

## Statistically Verifiable Ideal Standard for Air Pollution Based on Expected Number of Exceedances in Jaipur City

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**Abstract:** Air pollution is the most unsafe type of pollution. For controlling pollution, environment standard are developed. Environment standard is set to limit the impact of pollution to some acceptable level. Generally two types of standard are found in practice- Ideal and Realizable. . An ideal standard describe the general worldwide circumstance yet overlook the uncertainty and variation to look at compliance with it. Further Realizable standard is based on sample and does not consider any sampling variation which is natural. Since Ideal standard is statement randomly given does not involve any statistical meaning so there is a need to define statistically meaningful standard. Barnett and O'Hogan(1997) introduced the concept of Statistically Verifiable Ideal Standard (SVIS). The idea behind the SVIS is to combine ideal standard with statistically based rule of implementation. In this paper we construct SVIS for air pollutant based on expected number of exceedances. By an exceedance we mean that the number of times when the pollutant concentration is higher than that set down by the standard. This is accomplished with the help of Neyman Pearson hypothesis testing frame work. With help of this SVIS criterion we will check the compliance status of various monitoring sites in Jaipur city for which data is collected by Rajasthan Pollution Control Board (RPCB). Monitoring sites are Chandpole, JhalanaDungri, RIICO Office, Ajmeri Gate, Vidhyadhar Nagar, VKIA.

**Keywords:** Air pollution, Standard, Ideal standard, Realizable standard, Statistically Verifiable Ideal Standard (SVIS), Exceedances, Power Function.

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### I. Introduction

In this paper we shall discuss the construction of SVIS for air pollutant based on expected number of exceedances. This is accomplished with the help of Neyman Pearson hypothesis testing frame work. By an exceedance we mean that the number of times when the pollutant concentration is higher than that set down by the standard. Air pollution is the most unsafe type of pollution. It is caused when harmful gases, dust, smoke enters into atmosphere and makes difficult for living being to survive as the air become dirty. Air pollution also increases due to growth of urbanization and industrialization. For controlling pollution, environment standard are developed. Environment standard is set to limit the impact of pollution to some acceptable level. Generally there are two types of standard are found in practise- Ideal and Realizable. An ideal standard describe the general worldwide circumstance yet overlook the uncertainty and variation to look at compliance with it. Since Ideal standard is statement randomly given does not involve any statistical meaning so there is a need to define statistically meaningful standard.

Further Realizable standard is expressed in such a way that we can determine the compliance criteria that is whether a particular site is in compliance or not without uncertainty. It is based on sample and does not consider any sampling variation which is natural. For example in India the air pollution standard promulgated by NAAQS is realizable standard as it specifies that an upper limit of ambient pollutant concentration is "not to be exceeded more than 2% per time" at a given monitoring location. So rather than these two type of standards we will be using the concept of Statistically Verifiable Ideal Standard (SVIS) introduced by Barnett and O'Hogan. The idea behind the SVIS is to combine ideal standard with statistically based rule of implementation. Advantage of SVIS approach is that it specifies the quality of statistical verification required but does not specify the procedure by which that is to be achieved, allowing for development in technology and statistical theory without the need to reset the standard. The SVIS is statistically more meaningful than both the ideal or realizable standard for the reason that the ideal standard without a compliance criterion cannot really be used, as there is no method by which compliance may be monitoring. Realizable standards for different reasons are too limited. So for the following reasons we prefer SVIS.

After constructing SVIS with the help of hypothesis testing, we will be calculating Power function of the test and also drawing the power curve. By power function, we mean the probability of rejecting null hypothesis when it is not correct. Power function will be function of the true value of the parameter. In other words, we can define power function as power of test to reject  $H_0$  when the true value of parameter is  $\theta$  and is denoted by  $\beta(\theta)$ . The ideal power is 0 for null hypothesis and 1 for alternative hypothesis. The graph drawn using power function is known as a curve and is obtained by keeping the null hypothesis fixed and by varying the value of the true parameter. It gives relation between power (i.e. probability of rejecting  $H_0$ ) of test and the parameter value  $\theta$ .

In section 2, we shall discuss the construction of SVIS based on expected number of exceedances with the help of Neyman Pearson hypothesis testing frame work. In section 3, we will calculate the power function for the hypothesis testing and also draw the power curve. In section 4, we will be calculating the number of exceedances for all the monitoring sites for last three years. In section 5, with help of this SVIS criterion we will check the compliance status of various monitoring sites in Jaipur city for which data is collected by Rajasthan Pollution Control Board (RPCB). Sites are Chandpole, JhalanaDungri, RIICO Office, Ajmeri Gate, Vidhyadhar Nagar, VKIA.

## II. Material And Methods

### To Construct The Svis Based On Expected Number Of Exceedances, We Proceed As Below:

Note that by an exceedance we mean that the number of times when the pollutant concentration is higher than that set down by the standard. Let  $T$  denote 24 hourly concentration level of a particular pollutant observed on particular day. Let  $I$  denote the ideal standard set by regularity body for particular pollutant. If  $T > I$  at any particular day then we will say that it is one exceedance of the standard for a particular pollutant. If  $n$  is the number of observation for particular pollutant in particular year, and  $X$  is the number of exceedances in that year then for particular location, then  $X$  has binomial distribution with parameter  $N = 365$  and  $p = P [T > I]$ , i.e.,

$$X \sim B (N, p)$$

If we define  $\theta$  as expected number of exceedances then

$$E (X) = \theta$$

Now for the construction of SVIS, we will test the hypothesis

$$H_0: \theta \leq 2$$

against

$$H_1: \theta > 2$$

But we know

$$E(X) = \theta = Np$$

So hypothesis become

$$H_0: p \leq 2/365$$

against

$$H_1: p > 2/365$$

Now we define indicator random variable  $X_i$ ,  $i = 1, 2, \dots, n$  as below:

$$X_i = \begin{cases} 1 & \text{if exceedance occur at} \\ & \text{ith observation} \quad \dots(1) \\ 0 & \text{otherwise} \end{cases}$$

Then

$$X = \sum_{i=1}^n X_i = \text{total number of exceedances in a year out of } n \text{ observation}$$

If  $n$  is large,  $X = \sum_{i=1}^n X_i$  follows normal distribution with mean  $np$  and variance  $npq$  then to test the above hypothesis, we use UMP test  $\Phi(X)$  (say) has the following form:

$$\Phi(X) = \begin{cases} 1 & \text{if } \sum_{i=1}^n X_i \geq C \\ 0 & \text{if } \sum_{i=1}^n X_i < C \end{cases} \quad \dots(2)$$

Where  $c$  is some constant which is so obtained such that size of test  $\alpha$  (say) is obtained that is

$$P [\text{Reject } H_0 | H_0] = \alpha \quad \dots(3)$$

Now consider

$$P [\text{Reject } H_0 | H_0] = \alpha$$

$$\text{Or } P \left[ \frac{\sum X_i - np}{\sqrt{npq}} \geq C \mid p = 2/365 \right] = 0.05 \quad \dots(4)$$

$$\Rightarrow P \left[ Z \geq \frac{C - np}{\sqrt{npq}} \mid p = 2/365 \right] = 0.05 \quad \dots(5)$$

Putting n=96 and p= 2/365 we get

$$\Rightarrow P \left[ Z \geq \frac{365C - 192}{264} \right] = 0.05 \quad \dots(6)$$

Now to obtain C, we compare (6) with following equation:

$$P[ Z \leq Z\alpha ] = 0.95 \quad \dots(7)$$

$$\Rightarrow Z\alpha = 1.64$$

We get

$$\Rightarrow \frac{365C - 192}{264} = 1.64 \quad \dots(8)$$

$$\Rightarrow C = 1.71 \approx 2$$

### CONSTRUCTION OF POWER CURVE

To construct power function we will proceed as below:

By definition, power function  $\beta(\theta)$  is probability of rejecting null hypothesis when it is not correct. Mathematically we represent power function as:  
 $\beta(\theta) = \text{Power function} = P [\text{Reject } H_0 \mid p]$ , for various value of p

$$= P \left[ \frac{\sum X_i - np}{\sqrt{npq}} \geq C \mid p \right]$$

$$= P \left[ Z \geq \frac{C - np}{\sqrt{npq}} \mid p \right]$$

$$\text{Power function} = P [ Z \geq Z_\alpha \mid p ] \quad \dots(9)$$

Where  $Z_\alpha = \frac{C - np}{\sqrt{npq}}$

Now using equation (9), we calculate power function for various values of parameter p. Table below shows the power function value for parameter

Parameter "p"	Value	Power function (value of $\beta(\theta)$ )
1/365		0.00035
2/365		0.02079
3/365		0.08553
4/365		0.1764
5/365		0.27382
6/365		0.36748
7/365		0.45298
8/365		0.52898
9/365		0.59551
10/365		0.65325

11/365	0.7031
12/365	0.74598
13/365	0.7828
14/365	0.81436
15/365	0.8414
16/365	0.86455
17/365	0.88436
18/365	0.9013
19/365	0.91578
20/365	0.92817
21/365	0.93876
22/365	0.9478
23/365	0.95553
24/365	0.96213
25/365	0.96776
26/365	0.97257
27/365	0.97668
28/365	0.98018
29/365	0.98316
30/365	0.9857
31/365	0.98787
32/365	0.98971
33/365	0.99128
34/365	0.99262
35/365	0.99375
36/365	0.99472
37/365	0.99553
38/365	0.99623
39/365	0.99682
40/365	0.99732
41/365	0.99774
42/365	0.9981
43/365	0.9984
44/365	0.99865
45/365	0.99887
46/365	0.99905
47/365	0.9992
48/365	0.99933
49/365	0.99944
50/365	0.99953

Table 1: Calculated Power Function

Now a graph between different value of “p” and  $\beta(\theta)$ , gives power curve which is given in figure below:

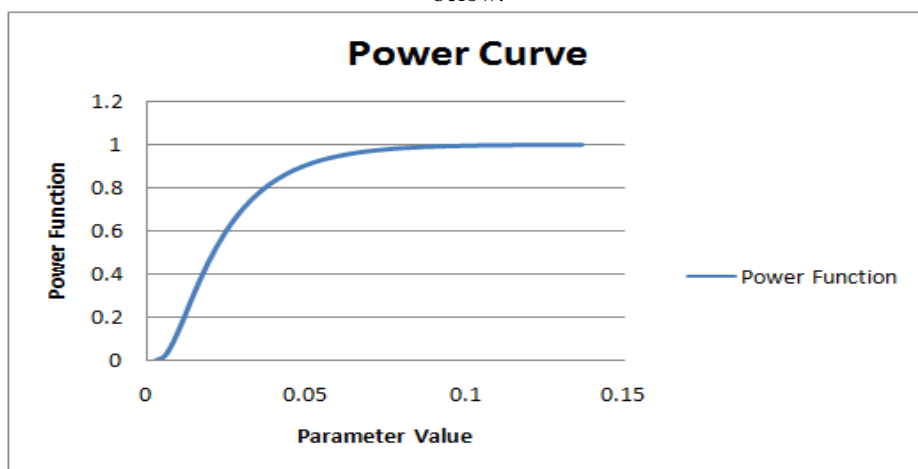


Figure 1

From the above graph, we can see that power function is leading to zero for  $p < 2/365$  and heading to 1 for  $p > 2/365$ . So, we see that probability of rejecting  $H_0$  when  $H_1$  is true tending to 1 so our test is consistent which is desirable property of test.

### III. Results And Discussion

From the data collected by RPCB for the year 2014, 2015, 2016, we compute the number of exceedances for year 2016 for each monitoring sites. The results are given below:

Site Name	Pollutant	Naaqs	Number Of Exceedances
Ajmeri Gate	No2	80	0
	Rspm	100	73
	So2	80	0
Jhalana Dungri	No2	80	0
	Rspm	100	43
	So2	80	0
Chandpole	No2	80	0
	Rspm	100	75
	So2	80	0
Riico Office	No2	80	0
	Rspm	100	46
	So2	80	0
Vdn	No2	80	0
	Rspm	100	90
	So2	80	0
Vkia	No2	80	0
	Rspm	100	95
	So2	80	0

**Table 2:** For Year 2014

Site Name	Pollutant	Naaqs	Number Of Exceedances
Ajmeri Gate	No2	80	0
	RSPM	100	73
	SO2	80	0
Jhalana Dungri	No2	80	0
	RSPM	100	55
	SO2	80	0
Chandpole	NO2	80	0
	RSPM	100	80
	SO2	80	0
Riico Office	No2	80	0
	RSPM	100	54
	SO2	80	0
Vdn	NO2	80	0
	RSPM	100	87
	SO2	80	0
Vkia	NO2	80	0
	RSPM	100	90
	SO2	80	0

**Table 3:** For Year 2015

Site Name	Pollutant	Naaqs	Number Of Exceedances
Ajmeri Gate	No2	80	0
	Rspm	100	83
	So2	80	0
Jhalana Dungri	No2	80	0
	Rspm	100	70
	So2	80	0
Chandpole	No2	80	0
	Rspm	100	85
	So2	80	0
Riico Office	No2	80	0
	Rspm	100	69
	So2	80	0
Vdn	No2	80	0
	Rspm	100	84
	So2	80	0
Vkia	No2	80	0
	Rspm	100	87
	So2	80	0

**Table 4:** For Year 2016

#### **IV. Conclusion**

From all the above 3 tables, we observed that all monitoring sites satisfy the compliance criteria with the standard for the pollutant SO<sub>2</sub> and NO<sub>2</sub>. But for the other pollutant RSPM all monitoring sites fails to meet the compliance criteria with standard. So there need to be some steps taken regarding pollution control due to pollutant RSPM as it's out of control.

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