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Optimal Decoupling Control Design for Multivariable Processes: The Quadruple Tank Application

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Abstract:*This research paper discusses different controller designs for the nonlinear Quadruple Tank Process. The process is a multi-input multi-output system designed to control the water level in the lower two tanks (1 and 2). To achieve this, we derived the mathematical model for the nonlinear Quadruple Tank Process based on physical laws, and consequently obtained the linearised state space equations. In other to make for practicality in real life control the linearised model dynamics was discretized. Furthermore, the Multivariable zero in the linearised model places a vital rule in that their zero location determines the dynamic behaviour (minimal phase and non-minimal phase) of the system. This depends on the position of the three-port valve parameters. In addition, the control model was used to design firstly a decentralized SISO control for the multivariable process utilizing the Proportional Integral (PI) controller and a Linear-Quadratic Proportional-IntegralPlus (LQ-PIP) controller for benchmark purpose. Secondly a centralized multivariable control using the MIMO LQ-PIP controller and a MIMO PIP decoupling by combined algebraic pole assignment were implemented. The controller performances were compared based on Integral of Absolute error (IAE) between output and set-point and control effort based on Integral of Absolute control (IAC). Analysis of the control results showed perfect tracking on the set-point trajectory.*

Key Word*:* Zero Location, multivariable process, Integral of Absolute error, Integral of Absolute control.

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I. Introduction

Multivariable systems have become so common in process industries, due to the fact that control process cannot be controlled with a single control loop. Systems with more than one control input and more than one output variables are known as multi-input multi-output (MIMO) or multivariable systems, and most control unit's operation in the process industries require these kinds of systems. Multivariable systems are applied todifferent aspects of control, for example, in chemical systems like process evaporator and chemical reactors, distillation columns and heat exchangers. in oil and gas industries, mechanical systems such as the helicopter model [1], motor generator set, the active magnetic bearing process [2] and in electrical system such as antennas, etc. Hence, the multivariable control techniques have received increasing interest in the industries [3]. In order to analyse and control multivariable systems, a Two-input Two-output (TITO) quadruple tank nonlinear process is selected for benchmark purposes of investigated control techniques. Quadruple Tank Process (QTP) is a system with four interconnected tanks and two pumps. The QTP has influenced different aspects of multivariable control since its development in 1996 at Lund Institute of Technology, Sweden by [4]. The multivariable dynamics (minimum phase and non-minimal phase) was performed to demonstrate the effect of the multivariable zero location for control design [4, 5].

The control of liquid levels in a system of interacting tanks is a major control problem in the chemical industries where liquids are pumped from one tank to another and stored. The liquid levels in these tanks always need to be controlled and flow between tanks must be regulated as well to avoid overflow which could be very dangerous most especially when chemicals like H2SO4, among others, are involved. Case studies of tank related accidents are the "bhopal tragedy" of 1984 in India, another is the "Flixborough disaster" of 1974 at a chemical plant near Flixborough, England, and the list goes on. On these heels, this research is motivated by the need to design an optimal control framework for liquid level control of a system of highly interacting quadruple tank system, subject to input-output constraints as would be the case in a practical, real-life scenario.

Traditional control systems based on PID controllers are such that the parameters are tuned to achieve the system objectives (e.g. stability), which values are not necessarily optimal. As such, the controller parameters can be tuned and re-tuned until a satisfactory performance is achieved. In certain cases, changing their parameters directly may yield unsatisfactory result or even unstable solution [9]. On the contrary, optimal control systems simultaneously guarantee the attainment of system objectives and the minimization of some performance index [8,9]. The parameters of an optimal control system are adjusted so that a performance index approaches an extremum value (i.e., minimum), as the quality of the control increases. The linear quadratic cost function is minimised in this work and closed-loop stability is guaranteed in the deterministic sense. In multivariable industrial processes where a number of output channels are controlled by a number of inputs, these output channels often exhibit a high degree of cross-coupling or interaction such that when there is the need to change an output specification, other outputs (which are not desired to be changed) are affected as well.

Motivated by the work done by [14,15] on the ideas in the control of multivariable systems, [4,5,6,8] developed a novel multivariable workshop process that comprises interlinked water tanks. In [13], the author used the physical data obtained in the laboratory to develop a mathematical model of the quadruple-tank. Contrary to the research work in [12], where a centralized controller is applied on the quadruple tank process, [11,12] applied decentralized PI control techniques on the model and it was illustrated that it is simple to control the process of the model in the minimum-phase configuration compare to that in non-minimum phase configuration. A combination of two control techniques, the predictive controller with smith predictor algorithm and fractional order controller, called Predictive Fractionalorder PI controller was applied on the quadruple tank process [16]. Their idea of the combination of the two control techniques include, the reduction of the influence of dead time through the application of the predictive controller with smith predictor algorithm and tuning tractability through the application of fractional order controller. The fractional order PI controller creates more chances for the adjustment of the dynamical characteristics of a control system. The controller technique developed in [13] was tuned by applying the Ziegler-Nichols gain shaping [3,4] methods.In their research work, they illustrated that the performance of the predictive fractional order PI control method with Hagglund tuning method on non-minimum phase system, is more effective in terms of rise time, peak time and settling time, than other methods. Furthermore, they illustrated the performance of the predictive fractional PI control method with gain shaping tuning method on minimum phase system, is more effective in terms of overshoot, peak time, rise time and settling time, than other methods.

II. The Physical Model System

A detailed understanding of the physical model of the system is very important for the design of any control process. This paper gives the fundamental knowledge of the multivariable process using the concept of a nonlinear Quadruple tank application. The TITO Quadruple Tank Process consists of four interconnected identical water tanks, two pumps and two valves that allow the inflow of water into the upper and lower tanks. The tanks are piled orderly in a vertical manner with one tank over another.

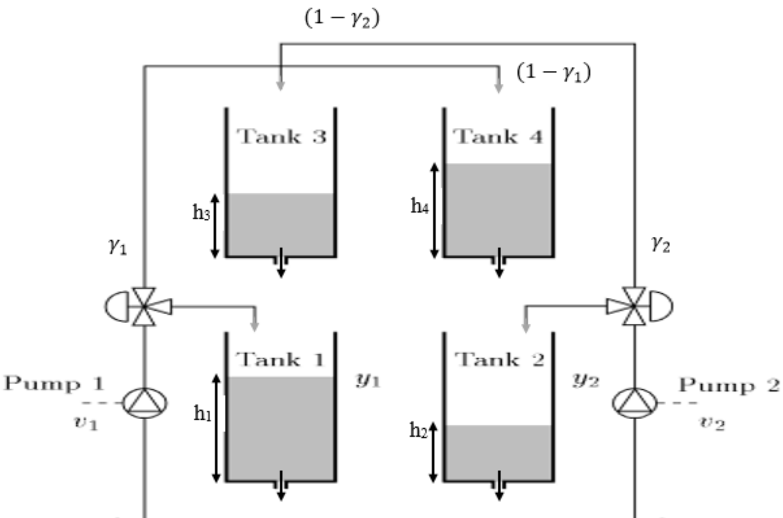

Figure 1. Schematic Diagram of the Quadruple Tank Process

The schematic diagram of the QTP is illustrated in Figure 1, each tank has an outlet hole at the button that allows the outflow of water from the upper tanks to the lower tanks and from the lower two tanks to the reservoir. The coupling/interaction between water tanks are regulated by two three-port valves. The process inputs are y_1 and y_2 (input voltages to the two pumps) and the outputs are y_1 and y_2 (voltages from level measurement devices). The height of each tank is 20cm with a diameter of about 6cm. A capacitive electrode is a measuring instrument used to measure the water levels.

Nonlinear Model

In fluid systems, the non-linearity relationship is usually between pressure and flow rate for a liquid under turbulent flow conditions [5]. The variation of volumetric flow of water, V in each tank can be expressed as

$$\frac{dV}{dT} = q_{in} + q_{out} \quad - \quad - \quad - \quad - \quad 1$$

From Torricelli's principle, given the outlet hole cross sectional area, a_i in each tank, the output flow rate from each tank can be derived as:

$$q_{out} = a_i\sqrt{g}h_i \quad - \quad - \quad - \quad - \quad -2$$

The acceleration due to gravity is denoted by g, and h_i is the water level in the tank. From Figure 1, the two pumps are connected such that the flow from pump 1 goes into tank 1 and 4 while the flow from pump 2 goes into tanks 2 and 3. Also, there are inflows from the upper tanks 3 and 4 into the lower tanks 1 and 2. The flow of water through pumps 1 and 2 is proportional to the applied input voltages V_1 and V_2 from pumps 1 and 2 and the corresponding flow is k_iv_i.

Where, k_i is the pump constant and v_i (where i= 1 to 2) denotes the input voltages. The flow to the upper two tanks 3 and 4 are $(1-\gamma_2)k_2v_2$ and $(1-\gamma_1)k_1v_1$ and flow to the lower tanks 1 and 2 are $\gamma_1 k_1v_1$ and $\gamma_2 k_2v_2$ respectively. This flow from the pumps 1 and 2 is influenced by the parameters γ_1, $\gamma_2 \in (0, 1)$ which determine the three-port valves positions. The dynamics of a nonlinear model for the Quadruple Tank Process [13] is derived based on the physical laws (the Mass balances and Bernoulli's law); and the nonlinear differential equation from 1 and 2 leads to equation 3.

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_1}v_1
\end{aligned} \quad - \quad 3$$

where:
A_i is the cross-sectional area of the tank *i*,
a_i is the cross-sectional area of the outlet hole in each tank,
h_i is the water level in the tanks, *i* = 1 to 4
The measured output levels are y_1 = kch1 and y2 = k_ch_2.
k_c is a proportional constantmultiplying the level signals h_1 and h_2.

Linearised State-Space Model Representation

The non-linear dynamic model equation 3 will be linearised around an operating point v_i^0 and h_i^0;
where the deviation variable $_{xi} = h_i - h_i^0$,and $u_i = v_i - v_i^0$.
The linearised statespace equation is derived using Taylor series explanation and expressed as equation 4.

$$\begin{aligned}
\dot{x}_1 &= -\frac{a_1}{A_1}\sqrt{\frac{g}{2{h_1}^0}}x_1 + \frac{a_3}{A_1}\sqrt{\frac{g}{2{h_3}^0}}x_3 + \frac{\gamma_1 k_1}{A_1}v_1 \\
\dot{x}_2 &= -\frac{a_2}{A_2}\sqrt{\frac{g}{2{h_2}^0}}x_2 + \frac{a_4}{A_2}\sqrt{\frac{g}{2{h_4}^0}}x_4 + \frac{\gamma_2 k_2}{A_2}v_2 \\
\dot{x}_3 &= -\frac{a_3}{A_3}\sqrt{\frac{g}{2{h_3}^0}}x_3 + \frac{(1-\gamma_2)k_2}{A_3}v_2 \\
\dot{x}_4 &= -\frac{a_4}{A_4}\sqrt{\frac{2g}{2{h_4}^0}}x_4 + \frac{(1-\gamma_1)k_1}{A_4}v_1
\end{aligned} \quad 4$$

where the time constant

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2{h_i}^0}{g}} \quad - \quad - \quad 5$$

$$(T_i)^{-1} = \frac{a_i}{A_i}\sqrt{\frac{g}{2{h_i}^0}} \quad - \quad 6$$

The general linear state space model from [17] is given as:

$\dot{x} = Ax + Bu$ - - - - - 7

$Y = Cx$ - - - - - 8

where A and B are Jacobian matrices, C is the output matrix, and U is the control input.

$$\mathrm{A} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_3 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} \quad \mathrm{B} = \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & -\frac{(1-\gamma_2)k_2}{A_2} \\ \frac{(1-\gamma_1)k_1}{A_1} & 0 \end{bmatrix} \quad \mathrm{C} = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} \qquad 9$$

The linearised state space model becomes

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_3 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & -\frac{(1-\gamma_2)k_2}{A_2} \\ \frac{(1-\gamma_1)k_1}{A_1} & 0 \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} \boldsymbol{x} \qquad 10$$

Converting the state space model in equations 7 and 8 to transfer function (TF) [5] by applyingLaplace transforms assuming zero initial condition. This results in equations 11 and 12.

$sX(s) = AX(s) + BU(s)$ - - - - 11

$Y(s) = CX(s)$ - - - - - 12

The Transfer function matrix which relates the output Y (s) to the input U(s) is obtained in equation 13.

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1} \qquad - \quad - \quad - \qquad 13$$

Like all SISO systems, a multivariable system can be represented as a TF; because of the difference in their matrices formation, it is called a transfer function matrix (TFM). Therefore, the accompanying TFM to the linearised Model is thus obtained by substituting A, B and C as denoted in equations 11 and 12 and the solution into equation 13. This results in equation 14.

$$G(s) = \begin{bmatrix} \frac{\gamma_1 T_1 K_1 k_c}{A_1(1-sT_1)} & \frac{(1-\gamma_2) T_1 K_1 k_c}{A_1(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1) T_2 K_2 k_c}{A_2(1+sT_4)(1+sT_2)} & \frac{\gamma_2 T_2 K_2 k_c}{A_2(1+sT_2)} \end{bmatrix} \qquad - \quad - \qquad 14$$

Based on literature, the following minimal phase parameter values shown in Table1 were used in this paper to design the QTP.

Substituting the parameter values from Table 1 into the equation 14, the QTP model is obtained in equation 15.

$$G(s) = \begin{bmatrix} \frac{2.6}{1+62s} & \frac{1.5}{(1+23s)(1+62s)} \\ \frac{1.4}{(1+30s)(1+90s)} & \frac{2.8}{A_2(1+90s)} \end{bmatrix} \qquad - \quad - \qquad 15$$

Table 1: Quadruple Tank Process Parameters

	Parameter values
A_1, A_3 $[cm^2]$	28
A_3, A_4 $[cm^2]$	32
a_1, a_2 $[cm^2]$	0.071
a_3, a_4 $[cm^2]$	0.057
h_1^0, h_2^0 $[cm]$	(12.4, 12.7)
h_3^0, h_4^0 $[cm]$	(1.8, 1.4)
υ_1^0, υ_1^0 $[V]$	(3.00, 3.00)
k_1, k_2 $[cm^3/Vs]$	(3.33, 3.35)
γ_1, γ_2	(0.70, 0.60)
T_1, T_2 $[s]$	(62, 90)
T_3, T_4 $[s]$	(23, 30)
k_c $[V/cm]$	0.5
g $[cm/s^2]$	981

Discretization of the system

The linearised TF model G(s) in equation 5 is discretised for control design purposes, using a zero-order hold (ZOH) discretization method and a sampling time, T=1[s]. The ZOH discretization operator is given as:

$$ZOH = \frac{1-e^{-sT}}{S} = \frac{1-Z^{-1}}{Z} = \frac{Z-1}{Z}.\frac{1}{S} \quad - \quad - \quad - \quad 16$$

The discretised TFM of the system model (equation 15) using equation 16 results in equation 17.

$$G(Z^{-1}) = \begin{bmatrix} \frac{0.0416Z^{-1}}{1-0.984Z^{-1}} & \frac{0.0002555\,Z^{-1}+0.0002517\,Z^{-2}}{1-1.956Z^{-1}+0.9565Z^{-2}} \\ \frac{0.0005156\,Z^{-1}+0.0005055\,Z^{-2}}{1-1.941Z^{-1}+0.9421Z^{-2}} & \frac{0.03094Z^{-1}}{1-0.9849} \end{bmatrix} \quad - \quad 17$$

Control Performance Measures

The control performance measures considered for the controller evaluation study are Integral of Absolute Error (IAE) and Integral of Absolute Control (IAC) as defined in equations 18 and 19:

$$IAE = \frac{1}{N}\sum_{k=1}^{N}|R(K) - y(K)| \quad - \quad - \quad - \quad 18$$

$$IAC = \frac{1}{N}\sum_{k=1}^{N}|U(K)| \quad - \quad - \quad - \quad - \quad 19$$

where:

N = the number of sampled data.

IAE provides a measure of the absolute error between the measured system output y(k) and the assumed set point R(k). IAC is then assumed to be proportional to the overall control effort u(k).

Minimum Phase TF Model Representation

In control design, the multivariable zero in the linearised model plays an important role. The zero location of the TFM in equation 15 has intuitive physical interpretations in terms of how the three port valve parameters 1 and 2 are set. The zeros of G(s) are the zeros of the numerator polynomial of the rational fraction. A multivariable zero that lies in the left half plane indicates a minimum phase system and when at least one zero lies in theright half plane the system is said to be non-minimum phase (Johansson 2000, Johansson et al. 1999). The zero to the TFM is determined as det G(s).

From the TFM, let

$$C_1 = \frac{T_1K_1k_c}{A_1} \quad - \quad - \quad - \quad - \quad - \quad 20$$

$$C_2 = \frac{T_2K_2k_c}{A_2} \quad - \quad - \quad - \quad - \quad - \quad 21$$

Finding the determinant of G(S) in 15 and substituting C_1 and C_2 gives;

$$detG(s) = \frac{\gamma_1\gamma_2C_1C_2}{(1-sT_1)(1-sT_2)} - \frac{(1-\gamma_1)(1-\gamma_2)C_1C_2}{(1-sT_1)(1-sT_2)(1-sT_3)(1-sT_4)} - \quad 22$$

$$detG(s) = C_1C_2\left[\frac{1}{(1-sT_1)(1-sT_2)}\right]\left[\gamma_1\gamma_2 - \frac{(1-\gamma_1)(1-\gamma_2)}{(1-sT_3)(1-sT_4)}\right] \quad - \quad 23$$

$$detG(s) = C_1C_2\left[\frac{1}{(1-sT_1)(1-sT_2)}\right]\left[\frac{\gamma_1\gamma_2(1-sT_3)(1-sT_4)-(1-\gamma_1)(1-\gamma_2)}{(1-sT_3)(1-sT_4)}\right] \quad - \quad 24$$

$$detG(s) = \frac{C_1C_2[\gamma_1\gamma_2(1-sT_3)(1-sT_4)-(1-\gamma_1)(1-\gamma_2)]}{(1-sT_1)(1-sT_2)(1-sT_3)(1-sT_4)} \quad - \quad 25$$

Equating det G(s) = 0, the denominator is eliminated, thereby making the numerator of the multivariable zero to take the form:

$$C_1C_2[\gamma_1\gamma_2(1-sT_3)(1-sT_4)-(1-\gamma_1)(1-\gamma_2)] \quad - \quad 26$$

$$(\gamma_1\gamma_2T_3T_4)s^2+(\gamma_1\gamma_2T_4)s+(\gamma_1+\gamma_2-1)=0 \quad - \quad 27$$

From equation 27, the time constant T_i (where i=1 to 4) and the parameters γ_1 and γ_2 are positive, in the quadratic equation there is a restriction on the constant term $\gamma_1+\gamma_2-1$which is the basis of concentration in this research.
For minimum phase (both zeros lie in the LHP);

$$\gamma_1+\gamma_2-1>0 \quad - \quad - \quad - \quad - \quad - \quad 28$$

and the range will be

$$1<(\gamma_1+\gamma_2)<2 \quad - \quad - \quad - \quad - \quad 29$$

For non-minimal phase (at least one of the zeros must lie in the RHP);

$$\gamma_1+\gamma_2-1<0 \quad - \quad - \quad - \quad - \quad - \quad 30$$

and its range therefore lies in

$$0<(\gamma_1+\gamma_2)<1 \quad - \quad - \quad - \quad - \quad 31$$

Relative Gain Array (RGA) Considerations
RGA is a control tool for the analysis of multivariable process control and was developed by [4]. It is used to select appropriate manipulated variable to controlled variable pairing in order to minimise the effect of loop interactions in a multi-loop system.
The most effective pairing of controlled and manipulated variable, the relative gain, $\boldsymbol{\lambda}_{ij}$ between a control variable, y_{ij} and a manipulated variable, u_j is defined as a ratio of two steady state gains;

$$\lambda_{ij}=\frac{\left(\frac{\delta y_i}{\delta u_j}\right)u}{\left(\frac{\delta y_i}{\delta u_j}\right)y}=\frac{open\ -loop\ gain}{closed\ \ -loop\ gain} \quad - \quad - \quad - \quad 32$$

for i = 1, 2, …….. n and j = 1, 2, ……..,n. Where $\left(\frac{\delta y_i}{\delta u_j}\right)u$denotes the partial derivative with all of the manipulated variables (input) except u_j held constant and $\left(\frac{\delta y_i}{\delta u_j}\right)y$the partial derivative with all the controlled variables (output) except y_i held constant.
Hovd &Skogestadin [9] demonstrated that RGA calculation can be expressed in matrix notation making computations for larger systems earlier. RGA denoted by $\boldsymbol{\Lambda}$ is given as;

$$\Lambda=\mathrm{G}(s).*(G^T(s))^{-1} \quad - \quad - \quad - \quad - \quad 33$$

Steady state RGA matrix is thus derived by setting the Laplace variable s = 0

$$\Lambda=\mathrm{G}(0).*(G^T(0))^{-1} \quad - \quad - \quad - \quad - \quad 34$$

where G is the process gain matrix,* represents element by element scalar multiplication (the asterisk often known as schur product or hadamard), T and -1 represent the transpose and inverse operation. The basic rule for optimal input-output pairing selection is to pair corresponding relative gains which are positive and close to one another as much as possible. The relative gain array $\boldsymbol{\Lambda}$ is arranged according to the form shown as follows:

$$\Lambda=\begin{matrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{matrix}\begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{n1} & \lambda_{n1} & \cdots & \lambda_{nm} \end{bmatrix}\begin{matrix} u_1 \\ u_2 \\ \cdots \\ \lambda u_n \end{matrix} \quad - \quad - \quad 35$$

SISO PIP and PID Control Techniques
A SISO system in control engineering is classified as a system with a single input and a single output and it happens to be the simplest and most commonly used control system. The PI/PID controller has played a vital rule in control Engineering. For over fifty years, the classical Proportional+Integral+Derivative (PID)

controllers exhibited maxi- mum quality that is widely accepted due to their no complicated structure and parameter tuning. In 1934, the first tuning rule was implemented on the PD controller and later extension was made for PI and PID controllers. Over 95% of controller today in the process industries are either implemented using the PI/PID controllers. Furthermore, the tuning of PID controllers was proposed by Ziegler and Nichols. In terms of block diagram PIP controller is a logical extension of conventional PI/PID controllers. The PIP control utilizes the non- minimal static space (NMSS) algorithm of linear, discrete time system. Their control algorithm is derived from controller polynomial using SVF pole assignment. The SVF pole assignment of a PIP Controller are controlled in predictive manner since the input and output are defined in terms of present and past values. Where the input to the controller is considered to be the present input value. The integral error state z_k integrates the error between the output and set point and ensures suitable model performance. The application for SISO PIP and the conventional Proportional Integral Derivative (PID) control techniques will be implemented in this chapter and emphasis is placed on decentralized PI controller and decentralized LQ-PIP controller. The feedback gains are obtained such that a Linear Quadratic cost function will be minimized. The PIP control method has tuning parameters represented by the weights of the LQ cost function.

Non-minimal State Space (NMSS) Model Representation

The NMSS model is the state space representation of general discrete time single input single output (SISO) transfer function model given as

$$y(k) = \frac{B(z^{-1})}{A(z^{-1})}u(k) \quad - \quad - \quad - \quad - \quad 36$$

where;

u(k) denotes the system input

y(k) is the measured system output

$A(z^{-1})$ and $B(z^{-1})$ are polynomials expressed as

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m} \quad - \quad 37$$

$$A(z^{-1}) = a_0 + a_1 z^{-1} + a z^{-2} + \cdots + a_n z^{-n} \quad - \quad 38$$

where n > m, a_0=1, $b_0 \neq 0$, z^{-1} is the backward shift operator defined as

$$B(z^{-1}) = x(k-1) \quad - \quad - \quad - \quad - \quad 39$$

The equivalent model of 3.1 in difference equation form is given as;

$$y(k) + a_1 y(k-1) + \cdots + a_n y(k-1) = b_1 u(k-1) + \cdots + b_m u(k-m) \quad -40$$

$$y(k) = -a_1 y(k-1) - \cdots - a_n y(k-1) + b_1 u(k-1) + \cdots + b_m u(k-m) \quad -41$$

The major advantage of NMSS is the simplicity of the state vector, which comprises only the present and the past sample values of the input and output values respectively. It is the natural state space representation of a DT-TF model. Thus, it is easy to show that the model in equation 36 can be written in the NMSS form;

$$x(k) = Fx(k-1) + gu(k-1) + dy_d(k) \quad - \quad - \quad 42$$

and the associated output equation:

$$y(k) = hx(k) \quad - \quad - \quad - \quad - \quad - \quad 43$$

The NMSS state vector x(k) defined in terms of present and past sampled outputs and the past sampled inputs and the integral-of error state variable z(k) is incorporated, ie

$$x(k) = [y(K) \;\; y(k-1) \ldots y(k-n+1) \;\; u(K-2) \ldots u(K-m+1) z(K)]^T \quad - \quad -44$$

where z(k) is defined as

$$z(K) = z(k-1) + y_d(k) \quad - \quad - \quad - \quad - \quad 45$$

Linear Quadratic LQ-PIP controller for SISO Control

This is a type of optimal control design whereby the control parameters are adjusted to achieve the minimum of a performance index. In this case, a linear-quadratic type of performance criterion is minimised. Thus, the PIP gain vector k is designed to minimize the quadratic cost function.

$$J = \sum_{k=0}^{\propto} x(k)^T Qx(k) + r(u(k)^2) \quad - \quad - \quad - \quad 46$$

where Q is square symmetric, positive semi-definite matrix and r is a positive scalar. x(k) and u(k) are the state vector and control input respectively. Equation 46 is the infinite time optimal LQ cost function for a SISO system.

Implementation of SISO LQ-PIP control law

The weighing matrix Q is taken as a purely diagonal vector whose diagonal elements are generally set to some user defined values,

$$Q = \mathrm{diag}(q_1 \; q_2 \cdots q_n \; q_{n+1} \cdots q_{n+m-1} \; q_{n+m}) \quad - \quad 47$$

The user defined output weighting parameters q_1, q_2,$\cdots$ q_n and input weighting parameters $q_{n+1},\cdots,q_{n+m-1}$ are generally set to common values of q_y, q_u and q_e respectively, while q_{n+m}denoted as q_e to indicate that it provides a weighting constraint on integral of error variable z(k).
The diagonal LQ weightings are given as

$$Q = diag(q_y \cdots q_y \; q_u \cdots q_e) \qquad 48$$

In the model in equation 46, input weighing r is typically set to q_u. The partial weightings on the output, input and integral of error variables in the NMSS vector x(k) are usually defined as

$$q_y = \frac{W_y}{n} \qquad 49$$

$$q_u = \frac{W_u}{m} \qquad 50$$

$$q_e = W_e \qquad 51$$

The three scalar weightings W_y, W_u, and W_e or the diagonal LQ weightings are cautiously chosen and manual tuned in order to achieve the desired closed loop performance. Having derived the weighting Q, the control input weighing r; the state transition matrix F, and input vector g obtained from the NMSS model. The SVF gain vector k^T can now be determined as

$$k^T = (r+g^T P g)^{1} g^T PF \qquad 52$$

Apparently from this theoretical outcome of the SVF gain vector, the solution depends upon the choice of the weighting matrix Q and r. From equation 52, the control law for a SISO LQ-PIP is derived by substituting the SVF gain vector k^T.

$$u(k) = -k^T x(k) = -(r + g^T P g)^{-1} g^T P F x(k) \qquad 53$$

P is the steady-state solution of the discrete time matrix Riccati equation as shown below:

$$P - F^T PF + F^T Pg(r+g^T Pg)^{-1} g^T PF - Q = 0 \qquad 54$$

Recursive solution to Riccati equation for the gain vector k solved are;

$$k^T(i) = (r + g^T P(i + 1)g)^{-1} g^T P(i + 1)F \qquad 55$$

$$P(i) = Q + F^T P(i+1)F - F^T P(i+1)gk^T(i) \qquad 56$$

PI Pole-Placement Control

Pole Placement control (PPC) is a major approach employed to control the performance of a system. The basic aim is to control and modify the stability of a closed loop poles of a feedback system to a desired location. The placement of the poles can only be achieved in a dynamic system that is controllable. Different PI controller structures have been identified and their design vary from the choice of controller structure to its controller tuning, which one approach may function well on one architecture and may work poorly on another. Considering the PI controller expressed in terms of two gains and of the form.

$$u(k) = K_P \left[e(t) + \frac{1}{Ti} \int_t^0 e(t)dt \right] \qquad 57$$

where
K_P = Proportional gain
Ti =Integral time

$$e(k) = y_d(k) - y(k) \qquad 58$$

$$Ti = \frac{K_P}{K_I} \qquad 59$$

K_I denotes the integral gain
The additional integral mode (often referred to as reset) corrects for any offset (error) that may occur between the desired value (set-point) and the process output automatically over time. The control law in its continuous form is given as;

$$u(k) = K_P e(k) + K_I \frac{1}{s} e(k) \qquad 60$$

Multiplying both sides by s ($s = \frac{1-z^{-1}}{h} = \frac{\Delta}{h}$) and discretize the system using Euler back discretization method equation 60 becomes:

$$su(k) = K_P se(k) + K_I e(k) \qquad 61$$

$$\frac{\Delta}{h} u(k) = \frac{K_P}{h} \Delta e(k) + K_I e(k) \qquad 62$$

$$u(k) - u(k-1) = K_P e(k) - K_P e(k-1) + K_I e(k) \qquad 63$$

$$u(k) = u(k-1) + (K_P + K_I h)e(k) + (-K_P)e(k-1) \qquad 64$$

$$u(k) = u(k-1) + q_0 e(k) + q_1 e(k-1) \qquad 65$$

The transfer function of the discrete PI controller derived from the control law is given by

$$\frac{u(k)}{e(k)} = \frac{q_0+q_1Z^{-1}}{1-z^{-1}} = \frac{Q(z)}{P(z)} \quad - \quad - \quad - \quad - \quad 66$$

Q(z) and P(z) are controller gain polynomials express

$$Q(z) = q_0 + q_1Z^{-1} + \cdots + q_rZ^{-r} \quad - \quad - \quad - \quad 67$$

$$P(z) = 1 + p_1Z^{-1} + \cdots + q_eZ^{-e} \quad - \quad - \quad - \quad 68$$

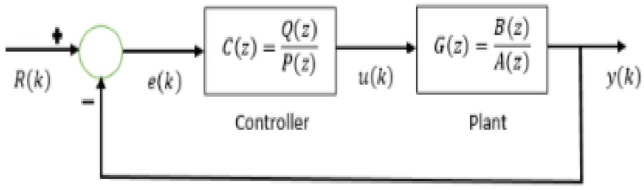

Figure 2: Block diagram for the Design of PI Controller

The CLTF to the PI controller using block diagram reduction method of figure 2 is obtained as

$$G_{PI} = \frac{QB}{PA+QB} \quad - \quad - \quad - \quad - \quad - \quad 69$$

$$B = b_1z^{-1} \quad - \quad - \quad - \quad - \quad - \quad 70$$

$$A = 1 + a_1z^{-1} \quad - \quad - \quad - \quad - \quad - \quad 71$$

$$P = 1 + p_1z^{-1} \quad - \quad - \quad - \quad - \quad - \quad 72$$

$$Q = q_0 + q_1z^{-1} \quad - \quad - \quad - \quad - \quad - \quad 73$$

To obtain the characteristic equation, equations 70, 71, 72, and 73 are substituted into the denominator of equation 69 resulting in;

$$(1 - z^{-1})(1 + a_1z^{-1}) + b_1z^{-1}(q_0 + q_1z^{-1}) = D(z) - \quad 74$$

$$1 + a_1z^{-1} - z^{-1} - a_1z^{-2} + b_1q_0z^{-1} + b_1q_1z^{-2} = D(z) \quad - \quad 75$$

$$1 + (a_1 - 1 + b_1q_0)z^{-1} + (b_1q_1 - a_1)z^{-2} = D(z) \quad - \quad 76$$

The dynamics of the closed loop system are governed by their denominators. We wish that the characteristic equation, which is the denominator of equation 69, equals the desired closed loop characteristic polynomial D(z).

$$D(z) = 1 + d_1z^{-1} + d_2z^{-2} + \cdots + d_nz^{-n} \quad - \quad - \quad 77$$

where z^0 : 1 = 1 - - - - - 78

$$z^{-1}: a_1 - 1 + b_1q_0 = d_1 \quad - \quad - \quad - \quad - \quad 79$$

$$z^{-2}: -a_1 + b_1q_1 = d_2 \quad - \quad - \quad - \quad - \quad 80$$

III. Control Design Simulation Results

The simulation of PI controller in figure 3 and LQ-PIP controller in figure 4 shows the response of the simulated nonlineaar model, the computation of the control actions, and their controller gains. The chosen controller gains are able to stabilize the process and guarantee satisfactory closed loop performance for all valves positioning corresponding to a minimum phase system i.e. 1 < γ 1 + γ 2 < 2. As Illustrated in figures 3 and 4, the water levels in tanks 1 and 2 were well regulated on the set-point trajectories, although very little overshoots and undershoots as shown with the little circles in both plots are observed. This corresponds to interactions from the upper tanks. The interactions are however greatly minimised in the optimal LQ-PIP controllers (figure 4) relative to that obtained from the classical PI controller (figure 3). Hence disturbance rejection in the water level control is more pronounced by the use of multi-loop LQ-PIP controllers. Furthermore, from the performance indices given in figures 5 and 6, (IAE) valves are smaller for LQ-PIP with respect to the consisted valves positions while the control eff ect (IAC) varies. This is not surprising as good control performance (small IAE) comes with a high demand on the controller eff ort.

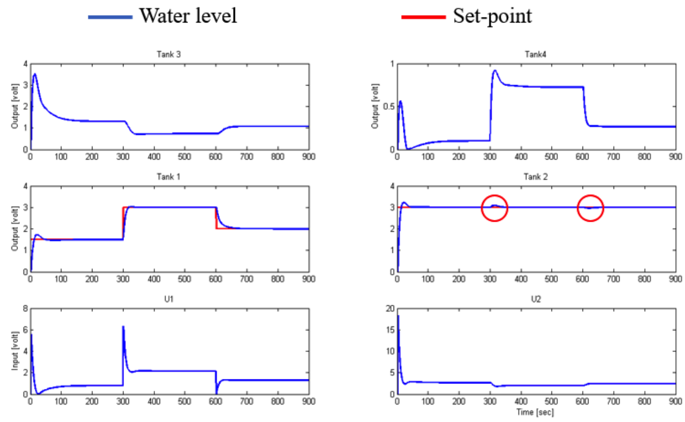

Figure 3: Simulation response of the QTP (minimum phase characteristics) with a Decentralised SISO PI Controller

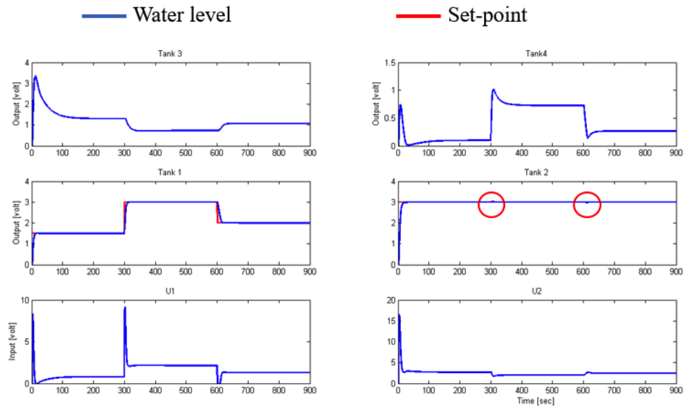

Figure 4: Simulation response of the QTP (minimum phase characteristics) with a Decentralised SISO LQ-PIP Controller

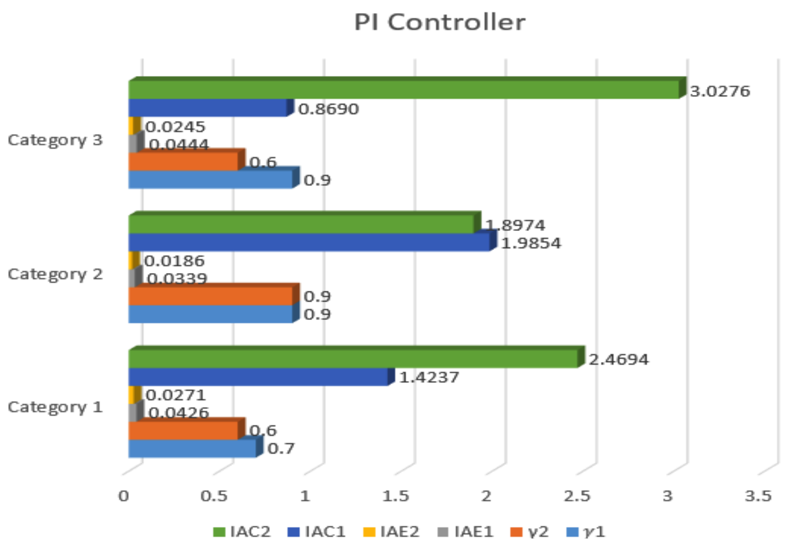

Figure 5: Representation of IAE and IAC for Decentralised SISO Controller (PI)

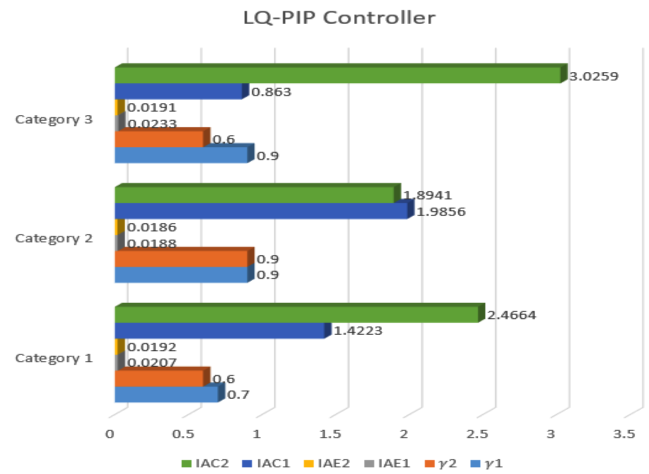

Figure 6: Representation of IAE and IAC for Decentralised SISO Controller (LQ-PIP)

IV. Conclusion

To evaluate the efficiency of the two decentralised SISO controllers, the LQ-PIP controller is compared with the PI controller. From simulation results LQ-PIP controller exhibits considerably superior control p. The LQ-PIP trajectory responses followed the set-point tracking, also being able to minimise the overshoots relatively. QTP depends solely on the three-port valve parameter γ_1 and γ_2 position. Based on the control performance measures, LQ-PIP was observed to have smaller IAE than PI controller. Although their IAE values where almost similar with little changes to some decimal. and PI controller exhibited a smaller control eff ort as compared to the LQ-PIP controller.

References

[1] Akesson, M., Gustafsson, E., Johansson, K. H., Control design for a helicopter lab process, in ‘IFAC World Congress, San Francisco, CA,1996.
[2] Alamelumangai, V., Natarajan, S., Tuning methods of multi-loop controller for a multivariable process, Indian Journal of Science and Technology, 2010, 3(2):131–138.
[3] Astr¨om, K., H¨agglund, T. Revisiting the ziegler–nichols step response method for PID control, Journal of process control,2004, 14(6): 635–650.
[4] Bristol, E., On a new measure of interaction for multivariable process control, IEEE transactions on automatic control, 1996, 133–134.
[5] Burns, R., Advanced control engineering, Butterworth-Heinemann, 2001.
[6] Falb, P. L., Wolovich, W. et al., Decoupling in the design and synthesis of multi-variable control systems, Automatic Control, IEEE Transactions 1967, 12(6):651–659.
[7] H¨agglund, T., A predictive PI controller for processes with long dead times, Control Systems, IEEE,1992, 12(1):57–60.
[8] H¨agglund, T., An industrial dead-time compensating pi controller, Control Engineering Practice, 1996, 4(6):749–756.
[9] Hovd, M., Skogestad, S., Simple frequency-dependent tools for control system analysis, structure selection and design, Automatica, 1992, 28(5):989–996.
[10] Hovd, M., Skogestad, S., Sequential design of decentralized controllers, Automatica, 1994, 30(10):1601–1607.
[11] Johansson, K. H., Relay feedback and multivariable control, PhD thesis, Lund University, 1997.
[12] Johansson, K. H. (2000), The quadruple-tank process: A multivariable laboratory process with an adjustable zero, Control Systems Technology, IEEE Transactions, 2000, 8(3):456–465.
[13] Johansson, K. H., Horch, A., Wijk, O., Hansson, A., Teaching multivariable control using the quadruple-tank process, in Decision and Control, 1999. Proceedings of the 38th IEEE Conference on, 1999, 1:807–812.
[14] keith B, T. L., Advanced informatic and control, 2011.
[14] Kheir, N., Astr¨om, K. J., Auslander, D., Cheok, K. C., Franklin, G. F., Masten, M., Rabins, M., Control systems engineering education, Automatica, 1996, 32(2):147166.
[15] Lees, M., Young, P., Chotai, A., Tych, W., A non-minimal state variable feedback approach to multivariable control of glasshouse climate, Transactions of the Institute of Measurement and Control, 1995, 17(4): 200–211.
[16] McAvoy, T. J., Interaction analysis: Principles and applications, 1983.
[17] Nise, N. S., Control Systems Engineering, John Wiley & Sons, 2007.
[18] Nunes, J., Modeling and control of the quadruple-tank process, PhD thesis, Master thesis, Dept. Automat. Contr., Lund Inst. Technol., Lund, Sweden, 1997.
[19] Pappas, T., Laub, A. J., Sandell Jr, N. R., On the numerical solution of the discrete-time algebraic riccati equation, Automatic Control, IEEE Transactions, 1980, 25(4):631–641.
[20] Gilbert, E. G., The decoupling of multivariable systems by state feedback, SIAM Journal on Control, 1969,7(1):50–63.
[21] Isa. M. Jr, B. S., The synthesis of linear multivariable systems by state-variable feedback’, Automatic Control, IEEE Transactions, 1964, 9(4):405–411.

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