

Multifactor CAPM for Risk/Investment Analysis in International Business/Trade with FOREX Volatility Factor

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Abstract:

Background: In regions, where interest rate parity exists, Forex rate, prices of options, change rapidly, adjusting the economic differences. The risk/investment analysis in international business or trade need to be assessed taking into consideration of Forex volatility, of the country. There is no specific, established model exists, integrating better Forex volatility factor with other CAPM factors, to realize Multifactor CAPM for Risk/Return analysis of International Businesses and International Finance investment decisions.

Method Used: Theoretical evolution of better Forex volatility factor, over earlier Forex volatility models and 5 Factor Fama French Model and applying, Statistical Analysis, for various conclusions, to build integrated CAPM with Forex volatility factor.

Result: Review of Forex volatility models and evolution of Three Component Forex Volatility factor, integrated in to Multi Factor CAPM of Fama-French. It is also evidently or empirically concluded that earlier Forex volatility models ie First movements and the mimicking Forex volatility two factor model are insufficient, using available BM 25 statistical data.

Key words: CAPM, Multifactor CAPM, Forex Volatility, 5 factor Fama-French model,

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I. Introduction

In a globalized scenario in which concept of domestic only firm has vanished or fast eroding, investment and costs are always both in terms of local currency and foreign currency, in a globalized, international business. If, local domestic firms are not meeting needs competitively both economically and financially, the needs are met by international firms, via cross border. In this context, research need to focus on introducing, factors representing Forex volatility, in the Capital Asset Pricing Model and becomes essential. Adler and Dumas (1983) suggests that the covariance of stock returns with foreign exchange changes should be a priced factor when purchasing power parity is violated. Stock returns, in an efficient market are proportional to the performance of investment return in the respective businesses. So, it is logical to extend, this argument to covariance of investment returns with forex volatility. Motivated by this insight, studies on foreign exchange volatility risk typically adding factors corresponding to foreign exchange volatility to the standard asset-pricing models as an additional factor, becomes essential, in globalized business's Capital Asset Pricing Model. Every firm's model need to be competitive, integrating with global supply chain or value chain, including financial and currency chain, towards it's risk/return on investment decisions.

II. Background

There are two movements in Foreign Exchange. Foreign Exchange changes (first moments) are important for Capital Asset Pricing and it may be foreign exchange volatility (second moments) not foreign exchange changes (first moments) that matter for the cross-section of Forex Cash flow operations. The Recently, Menkhoff, Sarno, Schmeling, and Schrimpf (2011) (MSSS) find that Global Forex volatility is a key driver of risk premia in the cross-section of carry trade returns. The pricing power of volatility also applies to other cross-sections, such as a common FX momentum strategy, individual currencies' excess returns, domestic corporate bonds, equity momentum, as well as Forex option portfolios and international bond portfolios. Bartov, Bodnar, and Kaul (1996), found that the market risk (beta) of multinational firms increases with the increase in foreign exchange volatility when a longer-horizon (5 years) is focused upon.

The first standard model is the CAPM of Sharpe (1964) and Lintner (1965) (MKT):

$$r_{it} = \alpha_i + \beta_i MKT_t + \varepsilon_{it} \quad \longrightarrow z$$

where r_{it} is the excess return on asset i in period t , and MKT_t is the excess market return.

If foreign exchange volatility is a priced factor, it should reduce pricing errors of the CAPM. So, there is a requirement to improve CAPM with corresponding factors, a two-factor model to augment the CAPM with Forex volatility factor, ie Market + Volatility is written as follows for excess return/risk computations of internationalized or globalized business, for sustainable global competitiveness.

Where VOL_t , is the volatility resulted out of commerce and economic innovations and measured by either the first difference or movement of the Forex volatility or the factor mimicking excess returns, as the average return on the two positive sensitivity capital assets minus the average return on the two negative sensitivity capital assets. So, as per the second process, the Forex (FX) volatility risk factor is

$$VOL_t = \{ BP_t + SP_t / 2 \} - \{ BN_t + SN_t / 2 \} \longrightarrow 2.2$$

where BP_t, SP_t, BN_t and SN_t are the returns on large and positive sensitivity, small and positive sensitivity, large and negative sensitivity and small and negative sensitivity assets, respectively.

Now, consider an enhanced or augmented version of the CAPM, a two factor model, which augments the basic CAPM above at 2.1, with a Foreign Exchange (FX) volatility factor (MKT + VOL)

$$R_{it} = \alpha_i + b_iMKT_t + c_iVOL_t + \epsilon_{it} \longrightarrow 2.3$$

In the literature, previous work shows that average returns on common stocks are related to firm characteristics like size, earnings/price, cash flow/price, book-to-market equity, past sales growth, long-term past return, and short-term past return. Because these patterns in average returns apparently are not explained by the CAPM, they are called anomalies (15). The empirical tests of the 5 factor Fama French model's objective, improved from 3 factor model, is to explain average returns on portfolios formed to produce large spreads in Size of the firm, Book to Market ratio, Profitability and Investment. The three-factor risk-return relation is, however, just a model. It surely does not explain expected returns on all securities and portfolios. Fama and French use the Dividend Discount model to get two new factors from it, investment and profitability (Fama and French, 2014). Dividend Discount model is an approximate approach, based on the current growth in Dividends, to find somewhat realistic value from historical data approach. In these empirical tests, Firstly, the model is applied to portfolios formed on Size, Book to Market ratio, Profitability and Investment. The portfolio returns to be explained are from improved versions of the sorts that produce the factor. Secondly, the five-factor CAPM model's performance is compared to the 3-factor CAPM model's performance with regards to explaining average returns associated with major anomalies not targeted by the model (Fama and French 2014). Five factor model, time series regression with addition of Profitability and Investment factors has the equation below:

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + \epsilon_{it} \longrightarrow 2.4$$

Where:

R_{it} is the return in month t of one of the portfolios

R_{Ft} is the risk free rate

$R_m - R_f$ is the return spread between the capitalization weighted stock market and cash

SMB is the return spread of small minus large stocks (i.e the size effect)

HML is the return spread of cheap minus expensive stocks (i.e the value effect)

RMW is the return spread of the most profitable firms minus the least profitable

CMA is the return spread of firms that invest conservatively minus aggressively

The test is to observe, whether 5 factor model captures average returns on the variables and to see which variables are positively or negatively correlated to each other and additionally identifying the size of the regression slopes and how all these factors are related to and affect average returns of stocks values. The tests done by Fama and French (2014) show that the value factor HML is redundant for describing average returns when profitability and investment factors have been added into the equation and that for applications where sole interest is abnormal returns, a four or five factor model can be used but if portfolio tilts are also of interest in addition to abnormal returns then the five-factor model is best to use. The results also show that the Fama-French five-factor model explains between 71% and 94% of the cross-section variance of expected returns for the size, value, profitability and investment portfolios. It has been proven that a 5 factor model directed at capturing the size, value, profitability and investment patterns in average stock returns performs better than the 3 factor model in that it lessens the anomaly average returns left unexplained. The five-factor model has yet to be proven as an improvement compared to previous models however it has left room for better models to be further developed from it in the future. Most investors still use the famous three-factor model but as methods seem to take some years before people start using, as industry personnel always have doubts (19). The five-factor

model's main problem is its failure to capture the low average returns on small stocks whose returns behave like those of firms that invest a lot despite low profitability (23).

Now, consider an enhanced or augmented version of the CAPM, given by Fama French a five-factor model, which augments the five factor model above at 2.4, with a Foreign Exchange (FX) volatility factor (MKT + SMB + HML + WML + VOL)

$$R_{it} = \alpha_i + b_i MKT_t + s_i SMB_t + v_i HML_t + m_i WML_t + c_i VOL_t + \epsilon_{it} \longrightarrow 2.5$$

III. Sub-Components of Volatility

In the literature, the added Forex Volatility factor is further sub-divided in to sub-components. The pervasiveness of the pricing power of foreign exchange volatility across a variety of test assets (documented in Menkhoff, Sarno, Schmeling, and Schrimpf, 2011) suggests a potentially promising approach to understanding foreign exchange risk in the equity market. It may be foreign exchange volatility (second moments) not foreign exchange changes (first moments) that matters for the cross-section of stock returns. This perspective also has a number of theoretical justifications. Motivated by these empirical and theoretical considerations, there has been conclusions. Single component, foreign exchange volatility has no power to explain either the time-series or the cross-section of stock returns. We do not differentiate different components of volatility (i.e. long-run and short-run components), which need to be further focused. Bartov, Bodnar, and Kaul (1996) find that the market risk (beta) of multinational firms increases with the increase in foreign exchange volatility when a longer-horizon (5 years) is focused upon. Adrian and Rosenberg (2008) find differential effects of the long-run and short-run components of stock market volatility on expected return of Assets. Similar argument is extendable to long-run volatility of Forex on expected return of Assets. Over and above, it is medium term volatility in Forex that effects the returns more.

So, in the present work, it is adding sub-components to the Forex Volatility to assess Risk/Return of assets. There are 3 sub-components, to be added corresponding to volatility of short run, medium run and long run. This model is based on the heterogeneous market hypothesis Muller et al (14-15 October, 1993), which implies that lower frequency volatility, i.e., weekly effects higher frequency volatility, i.e., daily, but not vice versa, for various financial parameters.

Similarly, if we apply Heterogenous AutoRegressive (HAR) model, to Forex volatility

$$RV_{t+1}^d = c + \beta^d RV_t^d + \beta^w RV_t^w + \beta^m RV_t^m + \epsilon_{t+1}^d \longrightarrow 3.1$$

where RV_t^d , RV_t^w and RV_t^m are daily, weekly and monthly realized variance, respectively at period 't'.

$$RV_t^w = \frac{RV_t^d + \dots + RV_t^d}{5} \longrightarrow \text{Weekly}$$

Similarly, the monthly realized variance is computed as the average of daily variances over 22 days. Although the HAR model is able to capture long memory and volatility clustering, it cannot explain abrupt changes in regimes. Indeed, recent subprime mortgage crises, European debt turmoil and a number of other financial calamities led to significantly, different behavior in the dynamics of the financial parameters realized variance during "good" and "bad" times, of the economic and financial upheavals. Therefore, it is proposed to extend the benchmark HAR model and allow the possibility of multiple regimes, governed by the volatility variables. We define the threshold HAR model with two regimes as follows:

$$RV_{t+1}^d = c_1 + \beta^d RV_t^d + \beta^w RV_t^w + \beta^m RV_t^m + \epsilon_{t+1}^d, \text{ if } T_{t-1} < \tau$$

$$= c_2 + \beta^d RV_t^d + \beta^w RV_t^w + \beta^m RV_t^m + \epsilon_{t+1}^d, \text{ if } T_{t-1} \geq \tau$$

Where T_{t-1} is a trigger variable with some lag 'l' and τ is the value of a threshold.

IV. Model Assumptions

This review starts by applying the CAPM framework to firm's production assets and then derive from it equity security dynamics consistent with credit risk as a state variable. The initial CAPM representation defines the excess return on a generic individual risky unlevered equity (or production asset: used unlevered equity,

production asset, real asset and simply asset as synonyms) as proportional to the excess return on the market risky unlevered equity:

$$E_t(R_{i,t+dt}) - R_{ft+dt} = \beta[E_t(R_{m,t+dt}) - R_{ft+dt}] \quad \text{---} \quad 4.1$$

Expected Return On a Security - Risk Free Rate = Beta of the Security (Difference between expected return on market and risk free rate)

where r_i indicates gross returns, $\beta = \text{Cov}(R_i, R_m) / \text{Var}(R_m)$, and everything is based on information at time t (i assumes, everything occurs at time t and i will drop the time subscript. In addition, i will assume r rates are instantaneous and will not use dt unless in the context of a larger expression that requires it for the sake of clarity). It relates firm production processes, not non-linear securities: it is about one real asset's (log)normal returns in relation to the market's (log)normal returns. Ferguson and Shockley (2003) show that the Beta of individual equity should be calculated using the market asset (and not the equity index), but their model should also be amended to take into account credit risk of the individual firm because the behavior of firm leverage changes through time, hence the need to take it into account. Other important elements consider here are the recovery factor and the term structure of debt, crucial, also to understand why certain industries allow for more leverage than others.

In the model section, it is modelling both the firm and market asset dynamics independently, starting from the unlevered equity level to derive the true asset Betas of various classified segments, then use the asset Betas to describe the non-linear behavior of levered equity returns as a function of the asset Beta, leverage, recovery and debt maturity. This specification reconciles the one-factor CAPM model for real assets with a conditional CAPM model for securities (equity, in this case).

To the model, Merton's model for Corporate Debt, assumptions also apply in addition to conditional CAPM assumptions, So, the following key assumptions are set and applicable to the model:

1. There are no transaction costs, taxes, or problems with indivisibilities of assets.
2. There are a sufficient number of investors with comparable wealth levels such that each investor believes that he can buy and sell as much of asset as he wants at the market price.
3. There exists an exchange market for borrowing and lending at the same rate of interest.
4. Short sales of all assets, with full use of the proceeds, are allowed.
5. Trading in assets takes place continuously in time.
6. The Modigliani-Miller (MM) theorem that the value of the firm is invariant to its capital structure.
7. The terms structure is flat and known with certainty; i.e, the price of riskless discount bond that promises payment of \$ at time T in the future is $P(t, T) = e^{-r(t-T)}$ where r is the (instantaneous) riskless rate of interest, the same for all time.
8. The dynamics for the value of the firm, V , through time can be described by a diffusion type stochastic process.

Merton notes that the perfect market assumptions (the first four) are easily relaxed. Assumption 7 is made to focus attention on default risk: Merton notes that introducing stochastic interest rates will make a fairly innocuous modification [where in one replaces $e^{-r(t-T)}$ by the pure discount bond price $P(t, T)$, which will now depend on relevant state variables of the economy of his main insights. Then, we are left with 5,6,8 as the key assumptions. Assumption 5 is used in practically all the Merton based model publications in general. Assumption 8 has been relaxed in some papers, below. Assumption 6 is derived in the publications with bankruptcy, but more importantly, Merton(1974,p.460,sectionV) notes the following: "If, for example, due to bankruptcy costs or corporate taxes, the MM theorem does not obtain and the value of the firm does depend on the debt-equity ratio, then the formal analysis of the paper is still valid." He then notes that the debt value and the firm value must be simultaneously obtained. It is clear from the foregoing statement that Merton was providing the analytical machinery needed to solve for the optimal capital structure, although he did

Assumptions under conditional CAPM:

1. Firms consist of equity and debt only: simplify data to accommodate this definition.
2. Debt maturity is allowed to change deterministically through time.
3. Assume that Debt consists of coupon paying instruments with a fixed coupon rate.
4. Shareholders who manage the company can make credible threats to debt holders and achieve debt forgiveness.

5. Liquidation occurs when the value of assets is lower than a critical value, which is, in general, lower than the total liabilities: equity often loses control when debt is already highly risky.

6. When a company is liquidated, it incurs loss of productivity and other costs are idiosyncratic to the firm or industry: these costs are incurred even if the company is sold in distress to a third party.

V. Capital Structure of CAPM

In the medium term, the CAPM (Capital Asset Pricing Model) method through RF (Return Risk-Free Assets) is more accurate in predicting stock returns than the APT (Arbitrage Pricing Theory) method. In the long run, the CAPM (Capital Asset Pricing Model) method is also more accurate in predicting stock returns than the APT (Arbitrage Pricing Theory) method(16).

Greeks (Δ , γ , θ) and leverage for both the individual firm and the market, one can calculate and use the CAPM to explain equity price movements conditional on those Greeks. The Greeks, in turn, are functions of credit risk and recovery. The first step to explain equity returns is to write the CAPM for assets and express individual unlevered equity returns as their Beta to the unlevered market:

$$E(r_{ai}) - r_f = \beta_{ai} [E(r_{am}) - r_f],$$

$$E(dA_i) = A_i \beta_{ai} [E(r_{am}) - r_f] + A_i r_f \quad \text{---5.1}$$

After working out production asset dynamics and Betas, one can use them to determine the levered equity dynamics, thanks to Ito's Lemma of Brownian Motion and Stochastic Differential Equations.

Following Merton, equity can be modelled as an option on the underlying firm's asset value:

$$E_i = \max(A_{i,T} - K_i, 0)$$

$$= A_i e^{-d(T-t)} N(d1) - K_i e^{-r(T-t)} N(d2) \quad \text{---5.2}$$

with T the maturity of debt instruments, r the return of a zero coupon bond, and d the dividend rate. The terms d1 and d2, under the risk-neutral measure, are equal to

$$d1 = \frac{\ln A_i / K_i + (r + 1/2 \sigma_{ai}^2)(T - t)}{\sigma_{ai} \sqrt{T - t}}$$

and

$$d2 = d1 - \sigma_{ai} \sqrt{T - t}$$

Applying Ito's lemma to 5.2, directly without giving details of the lemma here, the return in terms of Greeks

$$R_e \sim dE_i/E_i = 1/E_i [\partial E_i / \partial t dt + \partial E_i / \partial A_i dA_i + 1/2 \partial^2 E_i / \partial A_i^2 (dA_i)^2]$$

$$= 1/E_i [\theta_i dt + \Delta_i A_i [r_f dt + \sigma_a dB^*] + 1/2 \Gamma_i \sigma_a^2 A_i^2 dt]$$

$$= \theta_i / E_i dt + 1/E_i [\Delta_i A_i r_f + 1/2 \Gamma_i \sigma_a^2 A_i^2] dt + \Delta_i / E_i \sigma_a A_i dB^* \quad \text{---6.3}$$

and taking expectations, under the risk neutral measure. For every firm i,

$$E^*(r_{ei, t+dt}) \sim E^*(r_{ei}) = \theta_i / E_i dt + [\Delta_i A_i / E_i r_f + 1/2 \Gamma_i \sigma_a^2 A_i^2 / E_i] dt \quad \text{---6.4}$$

This representation of the equity dynamics shows that, when the capital structure of the company changes (equity delta or leverage amount), the equity return, traditionally used in CAPM tests on levered equity, will move away from the true asset return, thereby distorting the measurement of Beta.

This expression (6.4) is more detailed and intricately complex than the standard CAPM but conveys a simple message: equity returns are a function of the firm production asset Betas adjusted for the firm's credit risk, as determined by the firm's leverage, debt maturity, and volatility.

This can be better seen by rearranging the expression as the traditional CAPM.

$$E(r_{ei}) - r_f = [\underbrace{(D_i/E_i)}_{\text{Leverage}} r_f + \underbrace{\theta_i/E_i}_{\text{Debt Maturity}} + \underbrace{\Gamma_i A_i^2 \sigma_{ai}^2 / 2 E_i}_{\text{Volatility}}] dt$$

----- Additional Factors -----

$$+ \underbrace{\Delta A_i / E_i}_{\text{Market Asset Beta}} \beta_{ai} \times [E(r_{am}) - r_f] \quad \text{Market Factor}$$

Equation above also highlights that the Beta of the single stock to the market asset returns is a function of the in-the-moneyness of the equity call option on the company assets as well as the level of leverage of the company.

In the literature, there are findings that simple volatility of Forex is not effective to capture, returns on Assets, so a 3 component Forex Realized Variance indicated at 3.1 is also incorporated in the additional Factors, to represent International Businesses.

Thus the final expression/model for Asset returns are a function of the firm production asset Betas adjusted for the firm's credit risk, as determined by the firm's leverage, debt maturity, volatility of stock and Realized Forex Variance.

$$E(r_{ei}) - r_f = [\underbrace{(D_i/E_i)}_{\text{Leverage}} r_f + \underbrace{\theta_i/E_i}_{\text{Debt Maturity}} + \underbrace{\Gamma_i A_i 2\sigma_{ai}^2 / 2E_i}_{\text{Volatility}} + \underbrace{RV^d}_{\text{Forex Volatility}}] dt$$

----- Additional Factors -----

$$+ \underbrace{\Delta A_i/E_i}_{\text{Market Asset Beta}} \beta_{ai} \times [\underbrace{E(r_{am}) - r_f}_{\text{Market Factor}}]$$

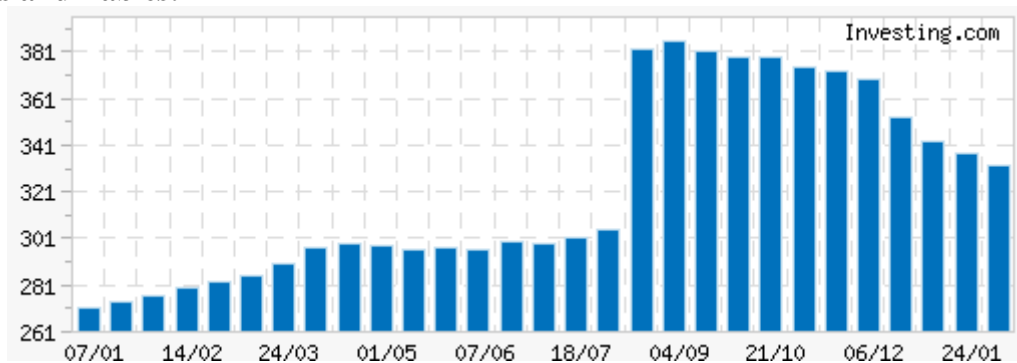
VI. Consolidation of Results

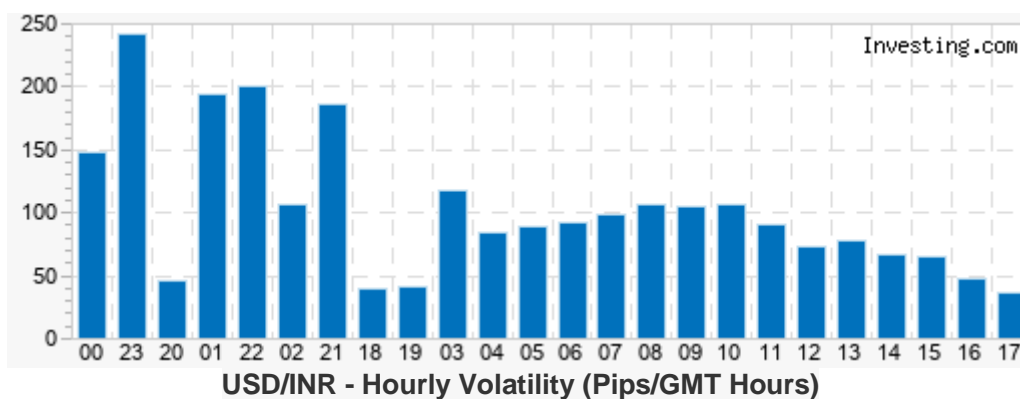
The research review undertaken, to include, essential Forex volatility, as an additional factor for return/risk analysis, in international business has yielded a model with 3 sub- component Forex volatility factor, for inclusion, in Multi Factor CAPM. The empirical evidence in the Table 1 and Table 2, at the end is establishing, there is no difference in including volatility with first difference of volatility or mimicking volatility as indicated by equation 2.2, as there is no difference in 'F' test or R² values after adding earlier Forex models, 'VOL' factor, thus the requirement of a 3 sub component FX volatility factor. It is medium term volatility component, which is more significant of all and effects the return more, in a globalized business or trade, in the international finance, investment decisions. There is no BM 30 industries for Indian region, to evaluate the evolved multi factor CAPM with 3 component Fx volatility factor, applying it to Indian region. In course of time, there is scope to take up, further research, identifying BM 30 for Indian region, to further empirically establish the evolved, better Forex volatility factor included in Multifactor CPAM for Indian region, if a time series or stock data base is available with support of the currently used IBM SSPS tool.

VII. Conclusion

The Forex volatility model is complete and is integrated into Multi Factor CAPM. The results and conclusion on earlier Forex volatility model and it's insufficiency for integration into Multifactor CAPM is also established. Empirical evidence of the Forex volatility model, in Indian region for international businesses would be established in course of time, as database for the region gets built. There is ample scope for transforming the Forex volatility model, into a multifactor model for Forex volatility alone, in case of requirement, while establishing further empirical evidence, of the evolved model.

Charts and Tables:





Calm Before Storm

Past FX volatility slumps have preceded large 6-month moves in the dollar



Descriptive statistics of the FX volatility and it's innovations

	Mean	Std Error	Minimum
Forex Volatility	0.3539	0.1400	0.0258
Maximum			
1.3318			
First Difference of Forex Volatility	0.0007	0.1086	-0.4299
Factor-mimicking portfolio of the volatility			0.6155
Innovations	-0.0014	0.0192	-0.1083
0.1137			

Table 1

Summary statistics of time-series and cross-sectional regressions for 25 size – BM portfolios:

Model	Summary of time-series regressions			
	α	S(α)	R ²	F-
Test				
MKT	0.31	0.16	0.74	3.95
MKT + VOL (not RV)	0.31	0.16	0.74	3.95
MKT + SMB + HML + WML	0.10	0.08	0.91	2.64
MKT + SMB + HML + WML + VOL (not RV)	0.10	0.08	0.91	2.64

Cross Sectional Regressions

Model	R ²	Factor	γ	tEIV	tMIS
MKT	0.13	Alpha	1.38	3.03	2.96
MKT+VOL(not RV)	Alpha	MKT	-0.65	-1.29	-1.26
		MKT	1.59	3.24	0.17
		MKT	-0.89	-1.77	-1.59
		VOL	0.06	1.61	0.94
MKT+SMB+HML + WML	Alpha	0.69	1.68	1.02	0.71
		MKT	-0.18	-0.38	-0.25
		SMB	0.20	1.32	1.31
		HML	0.47	3.15	3.15
		WML	2.03	2,17	0.96
MKT+SMB+HML +WML+VOL(not RV)	Alpha	0.73	1.73	1.11	0.71
	MKT	-0.23	-0.47	-0.32	
		SMB	0.22	1.46	1.41
		HML	0.47	3.13	3.14
		WML	2.39	2.37	1.06
		VOL	-0.02	-0.98	-0.61

Table 2
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