

An Application Jeevan – Kushalaiah Method to Find Lagrangian Multiplier in Economic Load Dispatch Including Losses and Lossless Transmission Line

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Abstract: This paper discusses the non iterative method to calculate the Lagrangian Multiplier (λ). The economic load dispatch (ELD) is the on-line economic load dispatch (ELD) wherein it is required to distribute the load among the generating units. The system such manner as to minimize the total cost of supplying the minute -to- minute requirements of systems. Jeevan – Kushalaiah method or J-K method is a method to calculate maximum number of combination between n-elements starting from minimum order to maximum order. The J-K method modifies iterative flow charts to single line flow charts. This paper deals with Newton-Raphson method.

Index terms: Jeevan – Kushalaiah method, Incremental Fuel Rate, Incremental Transmission Cost

I. Introduction

In load flow studies, For a particular load demand the generation at all generator buses are fixed except at the generator bus known as slack bus or reference bus or swing bus. In case of Economic Load Dispatch (ELD) the generations are not fixed but they are allowed to take values again the within certain limits so as to meet a particular load demand with minimum fuel consumption.

The cost of generation will depend upon the system constraint for a particular load demand. This means the cost of the generation is not fixed for a particular load demand but depends upon the operating constrains of the generator. The various constraints in economic load dispatch are

Equality Constrains:

$$P_p - jQ_p = V_p^* I_p = V_p^* \sum_{q=1}^n Y_{pq} V_q \dots\dots \quad (i)$$

Where p = intial bus and q = terminating bus

$$V_p = e_p + jf_p \text{ and } Y_{pq} = G_{pq} - jB_{pq}$$

$$P_p = \sum_{q=1}^n \{e_p(e_q G_{pq} + f_q B_{pq}) + f_p(f_q G_{pq} - e_q B_{pq})\} \dots\dots \quad (ii)$$

$$Q_p = \sum_{q=1}^n \{f_p(e_q G_{pq} + f_q B_{pq}) + e_p(f_q G_{pq} - e_q B_{pq})\} \dots\dots \quad (iii)$$

The limits of p and q: $1 \leq p \leq n$ and $1 \leq q \leq n$

Inequality Constrains:

a) Generator Constraints:

$$P_p^2 + Q_p^2 \leq C_p^2, \text{ where } C_p \text{ is prespecified value.}$$

$$P_{pmin} \leq P_p \leq P_{pmax} \text{ and } Q_{pmin} \leq Q_p \leq Q_{pmax}$$

b) Voltage Constraints:

$$|V_{pmin}| \leq |V_p| \leq |V_{pmax}|$$

$$\delta_{pmin} \leq \delta \leq \delta_{pmax}$$

Network Security Constraints:

$$\text{Incremental Fuel rate} = \frac{(\Delta \text{ input})}{(\Delta \text{ output})} = \frac{d(\text{input})}{d(\text{output})} = \frac{dF}{dP} \text{ in Rs/Btu}$$

$$\text{Incremental Efficiency} = \text{reciprocal of Incremental fuel Rate} = \frac{dP}{dF}$$

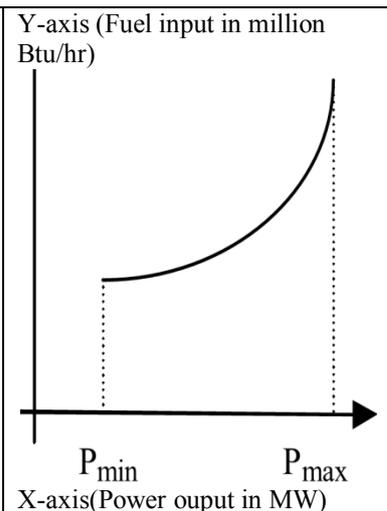


Fig.1: Incremental fuel rate vs. power output

II. Jeevan – Kushalaiah Method

Jeevan – Kushalaiah method is a method to find maximum number of possible combination between n-elements.

Let n-elements $a_1, a_2, a_3, a_4, \dots, a_{n-1}, a_n$

The maximum number of combination between elements are

$\{(1), (a_1, a_2, a_3, a_4, \dots, a_{n-1}, a_n), (a_1 a_2, a_1 a_3, a_1 a_4, \dots, a_1 a_{n-1}, a_1 a_n, a_2 a_3, a_2 a_4, \dots, a_2 a_{n-1}, a_2 a_n, \dots, a_3 a_4, a_3 a_5, \dots, a_3 a_{n-1}, a_3 a_n, \dots, a_{n-1} a_n), (a_1 a_2 a_3, a_1 a_2 a_4, \dots, a_1 a_2 a_{n-1}, a_1 a_2 a_n, \dots, a_{n-2} a_{n-1} a_n), \dots, (\dots), \dots, (a_1 a_2 a_3 \dots a_{n-1} a_n)\}$

..... (iv)

$= \{(\Theta_0), (\Theta_1), (\Theta_2), \dots, (\Theta_{p-1}), (\Theta_p)\}$

Here $\Theta_0 = 1$, $\Theta_1 = +, -, *, /$ of all elements

(Θ_p) = Addition or Subtraction or Multiplication or Division or a square Matrix.

Example - 1:

$(\Theta_p) = (a, b)$

- Addition = $a + b$
- Subtraction = $a - b$
- Multiplication = $a * b$
- Division = a / b
- Square Matrix = $\begin{bmatrix} a_{11} & 12 \\ 21 & b_{22} \end{bmatrix}$ (a and b are diagonal positions only.)

Commonly Multiplication and Addition is performed

$$\Theta_p = (n - (p-1))_{p-1} + \dots \quad (v)$$

total number of combinations of n-elements

$$= \Theta_0 + \Theta_1 + \sum_{p=2}^n \Theta_p \dots \quad (vi)$$

The value of $p \geq 2$

where $+$ is summator like factorial operation instead of multiplication addition is performed.

Example-2: $(5)_2 + = 5 + + = 5 + + 4 + + 3 + + 2 + + 1 + = 5 + 4 + 3 + 2 + 1 + 4 + 3 + 2 + 1 + 3 + 2 + 1 + 2 + 1 + 1 = 35$

III. Calculation of λ of ELD without Losses

The Load demand = P_D

The generation of n^{th} unit = P_n

Total fuel input to the system = F_T

The Fuel input tot n^{th} system = F_n

The system with without losses, $P_L = 0$

$$P_D = \sum_{k=1}^n P_k \dots \quad (vii)$$

The auxiliary function

$$F = F_T + \lambda (P_D - \sum_{k=1}^n P_k)$$

Where λ is Lagrangian multiplier

Differentiating F with respect to P_n and equating to zero

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \frac{dF_n}{dP_n} = \lambda$$

Where $\frac{dF_n}{dP_n}$ = incremental production cost of nth plant in

Rs. /MWhr

For a plant over a limited range

$$\frac{dF_n}{dP_n} = F_{nn} P_n + f_n = \lambda \dots \quad (viii)$$

Where F_{nn} = slope of total incremental production curve and

f_n = intercept of incremental production cost curve.

From equation number (viii), we have

$$P_n = (\lambda_{\text{assum}} - f_n) / F_{nn}$$

$$P_1 = (\lambda_{\text{assum}} - f_1) / F_{11}, P_2 = (\lambda_{\text{assum}} - f_2) / F_{22} \dots$$

$$P_{n-1} = (\lambda_{\text{assum}} - f_{n-1}) / F_{n-1n-1}, P_n = (\lambda_{\text{assum}} - f_n) / F_{nn}$$

Total power to be generated P_T is

$$P_D = P_1 + P_2 + \dots + P_n$$

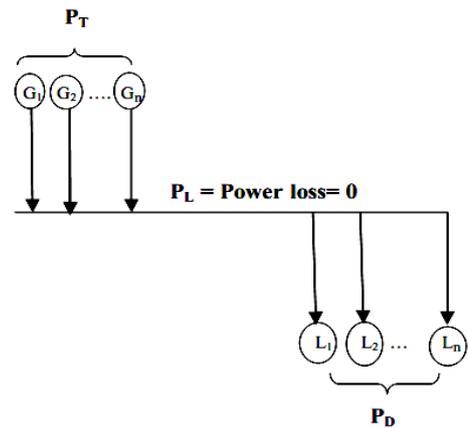


Fig .2: Generalized diagram of a power system without losses.

$$P_D = \{(\lambda_{\text{assum}} - f_1) / F_{11}\} + \{(\lambda_{\text{assum}} - f_2) / F_{22}\} + \dots + \{(\lambda_{\text{assum}} - f_n) / F_{nn}\} \dots \dots \dots \text{(ix)}$$

Rewriting equation-(ix) for direct calculation of λ

$$\lambda_{\text{assum}} = \{P_D (\sum_{j=1}^n F_{jj}) + \sum_{k=1}^n f_k(\phi_{n-k-1})\} / (\sum_{n=1}^n \phi n) \dots \dots \text{(x)}$$

Where ϕn = elements combination in Θ_{n-1}

IV. Calculation of λ of ELD without Losses

The system with without losses, P_L

$$P_D + P_L = \sum_{k=1}^n P_k \dots \dots \dots \text{(xi)}$$

The auxiliary function

$$F = F_T + \lambda (P_D + P_L - \sum_{k=1}^n P_k)$$

$$\frac{dF_n}{dP_n} + \lambda \left(\frac{\partial PL}{\partial P_n} \right) = \lambda \dots \dots \text{(xii)}$$

$\frac{\partial PL}{\partial P_n}$ = The Incremental Transmission Loss at plant – n

Loss formula approximately

$$P_L = \sum_j P_j \sum_k P_k B_{jk}$$

Assumptions

- The equivalent load current at any bus remains constant.
- The generator bus voltage magnitude and angles are constant.
- Power factor is constant

$$\frac{\partial PL}{\partial P_n} = 2 \sum_j P_j B_{jn} \dots \dots \text{(xiii)}$$

$$\frac{dF_n}{dP_n} = F_{nn} P_n + f_n \dots \dots \text{(xiv)}$$

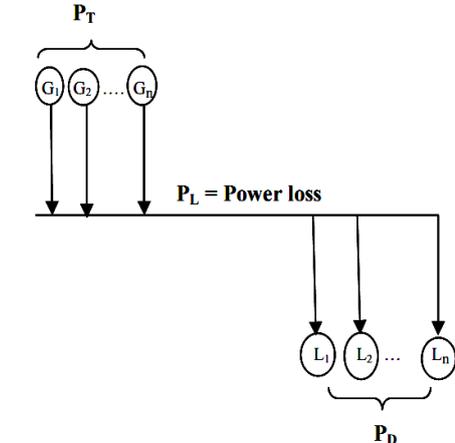


Fig . 3: Generalized diagram of a power system with losses.

Substituting equations (xiii)(xiv) in (xii) we get

$$F_{nn} P_n + f_n + 2\lambda \sum B_{mn} P_m = \lambda \dots \dots \text{(xv)}$$

Rewriting the equation - (xii)

$$\frac{dF_n}{dP_n} + \lambda \left(\frac{\partial PL}{\partial P_n} \right) = \lambda, \frac{\partial PL}{\partial P_n} = L_n$$

$$\frac{dF_n}{dP_n} + \lambda L_n = \lambda \implies \frac{dF_n}{dP_n} = \lambda (1 - L_n) \dots \dots \text{(xvi)}$$

From equation – (xiv) in (xvi)

$$F_{nn} P_n + f_n = \lambda (1 - L_n)$$

$$(F_{nn} P_n + f_n) / (1 - L_n) = \lambda$$

Generation of n-generator $P_n = \{\lambda (1 - L_n) - f_n\} / F_{nn}$

$$P_1 = \{\lambda (1 - L_1) - f_1\} / F_{11}, P_2 = \{\lambda (1 - L_2) - f_2\} / F_{22} \dots \dots \dots P_n = \{\lambda (1 - L_n) - f_n\} / F_{nn}$$

$$P_T = P_1 + P_2 + P_n = \{\lambda (1 - L_1) - f_1\} / F_{11} + \{\lambda (1 - L_2) - f_2\} / F_{22} + \dots \dots \dots + \{\lambda (1 - L_n) - f_n\} / F_{nn}$$

Rewriting the above equation

$$\lambda_{\text{assum}} = \{P_D (\sum_{j=1}^n F_{jj}) + \sum_{k=1}^n f_k(\phi_{n-k-1})\} / (\sum_{n=1}^n \phi n)(1-L_n) \dots \dots \text{(xvii)}$$

V. Algorithms and Flow Charts

V.a. Algorithm for ELD without Losses and ELD with losses

Step-I. Start and Read The Fuel input tot n^{th} system (F_n), F_{nn} = slope of total incremental production, The Load demand (P_D)

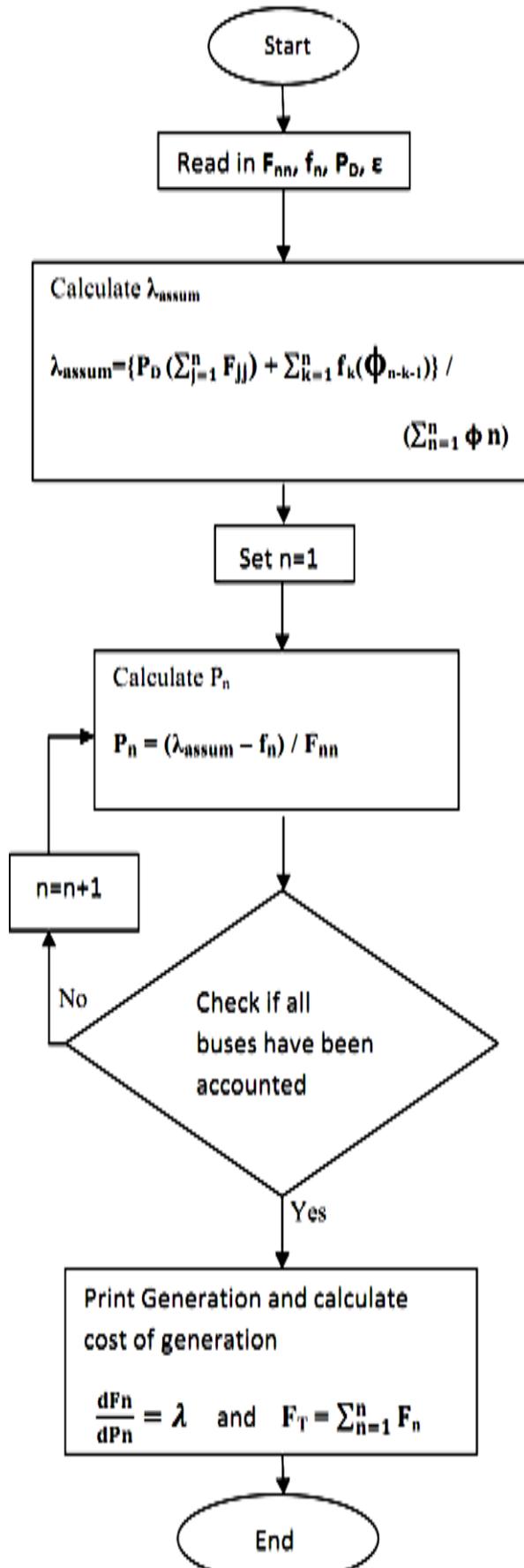
Step-II. Calculate loss. For ELD without losses $P_L = 0$

Step-III. Calculate λ_{assum} , and set bus number, $n = 1$, calculate P_n check all buses are completed or not, if yes go to next step, if not increment $n = n+1$ repeat step-III

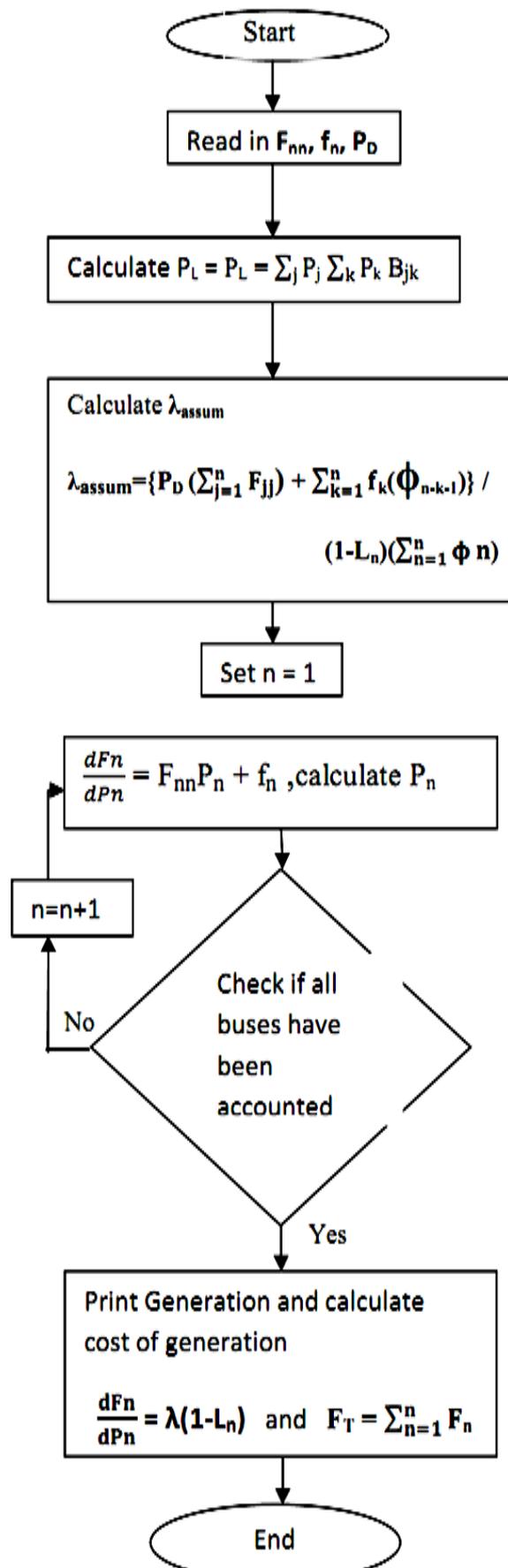
Step-IV Print Generation and calculate cost of generation.

Note: **Algorithm for ELD without Losses and ELD with losses changes according to their transmission line losses and operating conditions. In these algorithms are only for normal operation and not affected for atmospheric changes like raining and snow falling on the line.**

Flow chart for ELD without losses



Flow chart for ELD with losses



V. Conclusion

In this method, the Calculated values are within the

limits of voltage constraints and generator i.e.,

$$|V_{pmin}| \leq |V_p| \leq |V_{pmax}| \text{ and}$$

$$P_{pmin} \leq P_p \leq P_{pmax} \text{ and } Q_{pmin} \leq Q_p \leq Q_{pmax}$$

And there is iteration process to calculate Lagrangian multiplier λ , in the flow chart backward path is only for calculating required output at generating stations not to finding out the λ . This method applied for Thermal and Hydro power stations or connection of Hydro-Thermal Power station and Grid connections.

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