Applying Neural Network Learning Algorithms to Optimize Mobile Stations Positioning Accuracy

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Abstract:
The artificial neural network (ANN) uses the method of counterfeiting biological neural connection computing ability to find the optimal solution. In this paper, we apply neural network learning algorithms to approximate the value of weight geometric dilution of precision (WGDOP) to optimize wireless positioning accuracy in mobile communication system. By selecting the base-station (BS) set with the minimum of WGDOP to locate the mobile station position, it can reduce the effects of geometric distribution and improve positioning accuracy. In this paper, we choose the best combination of four base-stations for positioning the mobile station (MS), and each BS can measure signal arrival time. The architecture we proposed in this research can be applied to the global positioning system, wireless sensing network and mobile communication system.

Key Word: Artificial Neural Network (ANN); Weight Geometric Dilution of Precision (WGDOP); Mobile Station Positioning; Mobile communication system.

I. Introduction

In recent years, mobile positioning is an important issue in wireless communication. There are a lot of methods to locate the mobile station such as time of arrival (TOA), angle of arrival (AOA), time difference of arrival (TDOA), and signal strength (SS). TOA locate the MS position by measuring the signal arrival time, the signal be sent from the base-station (BS) to the MS, and the signal include the time information. When MS received the signal from the BS, it can compute the transmission time between the BS and the MS [1]. Using the transmission time gets the measuring distant. In two-dimension environments, we need a least three BS to estimate the location of the MS [2].

There are many ways to locate the MS position in wireless communication system, it can be generally divided into two major categories – handset-based methods and network-based methods. The advantages of handset-based methods are higher global converge rate and lower position error than the network-based methods. But the handset-based methods are more expensive because of device requirements and overall system technology integration. The global positioning system (GPS) [3], [4] is a handset-based method and it can provide the user’s location, relative speed and the time information. When equipped the GPS receiver, the handset-based methods need to modify the mobile device’s function to estimate its position.

In recent years, wireless communication technology has become an indispensable part of people's daily life because of the popularity of internet mobile devices. The mobile device can receive and send wireless signals. If there is a breakthrough in the technology of signal measurement and tracking, the user's location can be determined. With the improvement of wireless communication technology, the application level has also continued to increase. Wireless communication positioning technology and location-based services (LBS) have also attracted much attention. Many telecom service providers and academic units are studying wireless communication positioning and measurement.

There is still a lot of room for the future development of wireless positioning technology. Previous positioning systems let people linked think of outdoor location, but because of the popularity of mobile phones and other personal mobile devices, and related applications Internet of Things (IoT) is also increasing, making people more convenient. Indoor positioning technology is quite broad including: Wi-Fi [5], Bluetooth-Low-Energy (BLE) [6], Ultra-wideband (UWB) [7], ultrasonic positioning [8], etc.

The global positioning system (GPS) [9] is widely used in outdoor positioning technology. It is possible to integrate indoor and outdoor positioning technologies and provide good accuracy for the development of current research. Geometric dilution of precision (GDOP) is widely used for increasing the positioning accuracy. GDOP was first proposed to find the best satellite set with the higher locating accuracy in
the GPS.

In this paper, we applied the weight GDOP to both handset-based methods and network-based methods. The concept was originally applied for selecting the best satellite set, having the lowest positioning error in the three-dimensional environment. When choose the set which has the smallest value of the WGDOP, we can have higher position accuracy and reduce the influence of the bad geometric distribution. In cellular communication system, selecting appropriate BS set to estimate the MS location can greatly improve the positioning accuracy.

Since the MS will always communicate with the service BS, we can always select the service BS from the seven BSs into the BS set, and combine the others three BSs to form the different BS set, and the number of the BS set will reduce from 35 to 20. By estimating the value of all the different BS set, we can choose the best BS set having the smallest value of the WGDOP. The proposed method can be used for WGDOP approximation, GPS, WSN, and wireless communication system.

II. Weight Geometric Dilution of Precision

The Geometric dilation of precision is the criterion for selecting the best BS set and has higher positioning accuracy with better geometric distribution. To improve the positioning accuracy, we should minimize the value of GDOP [10].

In the three-dimensional environment, the distance between the satellite and the user can be expressed as

\[ r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 + c \cdot t_i + v_{ni}} \]  

where \((x, y, z)\) and \((X_i, Y_i, Z_i)\) are the user’s location and the position of satellite, \(c\) is the speed of light, \(t_i\) is the time offset, and \(v_{ni}\) is pseudo-range measurements noise.

Equation (1) can be represented with Taylor’s series expansion near the user’s position \((\hat{x}, \hat{y}, \hat{z})\). Defining \(\hat{r}\) as \(r\) at the position \((\hat{x}, \hat{y}, \hat{z})\) we can have

\[ \Delta r = r_i - \hat{r}_i = e_{1i} \delta_x + e_{2i} \delta_y + e_{3i} \delta_z + c \cdot t_i + v_{ni}, \]  

where \((\delta_x, \delta_y, \delta_z)\) are coordinate offsets of \((x, y, z)\):

\[ e_{1i} = \frac{\hat{x} - x_i}{\hat{r}_i}, \quad e_{2i} = \frac{\hat{y} - y_i}{\hat{r}_i}, \quad e_{3i} = \frac{\hat{z} - z_i}{\hat{r}_i}, \quad \hat{r}_i = \sqrt{\hat{x}^2 + \hat{y}^2 + \hat{z}^2}, \]  

\[ (e_{1i}, e_{2i}, e_{3i}), i = 1, 2, \ldots, n. \]  

Equation (3) presents the vector of line of sight from user \(\#i\) to the satellite. The linearized equations can be expressed as

\[ z = \begin{bmatrix} r_1 - \hat{r}_1 \\ r_2 - \hat{r}_2 \\ \vdots \\ r_n - \hat{r}_n \end{bmatrix}, \quad \delta = \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}, \quad v = \begin{bmatrix} v_{r1} \\ v_{r2} \\ \vdots \\ v_{rn} \end{bmatrix}, \quad H = \begin{bmatrix} e_{11} & e_{12} & e_{13} & 1 \\ e_{21} & e_{22} & e_{23} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ e_{n1} & e_{n2} & e_{n3} & 1 \end{bmatrix} \]  

\[ z = H\delta + v \]  

(4)

Using least-square (LS) algorithm, Eq. (4) can be solved to obtain the vector

\[ \delta = (H^T H)^{-1} H^T z \]  

Assuming that the pseudo-range errors are independent and have the same variances. Then, we can define the value of GDOP as

\[ GDOP = \sqrt{\text{tr}(H^T H)^{-1}} \]  

(6)

It can surely that can select the best satellite set when using inverse matrices calculate the value of GDOP, but the calculation is too complicated.

Assume that each measurement error has a different variations, especially a combination of different systems. The covariance matrix has some uncertainty in the measurement of virtual distances and has the following form:
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\[
E(v^T) = \begin{bmatrix}
\sigma_1^2 & 0 & 0 & 0 & 0 \\
0 & \sigma_2^2 & 0 & 0 & 0 \\
0 & 0 & \sigma_3^2 & 0 & 0 \\
0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \sigma_n^2
\end{bmatrix}
\]

\( W \) is a diagonal matrix, which is defined as a weight matrix:

\[
W = \begin{bmatrix}
1/\sigma_1^2 & 0 & 0 & 0 & 0 \\
0 & 1/\sigma_2^2 & 0 & 0 & 0 \\
0 & 0 & 1/\sigma_3^2 & 0 & 0 \\
0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & 1/\sigma_n^2
\end{bmatrix}
\]

\[
\hat{\delta} = (H^T WH)^{-1} H^T W z
\]

Considering that each measurement unit has different variances, the weighted least squares method can provide higher positioning accuracy than the least squares method. Not only the geometrical distribution is considered, but also the prior information of the error model is considered. The weighted geometric precision factor is used instead of the geometric precision factor as the basis for the selection of the measurement unit to effectively improve the positioning performance. The weighted geometric precision factor can be obtained from the following equation:

\[
WGDO\text{P} = \sqrt{\text{tr}(H^T WH)^{-1}} = \sqrt{\lambda_1^{-1} + \lambda_2^{-1} + \cdots + \lambda_n^{-1}}
\]

where \( \lambda_i \) is an eigenvalue of the matrix \( H^T WH \), \( i = 1, 2, \ldots, n \).

The weighted geometric of precision for each subset is calculated, and the most suitable measurement unit is selected using the minimum weighted geometric precision factor.

III. Neural Network

The artificial neural network (ANN) has the ability to learn, and it is an imitation of biological neural network information processing system, parallel decentralized processing based on the calculation model. Neural network mainly be formed by a large number of artificial neurons (neuron), also known as node (node) or processing unit (processing element, PE). By using several real value of the WGDOP and the elements in the matrix \( H \) as the training data for the neural network, we can significantly reduce the approximation error.

In general, the architecture of a neural network includes an input layer, an output layer, and a single or multiple hidden layers located between the input layer and the output layer. Input layers are used to show the network input variables, and no computing ability. Hidden layers are used to represent the interaction between input processing. Output layers are used to display the network output variable; the number of neurons also depends on the complexity of the problem. The neural network architecture was shown in the Fig. 1.
A number of hidden neurons can be decided by the following rules:

1. \(0.5(p+q)\);
2. \(p\);
3. \((2p+1)\);
4. \((3p+1)\).

Here, \(p\) is the number of the input node, and \(q\) is the number of the output node.

As we know, backpropagation neural network can learn and implement linear and nonlinear functions [13]. The process of backpropagation neural network can be considered as one of the gradient drops methods that minimize certain measures. Mean-square value of the difference between the actual output vector of network and the desired output vector. Define an error function \(F\):

\[
F = (1/2)\sum_k (T_k - O_k)^2 ,
\]
where \(T_k\) is the calculated output vector of the network while \(O_k\) is the desired output vector. Then, the gradient decent algorithm is employed to adapt the weights as follows:

\[
\Delta w_{ij}(t) = -\varepsilon \frac{\partial F}{\partial w_{ij}}(t)
\]
where \(\varepsilon\) is a predetermined learning rate, and \(w_{ij}\) denotes the weight connecting neuron \(i\) to neuron \(j\).

Theresilient propagation (Rprop) algorithm [11] provides faster convergence, usually more can escape from local-minimum. The Rprop algorithm is an efficient learning scheme in neural network that performs direct adaptation of the weight step based on local gradient information. It can overcome the disadvantages of pure gradient-descent for multilayer feedforward networks.

The main idea of Rprop algorithm is to reduce the potential false effects of partial derivatives on weight updates by retaining only the sign of the derivative as an indication of direction, where the error function will change by weight update. We introduce an individual update-value \(\Delta_i(t)\) for each weight, which only determines the size of the weight update. According to the following learning rules, this adaptive update value evolves during the learning process based on its local sight on the error function \(F\):

\[
\Delta_i(t) = \begin{cases} 
\eta^+ \cdot \Delta_i(t - 1), & \text{if } \frac{\partial F(t)}{\partial w_{ij}} > 0 \\
\eta^- \cdot \Delta_i(t - 1), & \text{if } \frac{\partial F(t)}{\partial w_{ij}} < 0 \\
\Delta_i(t - 1), & \text{else} 
\end{cases}
\]
where \(0 < \eta^- < 1 < \eta^+\). We can simply describe the adaptation rules as follows:

Each time the partial derivative error function \(F\) changes its sign with respect to the corresponding weight \(w_{ij}\), the update value \(\Delta_i\) is reduced by the factor \(\eta^+\). If the derivative retains its flag, the update value is slightly increased to speed up convergence in shallow regions. The weight update itself follows a very simple rule: if the derivative is positive (increasing the error), the weight is reduced by its updated value, and if the derivative is negative, the updated value is added to the weight:

\[
\Delta w_{ij}(t) = \begin{cases} 
-\Delta_i(t), & \text{if } \frac{\partial F(t)}{\partial w_{ij}} > 0 \\
+\Delta_i(t), & \text{if } \frac{\partial F(t)}{\partial w_{ij}} < 0 \\
0, & \text{else} 
\end{cases}
\]

IV. Network Architectures for WGDOP Approximation

The method of calculating the geometric dilution of precision by using the inverse matrix is easy to cause the computational burden. The articles [12] proposed the use of traditional back-propagation neural network learning measurement matrix and eigenvalue (eigenvalue) reciprocal relationship, in three-dimensional environment to estimate the geometric dilution of precision. In this paper, the method of approximating the geometric accuracy factor of the traditional backpropagation algorithm is extended to the approximate geometric dilution of precision by using the gradient descent adaptive learning rate training algorithm.

The following describes the six "Input/Output" mapping relationships in a two-dimensional environment. These architectures are based on a three-tier network architecture and can be expressed as "number of input layers - number of hidden layers - number of output layers". The relationship between the six
mapping mechanisms, as shown in Fig. 2.

![Diagram showing input/output mapping relationships for WGDOP approximation.](image)

Previously, Iwo and Chin [13] proposed the learning relationships between input and output in the traditional neural network using the measurement matrix and the eigenvalue to approximate the value of WGDOP without using inverse matrix. We used the mapping relationships between the input and output, and extend to the neural network. Furthermore, we proposed two mapping relationships for the simulation to compare the performance.

**Type 1:** three inputs are mapped to three outputs

\[ f_1(\lambda) = \lambda_1 + \lambda_2 + \lambda_3 = \text{trace}(H^TWH) \]
\[ f_2(\lambda) = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = \text{trace}((H^TWH)^2) \]
\[ f_3(\lambda) = \lambda_1 \lambda_2 \lambda_3 = \text{det}(H^TWH) \]

Input: \((f_1, f_2, f_3)^T\)
Output: \((\lambda_1^{-1}, \lambda_2^{-1}, \lambda_3^{-1})^T\)

**Type 2:** three inputs are mapped to one output

Input: \((f_1, f_2, f_3)^T\)
Output: WGDOP

**Type 3:** six inputs are mapped to three outputs

\[
H^TH = \begin{bmatrix}
B_{11} & B_{12} & B_{13} \\
B_{21} & B_{22} & B_{23} \\
\text{sym} & B_{33}
\end{bmatrix}
\]

Input: \((B_{11}, B_{12}, B_{13}, B_{22}, B_{23}, B_{33})^T\)
Output: \((\lambda_1^{-1}, \lambda_2^{-1}, \lambda_3^{-1})^T\)

**Type 4:** six inputs are mapped to one output

Input: \((B_{11}, B_{12}, B_{13}, B_{22}, B_{23}, B_{33})^T\)
Output: WGDOP

**Type 5:** twelve inputs are mapped to three outputs

Input:
\((e_{11}, e_{12}, e_{21}, e_{22}, e_{31}, e_{32}, e_{41}, e_{42}, k_1, k_2, k_3, k_4)^T\)
Output: \((\lambda_1^{-1}, \lambda_2^{-1}, \lambda_3^{-1})^T\)

**Type 6:** twelve inputs are mapped to one output

Input:
\((e_{11}, e_{12}, e_{21}, e_{22}, e_{31}, e_{32}, e_{41}, e_{42}, k_1, k_2, k_3, k_4)^T\)
Output: WGDOP

The proposed BS selection criterion with best GDOP can be modified for application in cellular communication systems. Since the MS will always communicate with the service BS, we can always select the service BS from the seven BSs into the BS set, and combine the others three BSs to form the different BS set, and the number of the BS set will reduce from C(7,4)=35 to C(6,3)=20. By estimating the value of all the different BS set, we can choose the best BS set having the smallest value of the GDOP. The proposed method can be used for GDOP approximation, GPS, WSN, and wireless communication system. To further simplify the process, the proposed BS selection criterion first chooses the serve BS and selects three measurements from the other six BSs to form the BSs set. In this way, GDOP is computed for 20 possible BSs sets and the one with the best and smallest value of GDOP is selected.

V. Simulation Results

Selecting the appropriate base-station to estimate the position of the mobile station can make the positioning error to a minimum. We choose the cellular wireless communication system as the simulation environment, which the size of each cell is the same, and the base-station providing the service is in the middle, and the geometric distribution of the base-station is as shown in Fig. 3.

![Fig. 3. Seven cells mobile system.](image)

Each cell’s radius is assumed 5 km, and the mobile station is uniformly distributed in the center of the cell. We can approximate the value of WGDOP for all BS sets, and select the BS combination with minimum WGDOP. By using the BS set with the minimum of WGDOP to estimate the mobile station position, we can reduce positioning error, which shows in Figure 4.

![Fig. 4. Using four base-stations with minimum of WGDOP for positioning.](image)
The error of the wireless communication positioning system mainly effects by the influence of the non-line-of-sight propagation effect. In this paper, and the uniform distributed noise model is used to simulate the two-dimensional environment. NLOS errors from all the BSs are different and assumed to be uniformly distributed over \((0, U_i)\), for \(i = 1, 2, \ldots, 7\), where \(U_i\) is an upper bound. The specific variables are chosen as follows: \(U_1 = 200\) m, \(U_2 = 400\) m, \(U_3 = 350\) m, \(U_4 = 700\) m, \(U_5 = 500\) m, and \(U_7 = 350\) m. The WGDOP residual is defined as the difference between the true WGDOP value and the estimated WGDOP value. The WGDOP residual can be used to evaluate the accuracy of the estimation.

Figure 5 shows the relationship between WGDOP residua and number of hidden neurons in two-dimensional environment. As the number of hidden neurons increases, the average difference of the WGDOP residual will also decreases. By the simulation result, the proposed mapping relationships for neural network, type5 and type6, need fewer number of hidden neurons than the other types to approximate the GDOP value and have better performance. This also means that the proposed method will have faster calculation speed and convergence time.

Table I shows the value of WGDOP residual with different number of hidden neurons. The average residua of the WGDOP decreases as the number of hidden layer neurons increases. When the number of hidden layer neurons is equal, it can provide a more accurate estimation result, which represents the number of input neurons. In other words, when the number of hidden layer neurons exceeds, the improvement in estimation accuracy is very limited. Based on the above simulation results, it can be known that neural network requires only \(2^p+1\) nodes in hidden layer neurons to accurately estimate the WGDOP.

<table>
<thead>
<tr>
<th>Neurons</th>
<th>Type 2</th>
<th>Type 4</th>
<th>Type 6</th>
<th>Type 1</th>
<th>Type 3</th>
<th>Type 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.5 \cdot (p+q))</td>
<td>0.3878</td>
<td>0.1503</td>
<td>0.0650</td>
<td>0.4151</td>
<td>0.1581</td>
<td>0.0732</td>
</tr>
<tr>
<td>(p)</td>
<td>0.2812</td>
<td>0.1028</td>
<td>0.0553</td>
<td>0.4151</td>
<td>0.1480</td>
<td>0.0616</td>
</tr>
<tr>
<td>(2 \cdot p+1)</td>
<td>0.1580</td>
<td>0.0745</td>
<td>0.0388</td>
<td>0.3587</td>
<td>0.0984</td>
<td>0.0458</td>
</tr>
<tr>
<td>(3 \cdot p+1)</td>
<td>0.1537</td>
<td>0.0538</td>
<td>0.0383</td>
<td>0.3128</td>
<td>0.0817</td>
<td>0.0408</td>
</tr>
</tbody>
</table>

Table II shows the value of WGDOP approximation with different base-stations combination. We apply neural network to approximate the value of WGDOP in mobile communication system and using type6 as input/output neural network mapping relationship with \(2^p+1\) hidden neuron to estimate WGDOP with different base-stations combinations. By selecting the BSs set with the minimum of WGDOP to locate the mobile station position, it can reduce the effects of geometric distribution and improve positioning accuracy.

<table>
<thead>
<tr>
<th>BS set</th>
<th>BS1</th>
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</table>
By selecting the BS set with of WGDOP for mobile positioning, we can improve location accuracy. Measuring the signal propagation time transmitted by the base-station to the mobile station, and multiplying by the propagation speed (generally calculated at the speed of light, about 3x10^8 m/sec), the distance from the mobile station to the base-station can be obtained. With this distance as the radius, the base-station can form a circle with the center of the circle. We select four base-stations combinations with minimum of WGDOP for the positioning, then generate four measuring circles, and estimate the position of the mobile station by the average of the circular intersection point. The location error using four base-stations with minimum of WGDOP for mobile positioning shows in Figure 6.

<table>
<thead>
<tr>
<th>BS set</th>
<th>BS1</th>
<th>BS2</th>
<th>BS3</th>
<th>BS4</th>
<th>BS5</th>
<th>BS6</th>
<th>BS7</th>
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<tr>
<td>BS set</td>
<td>BS1</td>
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<td>BS4</td>
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<tr>
<td>BS set</td>
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<tr>
<td>BS set</td>
<td>BS1</td>
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Fig. 6. Location error using four base-stations with minimum of WGDOP for mobile positioning.

VI. Conclusion

We propose using neural network architecture to approximate the value of WGDOP in a two-dimensional environment. The simulation results show that the WGDOP can be approximate by the proposed method, and it can reduce the number of node and the computational complexity. This architecture has no limit on the number of selected measurement devices, can be applied to global positioning systems, can also be used in wireless sensor networks, and cellular communication systems.

In this research, the approximation value of WGDOP by neural network can improve the localization efficiency and reduce the influence of geometric distribution. The proposed two type of input/output mapping used the element in the matrix H as the input data of the neural network. As the simulation result, we can get a lower approximation error by the proposed mapping relationship. It also shows that the proposed method has lower WGDOP residual and use fewer epochs than the other types to get a better performance. For cellular
wireless communication system, it will reduce the computational complexity and the location error by selecting the four base-stations with minimum of WGDOP to estimate the position of mobile station.

References
