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Comparison of Black - Scholes model based process and Variance Gamma process based model on Indian stock market

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Abstract
Geometric Brownian motion is used as a process for stock prices in the Black-Scholes model. Empirically, stock prices often show jumps caused by unpredictable events or news, also distribution of the log returns of asset exhibits the excess kurtosis. This leads to consideration of an alternative asset price model based on levy process instead of Wienner process. A three parameter stochastic process termed the variance gamma process that generalizes Brownian motion is developed as a model for the dynamics of log stock prices. The process is obtained by evaluating Brownian motion with drift at a random time change given by gamma process. The two additional parameters are the drift of the Brownian motion and volatility of the time change. These additional parameters provide control over skewness and kurtosis of return distribution. Closed forms are obtained for return density and the prices of European call options.

Keywords - *Geometric Brownian motion, Black - Scholes model, variance gamma process, Wienner process.*

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I. Introduction

In option pricing theory, the central problem is to find the fair value of the option.People trade option on the basis of behaviour of stock prices with respect to time.To find the value of European option there is a popular Black Scholes model which is based on some assumptions. Black Scholes model assumes that log returns of underlying asset follows normal distribution. Study on many traded assets showed that log returns of stock prices are not always normally distributed, it has characteristics like excess kurtosis, skewness, jump discontinuity etc. and those characteristics features cannot be express by Normal distribution. This is drawback of black Scholes model and to overcome it we have to find a new process which can capture all such features.Some studies show that a class of levy process, that is, the variance gamma process has those characteristics features.

In this article we have a model for the dynamics of the logarithm of the stock prices with a three parameter generalization of Brownian motion. The new levy process termed gamma process, is obtained by replacing time in Brownian motion with random time change given by a gamma process of unit mean and positive variance. In variance gamma model, the unit period continuously compounded return is normally distributed, conditional on gamma time change.

The variance gamma model for valuation of European option, given by Madan and Saneta in 1990, has two additional parameters compared to Black Scholes model that controls for (i) kurtosis (a symmetric increase in the left and right tail probabilities of the return distribution) (ii) skewness (that allows for asymmetry of the left and right tails of return density), allowing to price options with different strikes, without need to modify implied volatility or other parameters. We compare this variance gamma (VG) model with GBM model according to some performance criteria in NSE market.

II. Literature Review

Black Fischer, Myron Scholes (1973) in their theses "The pricing of options and corporate liabilities" derived a closed form formulae for a European option on an underlying asset with returns following Geometric Brownian motion, i.e. log returns of asset follows normal distribution.

Madan D. and Seneta E. (1990) in their theses derived The Variance Gamma (VG) model for share market returns which is based on levy process, i.e.Variance Gamma process.

R. Geske& W. Torous (1991) in their article named" Skewness, kurtosis and Black -Scholes option mispricing" showed that the distribution of stock returns exhibits deviations from normality ; in particular skewness and kurtosis.

Con R. and Tankov p. (2004) in their book titled " Financial Modeling with jump processes." Represent the properties and construction of levy processes and discussed multivariate modeling via Brownian subordination using levy process.

Sato, Ken-Iti (2011) in their book "Levy Processes and Infinitely Divisible Distributions." Represent processes with the properties of independent and stationary increments and made the link between such processes and their infinite divisible laws.

PermanaF.J ,Lesmono D. &Chendra E. (2014) in their paper titled " Valuation of European and American options under variance gamma process" conclude that the variance gamma model performs very well compared to the GBM model in Indonesian market. The VG model can match the first four moments including skewness and excess kurtosis.

ZoranIvanovski, Toni stojanovski&ZoranNarasanov (2015) in their price dynamics that exhibit heavy tails and excess kurtosis.Article concludes that the Geometric Brownian motion fails to capture the characteristic features of asset.

The VG process

The VG process is the class of a Levy process. The VG process is obtained by evaluating Brownian motion at a random time change given by a gamma process. The Gamma process G(t;μ,υ) with mean rate μ and variance rate ν is the process of independent gamma increments over non-overlapping intervals of time (t,t+h). By using subordinate of Brownian motion process, we define the VG process in terms of Brownian motion b(t;θ,σ) and gamma process G(t;1,υ) as follows:

$X(t;\sigma,\nu,\theta)= b(G(t;1,\upsilon);\theta,\sigma)$

The three parameters involved in the VG model are:
σ: volatility of Brownian motion which control volatility
υ: variance rate of gamma time change which controls kurtosis
θ: the drift in Brownian motion which control skewness

Under the risk-neutral process, the asset price dynamics following the VG process is given by:
$S(t)=S(0)\exp(rT + X(t;\sigma,\nu,\theta) +\omega t)$
Where $\omega = \frac{1}{v}\ln(1-\theta v-0.5\sigma^2 v)$

The Variance Gamma Model

In our proposed model, we will take the advantage of the conditional distribution property of the VG process introduced by Luciano and Schoutens[5].

Assuming the asset price process follows equation (1) then we can obtain the conditional distribution, conditionally on G(T)=x, as follows:

$\log\left(\frac{S(T)}{S(0)}\right) | G(T)$ = x is normally distributed with mean:

$cT+\theta x$ and variance : $\sigma^2 x$.

It mean that log(S(T))|G(T)=x is normally distributed with mean : log(S(0))+cT+θx and variance: $\sigma^2 x$.
In other words, (S(T)) | G(T)=x) is log normally distributed with location parameter
m(x)=logS(0) + cT + θx and scale parameter s(x)=σ $\sqrt{x}$

Then we can calculate the mean of S(T), conditionally on G(T) =x as follow:

$E(S(T) | G(T)=x)=S(0)\exp(cT+ \theta x + \tfrac{1}{2} \sigma^2 x)$.

Now, we can calculate the call option price with strike price K by the following integral:

$$C = \exp(-rt)\, E (\max(S(T)-K),0)$$

$$c = \exp(-rt)\int_0^{\infty}\int_K^{\infty}(y-K)\,f_y(y;m(x),s(x))\,g_x(x;\frac{T}{v},V)\,dydx$$

Where f_y (y;m(x),s(x)) is probability density function of log normal distribution and $g_x(x;\frac{T}{v},V)$.

III. Research Methodology

I. Objective:

1. To determine the theoretical prices of stock options by using variance gamma option pricing model.
2. To compare the performance of the GBM model and VG model with actual market data.

II. Sampling : For this study the three companies are selected from NIFTY. We are taken the data on 30 Jan 2020 for valuation of call option using variance gamma model.

III. Data Source : The data of asset price, strike price, implied volatility, time to expiration of shares and actual premiums are collected from the website www.nseindia.com.

IV. Methodology

We are taken the data of option chain of NSE market and choose the values of moment parameters by trial and error method.

For pricing option we used the Black- Scholes model calculator and also maple program of variance gamma model.

We compare the performance of both GBM and VG model with actual market price.

Data Analysis

We have computed fair values of option by using Black -Scholes model and variance gamma model. Model values and actual value of option are mentioned in given table. We observed that VG model is more relevant to actual price compare to Black - Scholes model.

Company / **Parameters**	**TECHM**	**ACC**	**TCS**
Current price (s) in Rs.	794.50	1535.10	2136.10
Stock price (K) in Rs.	800	1580	2120
Time to expiration (T) in year	0.1071	0.1071	0.1071
Interest rate (r) in %	10	10	10
Implied volatility (σ) in %	29.82	28.80	19.47
Call premium (c) in Rs.	25.95	33.60	66.05
Call premium from BS model	32.359	45.27	75.34
Call premium from VG model	25.94922	39.43	66.52

V. Conclusion

The data analysis shows that most of the Black-Scholes model prices are not near to the actual prices whether variance gamma model can approach to actual market value of option premium in Indian stock market. Variance gamma process based model helps to understand the variability of the market. Empirical data shows that the model based on variance gamma process is efficient compare to BS model. Variance gamma model helps the traders to observe the option price premium more efficiently.

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