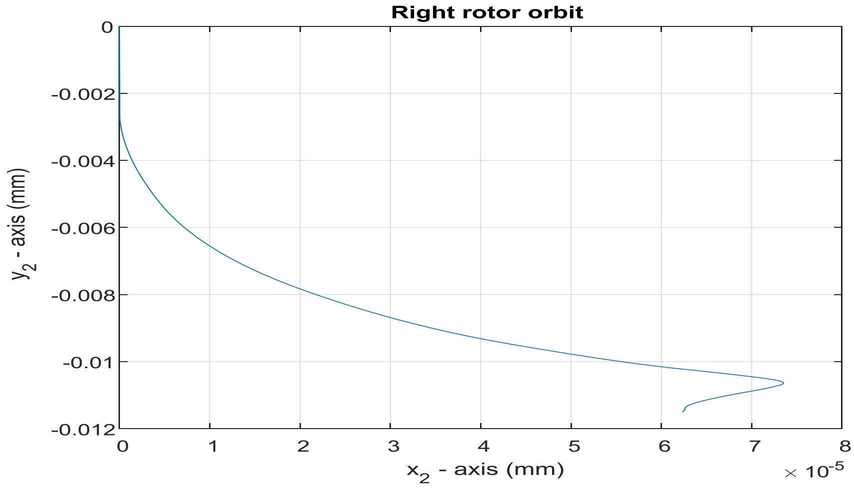


(d)

Figure. 2.2 The actual output responses, (a) y1(t) (b) y2(t) (c) y3(t) (d) y4(t)

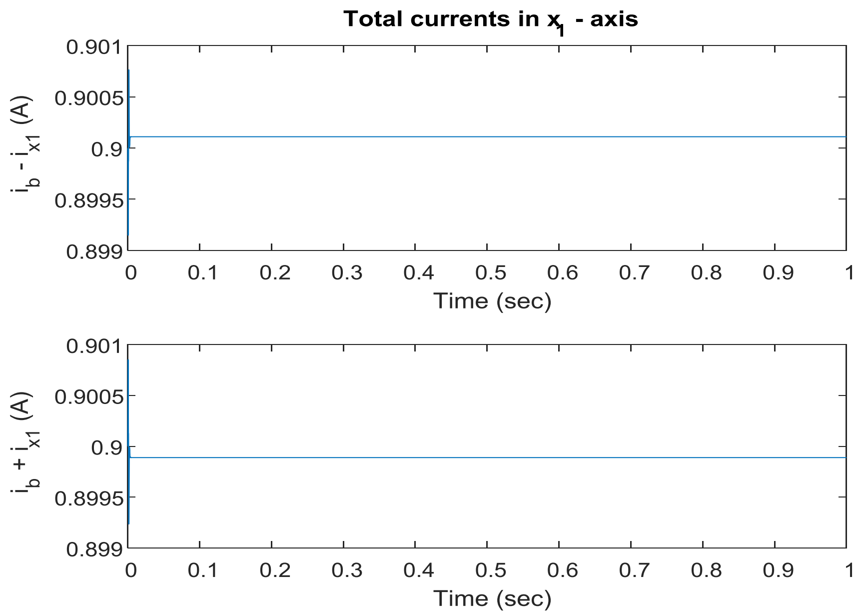


(a)

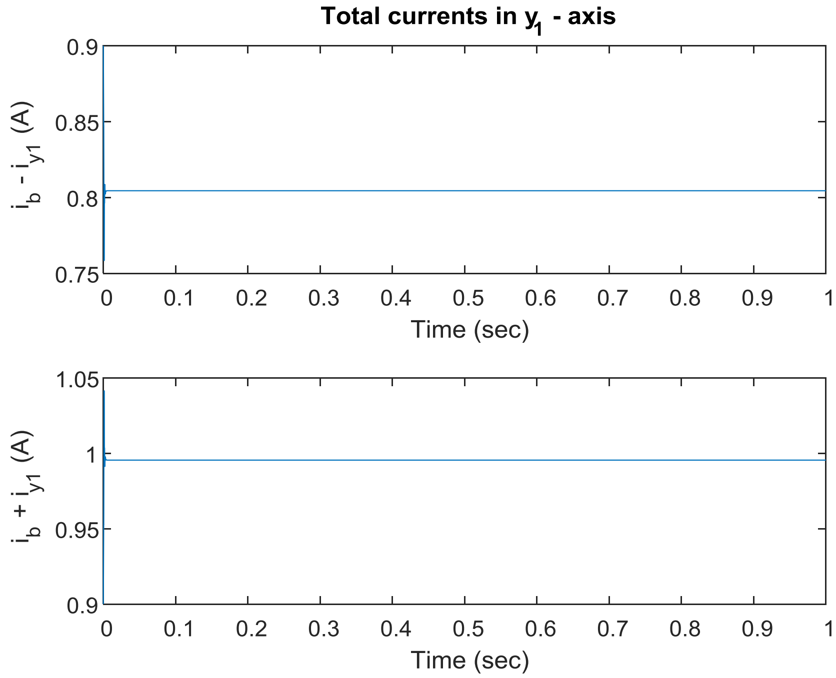


(b)

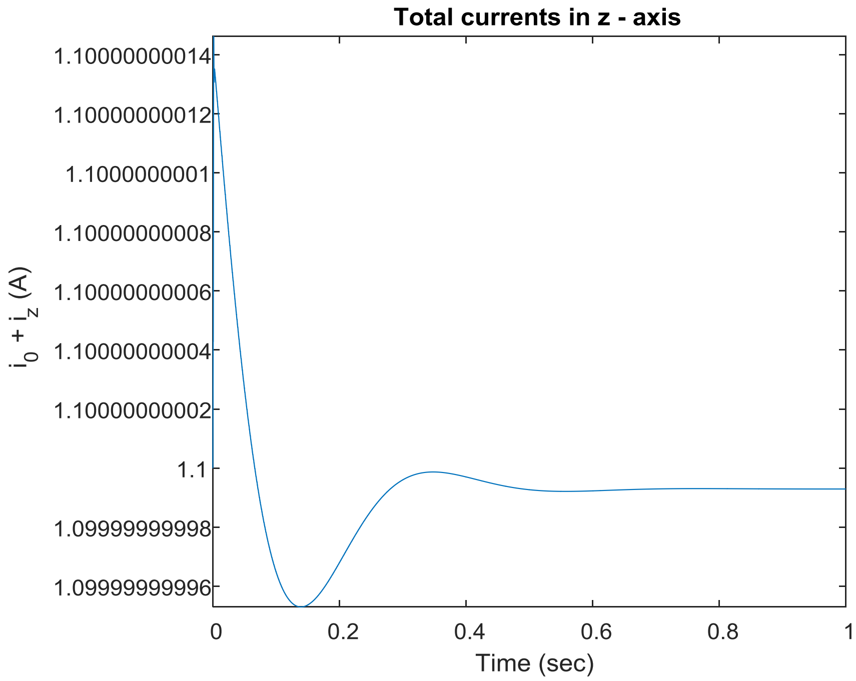
Figure. 2.3 System orbit, (a) left rotor orbit (b) right rotor orbit



(a)



(b)



(c)

Figure 2.4 control signals, (a) $i_b \pm i_{x1}$ (b) $i_b \pm i_{y1}$ and (c) $i_b + i_z$

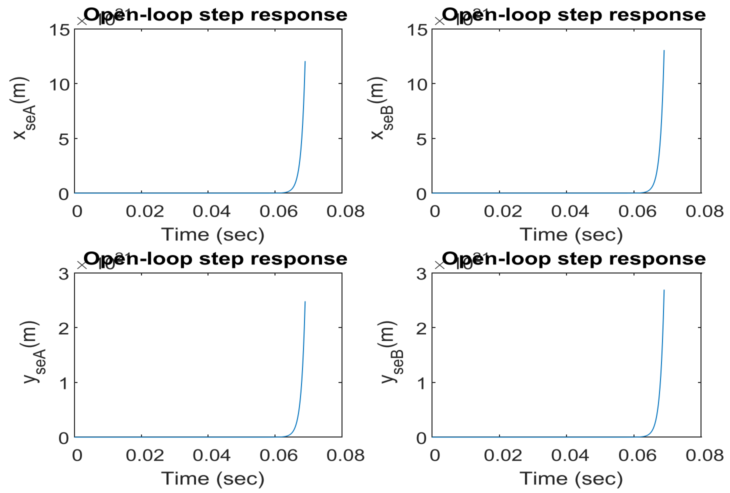


Figure 2.5 Open-loop step response

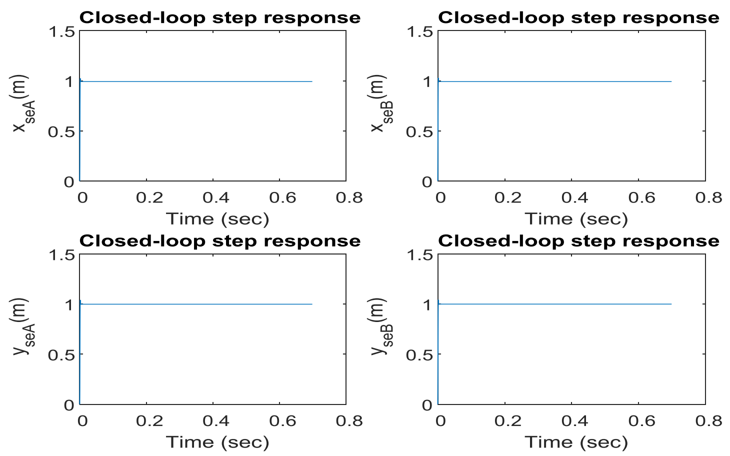


Figure 2.6 Closed-loop step response

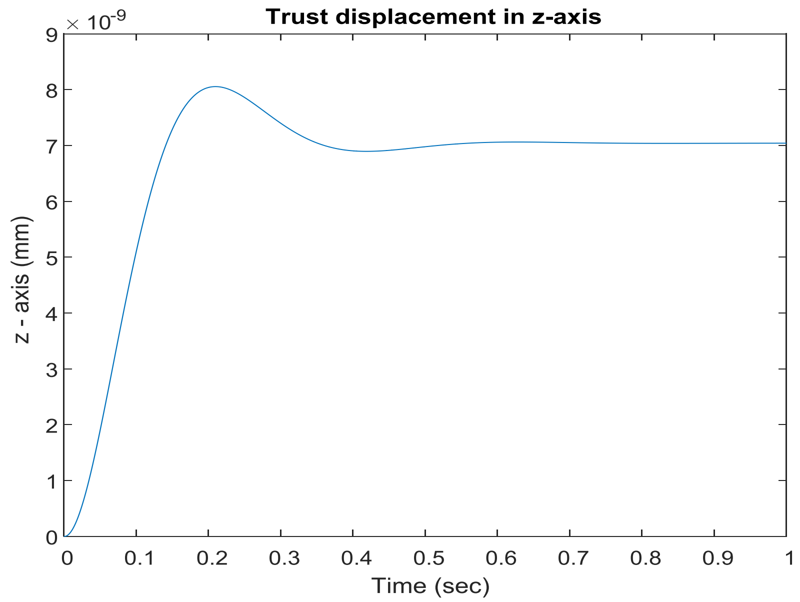


Figure 2.7 the z axis response

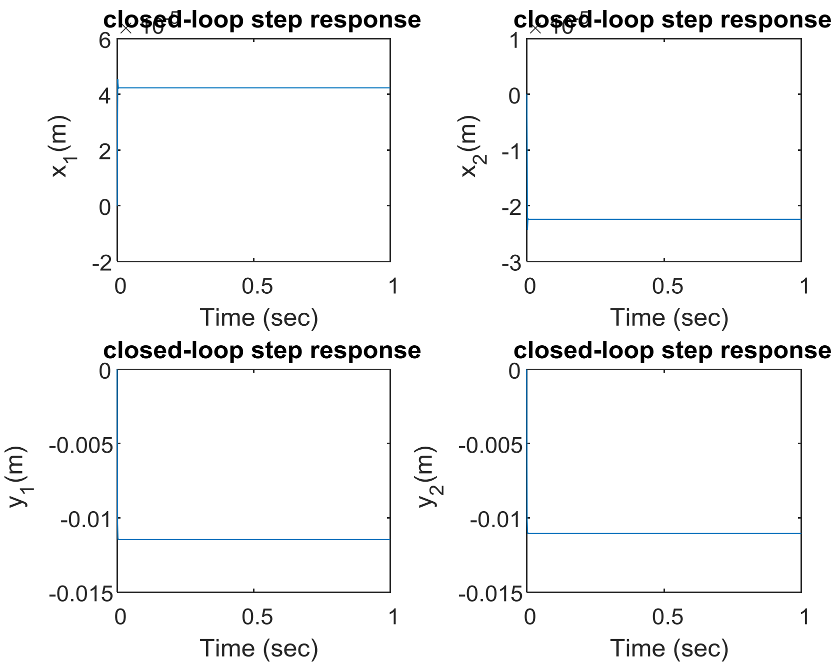


Figure 2.8 System state responses

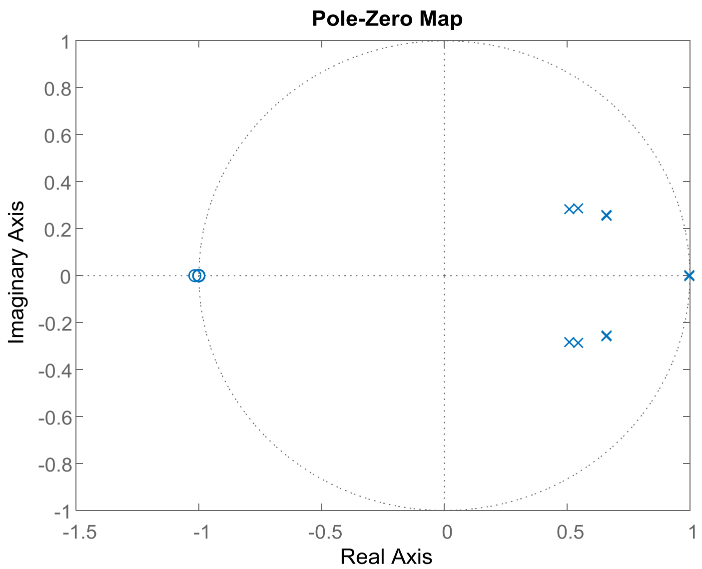


Figure 2.9 System poles

II. Summary

This study proposed the quite good controller for sampled-date active magnetic bearing system. To achieve the specified-goal this paper have to equipped some advanced technic as state-observer Kalman filter to estimate system state, use iterative learning control tool to reduce tracking error and develop the transient responses signal, and based on the linear quadratic digital tracker is built as background of this proposed controller. Then simulation results are given to demonstrate how effective of the proposed methodology on the system.

Table 3.1: The system parameters

Parameter	Description	Value	Unit
m	The mass of the rotor	2.56478	kg
L	The length of the rotor	0.505	m
τ	diameter of rotor	0.0166	M
J	The coefficient of inertia of rotor about X–Y axes	4.004e-2	kg m^2
J_z	The polar mass moment of inertia of rotor about Z-axis	6.565e-4	kg m^2
k_{ri}	The current stiffness of the RAMB	80	N/A
k_{rp}	The position stiffness of the RAMB	2.2e5	N/m
k_{ai}	The current stiffness of the TAMB	40	N/A
k_{ap}	The position stiffness of the AMB	3.6e4	N/m
a	The distance between CG and left RAMB	0.160	m
b	The distance between CG and right RAMB	0.190	m
c	The distance between CG and external disturbances	0.263	m
x_b, y_b	The nominal air gaps in X–Y axes of RAMB	0.4	mm
z_0	The nominal air gap in Z-axis of TAMB	0.5	mm

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