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Mathematical Programming In Portfolio Optimization: A Three-Part Review Of Classical, Advanced, And Emerging Techniques

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Abstract
This paper explores the framework of portfolio optimization from the perspective of mathematical programming techniques. It provides analysis of portfolio optimization techniques, classified into three categories: classical, advanced, and emerging methods. Classical portfolio optimization techniques - such as Mean-Variance Optimization (MVO), Linear Programming (LP), and Quadratic Programming (QP), form the foundation of portfolio theory and continue to be widely used because of their simplicity and interpretability. Advanced techniques, including Stochastic Programming (SP) and Mixed-Integer Programming (MIP) address complex challenges such as uncertainty and discrete constraints. Emerging techniques, such as Machine Learning (ML), Quantum Computing and Metaheuristic algorithms represent the cutting edge of portfolio optimization by offering innovative solutions. By bridging the past, present, and future of portfolio optimization, this review will help scholars and practitioners in navigating the dynamic area of mathematical programming in finance.
Keywords: *Portfolio optimization, Mathematical programming, Classical, Advanced, Emerging techniques*

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I. Introduction

In the ever-evolving landscape of financial decision-making, portfolio optimization serves as a critical tool to balance risk with return. The process of building a portfolio can be likened to selecting items at a buffet. At a buffet, diners face an array of options and must choose a combination of dishes that satisfy their preferences, dietary needs, and appetite within their constraints. Similarly, investors construct portfolios by selecting assets from a diverse menu, balancing return expectations, risk tolerance, and budgetary or regulatory constraints. Investors need to pick from a variety of assets, such as equities, gold, mutual funds, bonds, real estate, cash, and cash equivalents, each offering different levels of risk and return, with the ultimate goal of crafting a "plate" that satisfies their financial appetite.

Mathematical programming has become a cornerstone for solving portfolio optimization problems, offering a structured way to model and solve complex decision-making scenarios. Mathematical programming involves formulating optimization problems with mathematical equations and inequalities to find the best solution that satisfies certain constraints. In general, there are three main components to optimization problems. The first is the objective function that needs to be maximized or minimized. It defines what you want to optimize, such as maximizing profit, minimizing cost, or maximizing utility. The second component is a group of decision variables, whose values can be changed in order to optimize the objective function. A collection of constraints, or limitations on the possible values of the variables, constitutes the third component of an optimization problem. They are typically represented as inequalities or equalities involving the decision variables.

Mathematical programming has played a pivotal role in the evolution of portfolio optimization, providing the theoretical and computational tools needed to solve complex optimization problems. Harry Markowitz in 1952[31] was the first to use mathematical programming for portfolio selection. He developed Mean-Variance Optimization (MVO) model- a quadratic programming model known as Modern Portfolio Theory (MPT). It is a classic method which aims to find the optimal balance between expected return and risk. Common formulation of the Markowitz model is given below:

Let us assume that r_p be the expected return of the portfolio, r_i be the expected return of asset i, r_{target} be the target expected return of the portfolio, w_i be the weight of asset i in the portfolio (proportion of total investment allocation to asset i), σ_p be the standard deviation (risk) of the portfolio and MaxRisk limits the portfolio's risk by specifying a maximum allowable standard deviation.

Maximize $r_p = \sum_{i=1}^{n} w_i \cdot r_i$

Subject to:

Weight Constraint: $\sum_{i=1}^{n} w_i = 1$
Target Expected Return Constraint: $\sum_{i=1}^{n} w_i \cdot r_i \geq r_{target}$
Risk Constraint: $\sigma_p \leq$ MaxRisk
Non-negativity Constraint: $w_i \geq 0$ for all i

Since the inception of Markowitz's mean-variance framework in 1952[31], portfolio optimization has evolved to address increasingly complex and realistic scenarios. Mathematical programming techniques provide a rigorous framework for modeling diverse objectives and constraints, ranging from simple linear relationships to highly nonlinear and stochastic systems. Classical techniques, such as MVO and Linear Programming (LP), form the foundation of portfolio theory and are widely used for their simplicity and interpretability. However, these methods often struggle with real-world complexities, such as parameter sensitivity, non-normal return distributions, and dynamic market conditions. Advanced techniques, including Stochastic Programming (SP), Robust Optimization, and Mixed-Integer Programming (MIP) address these challenges by incorporating uncertainty, multi-stage decision-making, and complex constraints, and the latest wave of innovation in portfolio optimization is driven by emerging techniques such as Machine Learning (ML), Quantum Computing, and Metaheuristic Algorithms.

The main objective of this review paper is to survey a wide spectrum of mathematical programming approaches employed in the pursuit of optimal portfolio design and classified into three categories: classical, advanced, and emerging. By analysing these techniques, this paper aims to highlight their methodologies and key contributions, while also identifying research gaps and future directions. This review is therefore especially relevant at a time when the field is transforming due to the combination of machine learning and mathematical programming, the emergence of quantum computing, and the growing significance of hybrid algorithms in finance.

This review is organized as follows: First, we discuss classical techniques, focusing on their foundational principles and practical applications. Next, we explore advanced techniques, which extend classical methods to handle more complex and realistic scenarios. Finally, we present emerging techniques, which leverage recent advancements in computational power and algorithmic innovation. Throughout the paper, we emphasize the interplay between these categories and the potential for hybrid models that integrate multiple approaches.

II. Review Of Literature

Classical Techniques:

Markowitz (1952)[31] laid the foundation for portfolio optimization, a mathematical approach for constructing investment portfolios. He defined risk as the variance (or standard deviation) of portfolio returns and presented the mean-variance (MV) optimization model. He formalized the process of choosing a portfolio that either minimizes risk for a given expected return or maximizes expected return for a given amount of risk (variance of returns). His model produced an efficient frontier that represents the best risk-return combinations for portfolios. He mathematically demonstrated the benefits of diversification with the concept that holding two or more assets are less risky than holding one asset.

Martin (1955)[32] analyzed work of Markowitz with empirical data of securities for managing an investment portfolio. He proposed the use of linear programming (LP) for portfolio selection, contrasting with Markowitz's quadratic programming (QP) approach. By using LP, he aimed to simplify computational requirements while maintaining practical applicability and illustrated how optimization could guide portfolio selection decisions.

Sharpe (1963)[51] addressed the computational complexity of Markowitz's QP approach to portfolio optimization. He introduced the Single-Index Model to simplify Markowitz's Mean-Variance Portfolio Theory. He assessed his model on randomly selected securities from the New York Stock Exchange, analyzing their performance from 1940 to 1951. This paper laid the foundation for Sharpe's later development of the Capital Asset Pricing Model (CAPM) and remains influential in theory of portfolios.

Sharpe (1967)[50] suggested that the portfolio selection problem can be formulated as parametric LP problem. He utilized linear approximation to the quadratic formula for portfolio's risk and represented simple technique for evaluation of expected performance of portfolios.

Pogue (1970)[46] extended the foundational Markowitz model to consider investor's expectations on brokerage charges, price effects from large volume transactions, short-sale options, liability alternatives, and taxation considerations. He integrated variable transaction costs into the portfolio selection model using QP approach.

Lee et al. (1973)[29] focused on optimizing portfolio selection for mutual funds using a goal programming (GP) model. This model integrated the effectiveness of Markowitz's full covariance model and the simplified features of Sharpe's LP approach. They discussed the importance of the geometric mean of annual dividend yields and the variance of returns in assessing the risk associated with securities.

Konno et al. (1991)[27] presented the Mean-Absolute Deviation (MAD) model as an alternative to the classical MV approach developed by Harry Markowitz. They addressed computational and practical challenges of the MV framework, making it useful in real-world applications. They applied the MAD model to optimize portfolios on the Tokyo stock market, demonstrating its effectiveness and computational advantages.

Young (1998)[60] designed a model to address situations where investors are concerned about extreme downside risk rather than volatility. This model introduced a minimax approach to portfolio selection, offering an alternative to the traditional mean-variance framework by focusing on minimizing the worst-case return scenario. The proposed model's reliance on LP makes it computationally efficient and practical for large-scale applications.

Dias (2001)[17] focused on applying QP to modern portfolio selection, optimizing the trade-off between risk and return. He applied Wolfe’s algorithm for efficient frontier derivation and analyzed the performance of 24 portfolios generated by implementation of the algorithm, 12 in a bull market and the other 12 in a bear market.

Papahristodoulou et al. (2004)[45] explored the application of LP techniques to portfolio optimization. Using data from 67 securities over a 48-month time period, they developed two models: (i) maximin and (ii) minimization of mean absolute deviation. These models were then compared with the standard QP formulation.

Chang (2005)[11] presented a modified goal programming (GP) approach to address portfolio optimization using the MAD model. Incorporating practical limitations like budgetary restrictions and minimum investment thresholds, the modified GP framework made it possible to balance competing goals like maximizing returns and limiting risk.

Sun (2010)[56] focused on combining mean-variance optimization with LP to optimize stock portfolios in the Indonesia stock market, emphasizing risk-return trade-offs. He used Sharpe, Treynor and Jensen measurement to evaluate stock portfolio performance. He also utilized his set of portfolio to predict future return.

Tamiz et al. (2013)[57] focused on using a goal programming (GP) approach for selection of international mutual funds. They implemented GP with a variety of extended parameters and analyzed the historical performance data of twenty mutual funds from various worldwide areas. In order to acquire the international mutual fund portfolio they desired, they employed three different GP variations.

Siew et al. (2014)[52] worked on the portfolio composition and performance using GP approach in enhanced index tracking and compared it to the market index. Their approach considered multiple goals, including minimizing tracking error, controlling transaction costs, and achieving a return that outperforms the benchmark index.

Erdas (2020)[21] explored the use of LP in portfolio optimization by incorporating constraints like budget limits, sectoral diversity, and risk tolerance. He discussed MAD model theoretically and applied his model to Borsa Istanbul 30 Index, demonstrating its effectiveness in constructing optimal portfolios under real-world constraints.

Nath et al. (2020)[40] presented a multi-objective linear programming (MOLP) approach to portfolio optimization in the share market. They proposed two methods in the paper namely fuzzy method using Zimmermann technique and Min-max goal programming technique. They provided a real-world example using data from the Bombay Stock Exchange (BSE) to demonstrate the suggested procedures.

Oladejo (2020)[44] used Optimization techniques to find the best investment in a selected portfolio that yields highest returns with less inputs He conducted his research using secondary data provided by a certain company. The single-objective model maximized the return on the $15,000,000.00 that was available to invest in cash crops, mortgage securities, treasury bills, construction loans, certificates of deposit, fixed deposits, and crude oil.

Ling et al. (2023)[30] explored portfolio selection strategies in the context of Bursa Malaysia (the Malaysian stock exchange) using QP. They aimed to optimize portfolio selection by balancing risk and return, with a focus on practical applications for investors in the Malaysian market.

To summarize the key studies and techniques discussed in the literature, Table 1 gives an overview of the classical approaches to portfolio optimization, including the techniques used, key contributions, datasets, and performance metrics.

Table 1: Summary of classical portfolio optimization techniques

Paper Reference	Technique	Key Contribution	Dataset Used	Performance Metrics
Markowitz (1952)[31]	MVO	Concept of risk- return trade-off in portfolio optimization by MVO model with QP as computation tool.	N/A	Expected return, variance (risk), and efficient frontier
Martin (1955)[32]	QP,LP	Explored LP approaches to solve portfolio selection problems.	Simulated data	Risk return analysis

Sharpe (1963)[51]	QP	Simplified Mean -Variance Portfolio Theory by introducing Single-Index Model.	NY Stock Exchange historical data	Systematic risk and computational efficiency
Sharpe (1967)[50]	LP	Formulated parametric LP problem by utilizing linear approximation to the quadratic formula for portfolio's risk.	Secondary data (From another research paper)	Computational efficiency
Pogue (1970)[46]	QP	Extended Markowitz model to consider investor's expectations on brokerage charges, price effects from large volume transactions, short-sale options, liability alternatives, and taxation considerations.	Financial market data	Portfolio efficiency
Lee et al. (1973)[29]	GP	Optimal portfolio selection for mutual funds using GP model with integration of Sharpe's linear programming approach.	Mutual funds data from 61 companies	Risk-adjusted returns of portfolio, and computational efficiency
Konno et al. (1991)[27]	MAD	Reduced the computational complexity by introducing the Mean-Absolute Deviation model as another option to the mean-variance model.	TSE historical data	MAD risk and computational efficiency
Young (1998)[60]	LP	Proposed a minimax portfolio selection rule to minimize maximum loss, solved using LP.	Historical data	Minimax loss, portfolio performance under worst-case scenarios
Dias (2001)[17]	QP	Applied QP to modern portfolio selection, focusing on optimizing risk-return trade-offs.	Brazilian stock market historical data	Risk Adjusted Performance of portfolio, Sharpe ratio and Treynor ratio
Papahristodoulou et al. (2004)[45]	LP	Formulated two LP models (i) maximin, and (ii) minimization of mean absolute deviation for portfolio optimization.	Stockholm Stock Exchange historical data	MAD risk and computational efficiency
Chang (2005)[11]	GP	Proposed a modified GP approach for the MAD portfolio optimization model.	Secondary data (From another research paper)	Computational efficiency
Sun (2010)[56]	MVO and LP	Focused on selecting stocks into a portfolio using mean variance method combining with LP (solver).	Indonesia stock market historical data	Sharpe, Treynor and Jensen Measurement
Tamiz et al. (2013)[57]	GP	Developed a GP model for selecting international mutual fund portfolios.	International mutual fund data	Portfolio risk, return, diversification efficiency
Siew et al.(2014)[52]	GP	Applied GP to enhanced index tracking, optimizing portfolio performance relative to a benchmark index.	Malaysia stock market historical data	Tracking error, portfolio return, benchmark deviation
Erdas (2020)[21]	LP	Developed a portfolio optimization model using LP under specific constraints, such as budget limits and sectoral diversification.	Borsa Istanbul 30 Index historical stocks data	Portfolio return and MAD risk
Nath et al.(2020)[40]	GP	Applied multi-objective LP by fuzzy method using Zimmermann technique and Min-max GP to optimize portfolio selection.	Bombay Stock Exchange historical data	Portfolio semi-absolute deviation risk, return and efficiency
Oladejo (2020)[44]	LP	Explored how LP techniques can be used to optimize a firm's portfolio selection.	Firm-specific financial data	Portfolio risk, return, computational efficiency
Ling et al. (2023)[30]	QP	Explored portfolio selection strategies using QP, emphasizing risk minimization and return maximization.	Bursa Malaysia Stock Exchange historical data	Portfolio risk, return and tracking error

Note: MVO: Mean-Variance Optimization; QP: Quadratic Programming; LP: Linear Programming; GP: Goal Programming; MAD: Mean Absolute Deviation

Advanced Techniques:

Bertsimas et al. (1999)[7] presented a milestone in portfolio optimization by introducing Mixed-Integer Programming (MIP) techniques into real-world portfolio construction. They collaborated with Grantham, Mayo, Van Otterloo and Company (GMO), a prominent asset management firm to apply MIP methods to portfolios consisting of several sub portfolios and constructed 11 quantitatively managed portfolios representing over $8 billion in assets.

Ogryczak (2000)[41] introduced a multiple criteria linear programming (MCLP) model that allows investors to consider various objectives simultaneously, such as maximizing returns while minimizing risk, within a linear programming structure.

Konno et al. (2005)[26] formulated portfolio optimization problem as non-concave maximization problem under linear constraints using absolute deviation as a measure of risk. They used historical data of Tokyo stock exchange (TSE) for their study. They provided valuable insights into the application of global optimization and integer programming techniques in portfolio optimization under non-convex transaction costs.

Benati et al. (2007)[6] introduced a novel mixed-integer linear programming (MILP) approach to solve portfolio optimization problems. They incorporated the Value-at-Risk (VaR) as a risk measure.

Ibrahim et al. (2008)[25] proposed both single stage and two stage stochastic programming (SP) models for portfolio selection problems. They focused on minimizing the maximum downside deviation from the expected return. The models are applied to the optimal selection of stocks listed in Bursa Malaysia and the return of the optimal portfolio is compared between the two stochastic models.

Bertsimas et al. (2009)[8] presented a novel algorithm for solving cardinality-constrained quadratic optimization (CCQO) problems and addressed the computational challenges posed by the inclusion of discrete constraints. They compared their algorithms against CPLEX's quadratic mixed-integer solver and concluded that the proposed algorithms have computational advantages over a general mixed-integer solver.

Sawik (2010)[49] studied selected multi-objective methods for multi-period portfolio optimization problem. He used data set from Warsaw Stock Exchange for his study. Multi-objective MIP methods were used to find tradeoffs between risk, return, and the number of securities in the portfolio.

Xidonas et al. (2010)[59] developed a multi-objective MIP model for equity portfolio construction and selection. Their model aimed to generate Pareto optimal portfolios using an innovative version of the ε-constraint method and proposed methodology is tested through an application in the Athens Stock Exchange.

Cesarone et al. (2011)[10] studied performance of the portfolios obtained by Limited Asset Markowitz (LAM), Limited Asset Mean Absolute Deviation (LAMAD) and Limited Asset Conditional Value-at-Risk (LACVaR) models. They compared linear and quadratic optimization models for portfolio selection, providing their practical applicability and performance. They also analyzed the CVaR and MAD models with cardinality constraints and solved as mixed integer linear programming (MILP) models using CPLEX solver.

Moon et al. (2011)[38] presented a robust model for portfolio optimization focusing on the mean absolute deviation approach. They constructed a simple robust mean absolute deviation (RMAD) model which led to a linear program and reduced computational complexity of existing optimization methods. They tested the robust strategies on real market data and discussed performance of model based on financial elasticity, standard deviation, and market condition such as growth, steady state, and decline in trend.

Stoyan et al. (2011)[55] developed Stochastic-Goal Mixed-Integer Programming (SGMIP) approach for an integrated stock and bond portfolio problem. Their approach addressed uncertainties in asset returns and incorporated real-world constraints, such as transaction costs and minimum transaction lots.

Masmoudi et al. (2012)[33] presented a recourse goal programming approach to a multiple objective stochastic programming portfolio selection model. Their model utilized historical data of securities listed in the S&P100 index to determine the optimal investment proportions which resulted in a portfolio with a beta value of 1.68, heavily weighted towards banking, investment, and industrial companies.

Sawik (2012)[48] provided a focused exploration of bi-criteria portfolio optimization using mathematical programming, integrating percentile-based and symmetric risk measures. He proposed scenario-based portfolio optimization problems under uncertainty and formulated as a bi-objective linear, mixed integer or quadratic program and solved using commercially available software (AMPL/CPLEX) for mathematical programming.

Ghahtarani et al. (2013)[23] presented Goal Programming(GP) approach for the portfolio selection and addressed the uncertainty of the parameters by robust optimization approach. They considered 20 stocks from the Tehran stock exchange for empirical study of the robust optimization of GP in the portfolio selection problem.

Lam et al. (2017)[28] proposed a two-stage MIP model to improve the existing single-stage MIP model for tracking benchmark Index in Malaysia. They determined and compared the optimal portfolio performance of both models in terms of portfolio mean return, tracking error, excess return and information ratio. Their result concluded that the proposed model is able to outperform the existing index tracking model in tracking the benchmark index.

Babat et al. (2018)[3] addressed the computational challenges associated with optimizing portfolios based on Value-at-Risk (VaR), a widely used risk measure in finance. They formulated VaR portfolio optimization problem as MILP problem, enabling the application of integer programming techniques to find near-optimal solutions efficiently.

Aksarayli et al. (2018)[1] introduced a polynomial goal programming (PGP) model for portfolio optimization. Entropy and higher moments of the returns on assets (skewness and kurtosis) were included to accomplish a more comprehensive risk-return trade-off. They tested practicability of the suggested model on two real data sets, and the findings showed that the PGP model is particularly well-suited for portfolio models with higher moments.

Lam et al. (2020)[19] focused on using a two-stage MIP model to optimize portfolio selection aimed at tracking a benchmark index. They contributed to the literature on index tracking and portfolio optimization by combining tracking error minimization with constraints like transaction costs and cardinality.

Ohanuba et al. (2020)[43] focused on effective financial management through decision planning for investing in a competitive portfolio of stocks. They utilized a table-like method to address stock allocation problems in dynamic programming (DP). They concentrated on three primary concerns with the S&P 500 index: style risk, sector risk, and single stock concentration. The financial problem was solved using a table-like approach, which yielded optimal results that were comparable to traditional Modern Portfolio Theory but with simpler computations.

Fernández et al. (2021)[22] proposed the mean squared variance (MSV) portfolio as an alternative to traditional mean-variance (MV) strategy. They developed MILP formulation for MSV portfolio optimization and tested it empirically on eight portfolio time series problems. They also compared performance results of the MSV strategy with those of the standard MV strategy.

Sadri et al. (2022)[47] presented a comprehensive approach for choosing a capital portfolio under uncertain conditions. Their proposed model had three objectives: minimizing risk, maximizing liquidity, and maximizing returns. They extracted data from Tehran Stock Exchange and then used goal programming technique to construct a robust optimization model.

To summarize the key studies and techniques discussed in the literature, Table 2 gives an overview of the advanced techniques to portfolio optimization, including the techniques used, key contributions, datasets, and performance metrics.

Table 2: Summary of advanced portfolio optimization techniques

Paper Reference	Technique	Key Contribution	Dataset Used	Performance Metrics
Bertsimas et al. (1999)[7]	MIP	Constructed portfolio, incorporating constraints like transaction costs and liquidity. They implemented MIP model in FORTRAN using MIP solver.	Grantham, Mayo, Van Otterloo firm data	Portfolio performance, computational efficiency
Ogryczak (2000)[41]	MCLP	Developed MCLP model to select portfolio with multiple objectives like risk and return.	Warsaw stock market data	Portfolio efficiency, risk-return trade-off
Konno et al. (2005)[26]	Global Optimization and MIP	Global optimization and integer programming were compared to optimize the portfolio under non-convex transaction costs.	TSE historical data	Portfolio risk, transaction cost efficiency
Benati et al. (2007)[6]	MILP	Formulated the optimal mean/ VaR portfolio problem using MILP, balancing risk and return.	Milan stock market data	VaR, portfolio efficiency
Ibrahim et al. (2008)[25]	SP	Proposed both single stage and two stage SP models for portfolio selection problems using maximum downside deviation measure, focusing on minimizing downside risk.	Bursa Malaysia Stock Exchange Historical data	Downside risk, portfolio performance
Bertsimas et al. (2009)[8]	CCQO	Developed an algorithm of CCQO for portfolio selection with limited assets.	Simulated data	Portfolio performance, computational efficiency
Sawik (2010)[49]	MIP	Multi-objective MIP was used for multi-period portfolio optimization to find tradeoffs between total number of securities, return, and risk.	Warsaw Stock Exchange historical data	VaR, portfolio efficiency
Xidonas et al. (2010)[59]	MIP	Proposed a MIP approach for construction and selection of equity portfolio. The GAMS/CPLEX solver is used to solve a multi-objective problem with the augmented ε-constraint method.	Athens Stock Exchange historical data	MAD risk, Relative Price Earnings Ratio, portfolio return

Cesarone et al. (2011)[10]	LP ,QP	Compared linear and quadratic optimization models on data sets involving real-world capital market indices for portfolio selection models.	Yahoo finance historical data	MAD risk, CVaR and computational efficiency
Moon et al. (2011)[38]	Robust MAD Model	Proposed simple robust portfolio optimization model using mean absolute deviation techniques within a linear program framework.	NYSE, NADAQ, AMEX stocks historical data	MAD risk, portfolio risk, robustness
Stoyan et al. (2011)[55]	SGMIP	Developed SGMIP approach for integrated bond and stock portfolio optimization.	TSX historical data and Canadian bonds	Portfolio risk, return, goal achievement and computational efficiency
Masmoudi et al. (2012)[33]	Recourse GP	Presented a recourse GP approach to a multiple objective SP portfolio selection model. They solved their recourse goal program using the LINGO solver.	S&P100 securities historical data	Goal achievement, portfolio efficiency
Sawik (2012)[48]	LP, QP, MIP	Evaluated three distinct bi-criteria models for optimizing portfolios that combined symmetric and percentile-based risk measures.	Historical data	Risk measures, portfolio efficiency and computational efficiency
Ghahtarani et al. (2013)[23]	Robust GP	Applied robust GP for multi-objective portfolio selection, addressing uncertainty and multiple objectives.	Tehran stock exchange historical data	Portfolio return, systematic risk and goal achievement
Lam et al. (2017)[28]	MIP	Developed a two-stage MIP model for enhanced index tracking in portfolio optimization.	Malaysia stock market historical data	Tracking error, Information ratio, and portfolio efficiency
Babat et al. (2018)[3]	MILP	Proposed integer programming techniques for computing near-optimal Value-at-Risk portfolios.	USA Financial market data	VaR, portfolio performance
Aksarayli et al. (2018)[1]	PGP	Introduced PGP model to optimize portfolio based on entropy and higher moments.	USA Portfolio data, ISE historical data	Portfolio risk, return, and entropy measures
Lam et al. (2020)[19]	MIP	Applied a two-stage MIP model, where the first optimization step involved minimizing the tracking error and the second stage involved maximizing the portfolio mean return.	Malaysia stock market historical data	Tracking error, portfolio efficiency
Ohanuba et al. (2020)[43]	DP	Explored financial optimization using DP via the tabular method.	Simulated data	Portfolio performance, computational efficiency
Fernández et al. (2021)[22]	MILP	Proposed MILP formulation for the mean squared variance portfolio optimization problem.	Historical stock market data	Mean return, Sharpe ratio
Sadri et al. (2022)[47]	RMOMM	Developed a robust multi- objective mathematical model for optimizing stock portfolios.	Tehran Stock Exchange historical data	Portfolio return, CVaR and robustness

Note: MIP: Mix Integer Programming; MCLP: Multiple Criteria Linear Programming; MILP: Mix Integer Linear Programming; SP: Stochastic Programming; CCQO: Cardinality –Constrained Quadratic Optimization; LP: Linear Programming; QP: Quadratic Programming; SGMIP: Stochastic-Goal Mixed Integer Programming; GP: Goal Programming; PGP: Polynomial Goal Programming; DP: Dynamic Programming; RMOMM: Robust Multi- Objective Mathematical Model; VaR: Value at Risk; CVaR: Conditional Value at Risk; MAD: Mean Absolute Deviation

Emerging Techniques:

Oh et al. (2005)[42] proposed the index fund management scheme that used genetic algorithm (GA) to support portfolio optimization process. The Korea Stock Price Index (KOSPI) 200 was subjected to the proposed GA scheme between January 1999 and December 2001. Their results indicated that the GA procedure offers significant advantages over the traditional portfolio mechanism.

Soleimani et al. (2009)[54] emphasized the role of heuristic algorithms in solving complex and combinatorial problems efficiently. They introduced a new portfolio selection model based on Markowitz's framework with significant constraints: cardinality constraints, minimum transaction lots, and a novel constraint regarding market (sector) capitalization. The complexity of the problem is indicated by its classification as mixed-integer nonlinear programming (NP-Hard), and a genetic algorithm (GA) was suggested as a solution technique.

Deng et al. (2010)[16] presented an extension of Ant Colony Optimization (ACO) to the Markowitz mean-variance portfolio model comprising bounding and cardinality constraints. When they compared ACO to particle swarm optimization (PSO) on Cardinality Constrained Markowitz mean-variance Portfolio Optimization (CCMPO) problems, they found that ACO is significantly more reliable and efficient, particularly for low-risk investments.

Mousavi et al. (2014)[39] emphasized the application of genetic programming model designed for dynamic portfolio trading system. Genetic programming is introduced as an extension of Genetic Algorithm (GA) and used to capture dynamics of stock market prices through time. A multi-tree genetic programming forest was created in order to derive various trading principles from historical data. Their proposed model significantly outperformed conventional static and dynamic portfolio trading models in terms of portfolio return and risk-adjusted return in both emerging and mature markets.

Mittal et al. (2014)[36] proposed a multi-objective model of portfolio rebalancing problem considering return, risk and liquidity as key financial criterion. They developed a real-coded genetic algorithm (RGGA) to solve the portfolio rebalancing problem and built an optimal portfolio. They proposed model using data of National Stock Exchange (NSE), Mumbai and also considered nonlinear transaction costs.

Mishra et al. (2016)[35] introduced a novel prediction-based mean-variance (PBMV) model as an alternative to the traditional Markowitz MV model, aimed at addressing the constrained portfolio optimization problem. Low complexity heuristic functional link artificial neural network (HFLANN) model is used to predict the expected future returns in their proposed model. Multi-objective evolutionary algorithms (MOEAs) are then used to optimize the portfolio.

Dubinskas et al. (2017)[20] assessed the fitness of GA approach in optimizing the investment portfolio. After choosing four Lithuanian companies that were listed on the official list of the OMX Baltics Stock Exchange, they constructed the optimum investment portfolio utilizing MatLab software and a GA-based methodology. Their results suggested that the risk-return ratio of the genetic algorithm-based portfolio was superior to that of the portfolio optimized using stochastic and deterministic programming techniques.

Hidayat et al. (2018)[24] proposed a hybrid optimization method that combined LP models with GA for solving portfolio optimization problems. They explored the synergy between deterministic optimization techniques (LP) and heuristic methods (GA), aiming to overcome challenges such as the non-linearity and complexity of real-world portfolio optimization problems while maintaining computational feasibility.

Meghwani et al. (2018)[34] presented a tri-objective model for portfolio optimization with the objectives being risk, return and transaction cost. The suggested model incorporated a number of real-world constraints, such as cardinality, self-financing, quantity, pre-assignment, and cost-related constraints. They focused on employing multi-objective evolutionary algorithms (MOEAs) to handle equality constraints, such as the self-financing requirement and the constraints formed by the inclusion of transaction cost models. They proposed novel repair method supported by a theoretical proof to address a broader range of separable transaction cost models.

Díaz et al. (2019)[18] proposed a hybrid model that integrated transaction costs, stock weight, market capitalization, and sector diversity for solving the multi period index tracking problem. Their hybrid methodology used mixed-integer nonlinear programming (MINLP) to calculate the weights of the index tracking portfolio and the GA for selecting stocks. Their results showed that hybrid model can provide an index fund whose return rate is similar to the market index with significantly lower risk.

Cui et al. (2020)[15] introduced a two-stage stochastic portfolio optimization model that included a variety of practical trading constraints. They adopted Conditional Value at Risk (CVaR) as the risk metric in their model. They formulated a hybrid combinatorial method combining a hybrid algorithm with LP solver. They also presented how their hybrid approach effectively solves complex portfolio optimization problems by comparing the computational results of three distinct metaheuristic algorithms.

Chen et al. (2021)[13] developed a hybrid model based on machine learning (ML) for stock prediction and MV model for portfolio selection as part of their portfolio construction strategy. They proposed a hybrid model which predicts stock prices by merging an improved firefly algorithm (IFA) with eXtreme Gradient Boosting (XGBoost). Stocks with better potential returns are then selected to use the MV model. Their hybrid approach addressed the limitations of traditional MVO by improving predictive accuracy and enhancing portfolio performance.

Chen et al. (2021)[14] focused on using the interdependencies between variables in Evolutionary Algorithms (EAs) to solve Mixed-Integer Non-Linear Programming (MINLP) problems in the context of optimizing a multi-objective constrained portfolio. They proposed a Compressed Coding Scheme (CCS), which makes use of the dependence among the variables by compressing the two dependent variables into a single variable. They performed comparison studies for constrained portfolio optimization and tested new algorithms on 20 benchmark scenarios with varying asset numbers.

Banerjee et al. (2022)[5] obtained an optimal portfolio selection of Indian Equity Mutual Funds by maximizing return and minimizing risk using GA. They constructed portfolios based on the BSE 100 benchmark, optimizing fund weightage for enhanced investment decision-making.

Chaweewanchon et al. (2022)[12] applied convolutional neural network (CNN) with bidirectional long short-term memory (BiLSTM) as a prediction method for stock pre-selection and the Markowitz mean-variance model for optimal portfolio construction. They used two portfolio models, the mean-variance model and the equal-weight portfolio (1/N) model for demonstration with historical data from Stock Exchange of Thailand 50 Index. Their results concluded that pre-selection of stocks can improve Markowitz mean-variance model performance.

Ban et al. (2023)[4] investigated the prediction of financial assets with high volatility, such as Bitcoin and gold prices using Long Short-Term Memory (LSTM) models. Then they employed a dynamic programming model combined with the greedy algorithm to optimize daily trading strategies, resulting in a substantial increase in total assets over a five-year period.

Buonaiuto et al. (2023)[9] applied Portfolio Optimization by Variational Quantum Eigensolver (VQE) on real quantum computers. They translated the formulation of the general quadratic problem into a Quadratic Unconstrained Binary Optimization, which was mapped to a Hamiltonian. The optimal portfolio was represented by the minimum eigenvalue of this optimization, which was estimated by VQE. They highlighted potential of quantum computing for computational speed and efficiency in portfolio optimization.

Singh et al. (2023)[53] proposed a hybrid deep learning model incorporating Convolutional Neural Networks (CNN) and LSTM networks for selection of stocks and optimal portfolio formation using Markowitz MV model. They used metrics like the Sharpe ratio and cumulative return to validate their model's effectiveness in generating risk-adjusted returns. They also established statistical significance of the model using non-parametric tests and demonstrated the practical application of their model.

Vaziri et al. (2023)[58] presented a comprehensive and time-varying methodology for stock price forecasting and optimal portfolio formation. They used multi-objective mathematical programming (MOMP) combined with a bidirectional long short-term memory model and particle swarm optimization (PSO-BiLSTM) to forecast stock prices and to construct an optimal portfolio. They created more realistic portfolios by integrating deep learning with investment constraints and optimization under budget constraints.

Zarezade et al. (2024)[61] focused on optimization of cryptocurrency portfolio, addressing the high volatility and risks associated with the cryptocurrency market. They proposed a new mathematical formulation of Conditional Drawdown at Risk (CdaR) to enhance portfolio construction within high-risk financial environments. They transformed the model into a deterministic multi-objective approach by integrating chance-constrained programming (CCP) to handle market uncertainties.

Asgari et al. (2025)[2] introduced a self-adjusting algorithm for optimization of stock portfolio, leveraging both technical analysis and fundamental index analysis. They used Sharpe ratio index for comparison of portfolio and enhanced portfolio profitability and risk management by incorporating price-to-earnings ratio with technical analysis constraints. They validated that the suggested algorithm performed better than traditional models and provided robustness in a variety of market conditions.

To summarize the key studies and techniques discussed in the literature, Table 3 gives an overview of the emerging approaches for portfolio optimization, including the techniques used, key contributions, datasets, and performance metrics.

Table 3: Summary of emerging portfolio optimization techniques

Paper Reference	Technique	Key Contribution	Dataset Used	Performance Metrics
Oh et al. (2005)[42]	GA	Genetic algorithms have been used to assist index fund management with portfolio optimization.	Korea stock price index (KOSPI) 200 historical data	Tracking error volatility and Portfolio efficiency
Soleimani et al. (2009)[44]	GA	Developed a GA-based approach for Markowitz portfolio selection with constraints like cardinality, minimum transaction lots, and market capitalization.	Simulated data	Portfolio risk, return, constraint satisfaction and computational efficiency
Deng et al. (2010)[16]	ACO	Used ACO to solve Markowitz MV model including cardinality and bounding constraints.	Stock market index historical data	Portfolio risk, return, computational efficiency

Mousavi et al. (2014)[39]	Genetic programming	Suggested a dynamic portfolio trading system using multi-tree genetic programming.	Iranian and Canadian stock exchange historical data	Conditional Sharpe ratio, Portfolio performance and adaptability
Mittal et al. (2014)[36]	MOPRM	Developed MOPRM incorporating transaction costs with incremental discounts.	NSE India historical data	Portfolio risk, return, relative error and transaction cost efficiency
Mishra et al. (2016)[35]	MOEA	Introduced a prediction-based MV model using multi-objective evolutionary algorithms to select a constrained portfolio.	OR-library data and stock market index historical data	Portfolio performance, computational efficiency
Dubinskas et al. (2017)[20]	GA	Applied a genetic algorithm-based approach for optimization of portfolio.	OMX Baltics Stock Exchange historical data	Risk and return ratio, portfolio efficiency
Hidayat et al. (2018)[24]	GA with LP	Addressed the use of LP models based on genetic algorithms for investment portfolio optimization.	Indonesia capital market stocks data	Portfolio efficiency, computational time
Meghwani et al. (2018)[34]	MOHA	Developed MOHA to optimize and rebalance a practical portfolio, considering transaction costs.	Historical data from Fama and French Data Library	Portfolio risk, return, transaction cost efficiency
Díaz et al. (2019)[18]	GA with MINLP	Proposed a hybrid model combining GA and MINLP for index fund optimization.	S&P 500 historical data	Portfolio performance, Sharpe ratio and computational efficiency
Cui et al. (2020)[15]	Hybrid algorithm and LP	Developed a two-stage stochastic portfolio optimization model with uncertain asset values using hybrid combinatorial approach.	Historical data from OR-Library	Portfolio risk, return, computational efficiency
Chen et al. (2021)[13]	ML with MV model	Integrated ML based stock price prediction with mean-variance model for optimization of portfolio.	Shanghai Stock Exchange historical data	Portfolio return –risk ratio, prediction accuracy
Chen et al. (2021)[14]	MOEA	Used evolutionary algorithms for multi-objective constrained portfolio optimization, utilizing the dependence between variables.	OR-Library data and historical stock data from Yahoo Finance	Inverted Generational Distance (IGD), Inverted Hypervolume (IH), constraint satisfaction
Banerjee et al. (2022)[5]	GA	Applied GA for optimal portfolio selection of equity mutual funds, focusing on the Indian market.	Indian equity mutual fund historical data	Portfolio risk, return and efficiency
Chaweewanchon et al. (2022)[12]	ML with MV model	Combined ML for predictive stock selection with Markowitz MV portfolio optimization.	SET50 historical data	Sharpe ratio, mean return and risk, prediction accuracy
Ban et al. (2023)[4]	LSTM and DP	Optimized venture portfolios using LSTM for prediction and DP for decision-making.	Historical data of Gold from London Market and Bitcoin from NASDAQ	Portfolio risk, return and prediction accuracy
Buonaiuto et al. (2023)[9]	QC	Explored portfolio optimization using quantum computing by Variational Quantum Eigensolver.	Yahoo finance data	Portfolio efficiency, computational speed
Singh et al. (2023)[53]	Hybrid CNN-LSTM with MV model	Proposed a hybrid deep learning model incorporating CNN and LSTM networks for stock selection and MV model for portfolio optimization.	NSE India historical data	Accuracy, Sharpe ratio, Cumulative and Risk-adjusted return
Vaziri et al. (2023)[58]	PSO-BiLSTM with MOMP	Proposed a time-varying stock portfolio selection model combining PSO-BiLSTM and MOMP under budget constraints.	Historical data from TSE and OTC Iran	Portfolio profit to risk ratio, computational efficiency

Zarezade et al. (2024)[61]	CCP	Applied CCP for crypto currency portfolio optimization using CDaR.	Crypto-currency data	Portfolio risk, return and CDaR
Asgari et al. (2025)[2]	Self-Adjusting Algorithm	Proposed a self-adjusting algorithm based on GA for stock portfolio optimization, considering technical and fundamental index analysis.	Tehran stock market historical data	Sharpe ratio index, Mean of ideal deviation, Portfolio return and adaptability
Note: GA: Genetic Algorithm; ACO: Ant Colony Optimization; MOPRM: Multi-Objective Portfolio Rebalancing Model; MOEA: Multi-Objective Evolutionary Algorithms; LP: Linear Programming; MOHA: Multi-Objective Heuristic Algorithms; MINLP: Mixed-Integer Non Linear Programming; ML: Machine Learning; MV Model: Mean-Variance Model; LSTM: Long Short-Term Memory; DP: Dynamic Programming; QC: Quantum Computing; CNN: Convolutional Neural Networks; PSO-BiLSTM model: Particle Swarm Optimization- Bidirectional Long Short-Term Memory model; MOMP: Multi-Objective Mathematical Programming; CCP: Chance-Constrained Programming; CDaR: Conditional Drawdown at Risk				

To systematically trace the development of portfolio optimization methodologies, Table 4 categorizes significant research contributions along a timeline, divided into three primary domains: classical, advanced, and emerging techniques.

Table 4: Timeline of Portfolio Optimization Techniques

Timeline	Classical Techniques	Advanced Techniques	Emerging Techniques
1951-1955	Markowitz (1952)[31] [MVO] Martin (1955)[32] [QP,LP]	-	-
1956-1960	-	-	-
1961-1965	Sharpe (1963)[51] [QP]	-	-
1966-1970	Sharpe (1967)[50] [LP] Pogue (1970)[46] [QP]	-	-
1971-1975	Lee et al. (1973)[29] [GP]	-	-
1976-1980	-	-	-
1981-1985	-	-	-
1986-1990	-	-	-
1991-1995	Konno et al. (1991)[27] [MAD]	-	-
1996-2000	Young (1998)[60] [LP]	Bertsimas et al. (1999)[7] [MIP] Ogryczak (2000)[41] [MCLP]	-
2001-2005	Dias (2001)[17] [QP] Papahristodoulou et al. (2004)[45] [LP] Chang (2005)[11] [GP]	Konno et al. (2005)[26] [Global Optimization and MIP]	Oh et al. (2005)[42] [GA]
2006-2010	Sun (2010)[56] [MVO, LP]	Benati et al. (2007)[6] [MILP] Ibrahim et al. (2008)[25] [SP] Bertsimas et al. (2009)[8] [CCQO] Sawik (2010)[49] [MIP] Xidonas et al. (2010)[59] [MIP]	Soleimani et al. (2009)[44] [GA] Deng et al. (2010)[16] [ACO]
2011-2015	Tamiz et al. (2013)[57] [GP] Siew et al.(2014)[52] [GP]	Cesarone et al. (2011)[10] [LP ,QP] Moon et al. (2011)[38] [Robust MAD Model] Stoyan et al. (2011)[55] [SGMIP] Masmoudi et al. (2012)[33] [Recourse GP] Sawik (2012)[48] [LP, QP, MIP] Ghahtarani et al. (2013)[23] [Robust GP]	Mousavi et al. (2014)[39] [Genetic programming] Mittal et al. (2014)[36] [MOPRM]
2016-2020	Erdas (2020)[21] [LP] Nath et al.(2020)[40] [GP] Oladejo (2020)[44] [LP]	Lam et al. (2017)[28] [MIP] Babat et al. (2018)[3] [MILP] Aksarayli et al. (2018)[1] [PGP] Lam et al. (2020)[19] [MIP] Ohanuba et al. (2020)[43] [DP]	Mishra et al. (2016)[35] [MOEA] Dubinskas et al. (2017)[20] [GA] Hidayat et al. (2018)[24] [GA with LP] Meghwani et al. (2018)[34] [MOHA] Díaz et al. (2019)[18] [GA with MINLP] Cui et al. (2020)[15] [Hybrid algorithm and LP]

2021-2025	Ling et al. (2023)[30] [QP]	Fernández et al. (2021)[22] [MILP] Sadri et al. (2022)[47] [RMOMM]	Chen et al. (2021)[13] [ML with MV model] Chen et al. (2021)[14] [MOEA] Banerjee et al. (2022)[5] [GA] Chaweewanchon et al. (2022)[12] [ML with MV model] Ban et al. (2023)[4] [LSTM and DP] Buonaiuto et al. (2023)[9] [QC] Singh et al. (2023)[53] [Hybrid CNN-LSTM with MV model] Vaziri et al. (2023)[58] [PSO-BiLSTM with MOMP] Zarezade et al. (2024)[61] [CCP] Asgari et al. (2025)[2] [Self-Adjusting Algorithm]

III. Research Gap

Despite significant advancements in portfolio optimization using mathematical programming, several research gaps remain unaddressed. First, while classical quadratic programming methods like Markowitz (1952)[31] dominate theoretical frameworks but struggle to handle modern constraints and real-world regulatory limitations. The classical studies of Markowitz (1952)[31], Martin (1955)[32], Sharpe (1967)[50], Lee et al. (1973)[29], Konno et al. (1991)[27], and Young (1998)[60] rely on static frameworks, neglecting dynamic market conditions, investor behavior, and multi-period optimization. These models lack robustness to estimation errors, and ignore transaction costs which are prevalent in real-world financial data.

To tackle the limitations of static frameworks and estimation errors, Bertsimas et al. (1999)[7] and Bertsimas et al. (2009)[8] offered robust solutions for large-scale and complex portfolios by leveraging MIP and cardinality-constrained optimization. Advanced techniques of Konno et al. (2005)[26], Sawik (2010)[49], Sawik (2012)[48], Ohanuba et al. (2020)[43], and Sadri et al. (2022)[47] addressed gaps related to realistic market conditions, computational inefficiencies, and also integrated percentile and symmetric risk measures with alternative risk metrics. While these techniques improve portfolio optimization, they still face computational challenges for extremely large-scale portfolios or high-frequency trading environments.

Emerging techniques such as machine learning, quantum computing, hybrid combinatorial methods, genetic algorithms, and multi-objective heuristics etc. address these limitations by enabling dynamic adaptation, improving scalability and real-time strategies that respond to changing market dynamics and investor behavior. However, these emerging methods also face challenges, including high computational costs, hardware limitations, sensitivity to data quality, and a lack of interpretability in complex models.

Hidayat et al. (2018)[24] presented a novel integration of GA with LP for portfolio optimization but the computational efficiency of their hybrid model is not benchmarked against any modern alternatives which leaves scalability questions unresolved. Meghwani et al. (2018)[34] used heuristic algorithms that may not guarantee optimal solutions and can be sensitive to parameter tuning. Chen et al. (2021)[13] integrated machine learning models with portfolio optimization but these models may overfit to historical data, which would lead to poor out-of-sample performance. Buonaiuto et al. (2023)[9] experimented on real quantum devices, which are still in early stages and may not yet outperform classical methods for large-scale problems. Vaziri et al. (2023)[58] combined PSO, BiLSTM, and multi-objective programming which may be overly complex, making it difficult to interpret or implement. While these emerging techniques offer significant advancements, their limitations highlight the need for continued innovation to achieve robust, efficient, and interpretable portfolio optimization solutions.

As most of the existing research is based on static market conditions, multi-market portfolio optimization remains underexplored. Also current research predominantly examines single-asset-class optimization (e.g., stock portfolios using factor models or bond portfolios using duration matching), while multi-asset-class hybrid portfolios are little known. There is a lack of comprehensive studies that explore the application of hybrid asset class data in portfolio optimization models. But as new asset classes are emerging like – crypto, and tokenized securities, integration of hybrid portfolio asset classes will develop more robust and adaptive optimization strategies. While stochastic and robust optimization methods attempt to address uncertainty, they are rarely tested in live trading environments, limiting their practical applicability. To bridge the gap, future research must develop adaptive and locally constrained models using techniques like regime-switching stochastic programming and hybrid AI-OR (Artificial Intelligence-Operations Research) methods. An actionable roadmap can include collaborating with asset managers to conduct live testing, and enabling near real-time optimization that accounts for real-world market frictions such as transaction costs and taxes.

IV. Conclusion And Scope For Future Research

Portfolio optimization has come a long way since the introduction of Mean-Variance Optimization (MVO) in 1952. Over the decades, the field has evolved significantly, driven by advancements in mathematical programming, computational power, and data-driven approaches. This review paper has provided a comprehensive and structured overview of portfolio optimization techniques, classified into three categories: classical, advanced, and emerging. From classical techniques to advanced and emerging techniques mathematical programming has enabled researchers and practitioners to address a broad range of portfolio optimization challenges. Each technique has its strengths and limitations, and the choice of method often depends on the specific problem context, such as the presence of constraints, the need for computational efficiency, or the handling of uncertainty. Overall, mathematical programming continues to be an important tool for investors, empowering them to make informed decisions in dynamic and volatile markets.

The future of portfolio optimization lies in the integration of classical, advanced, and emerging techniques with algorithmic innovation and more adaptive, resilient, and inclusive portfolio optimization strategies. Hybrid models, which combine different approaches, can potentially be the future as well, by achieving improved performance based on the synergy of classical, new-age, and different approaches. As portfolio optimization models become more complex, there is a growing need for transparency and interpretability and therefore we require continued innovation and interdisciplinary collaboration from mathematics, computer science, and finance.

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