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The Three-Body Problem: Art, Science, Science Fiction And Epistemology To Comprehend The Nature

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I. The Three-Body Problem In Cixin Liu's Literature And The Netflix Series – An Ode To Science

The encounter between Physics and Cinema is something that draws the attention of the public and the Academy, both to hard physics and to its instrumental use in Physics Teaching. The encounter began in the 20th century, when, in 1902, the first science fiction film was released: "Le Voyage dans la Lune" (fig, 1), directed by Georges Méliès, inspired by the books of great science fiction writers, especially Jules Verne. Despite being a short film (13 min.), silent and in "black and white", it brought to the cinema the first speculations regarding the future of humanity. It portrays a trip from Earth to the Moon using a "projectile" as a means of locomotion, which is clearly inspired by the book "From the Earth to the Moon" by Jules Verne. The film addresses the surprise of humans upon arriving on the Moon and encountering extraterrestrial beings. This second part is inspired by the book "The First Men in the Moon" by H.G.Wells.

Fig. 1. "Voyage dans la Lune" – Georges Mèlies.

Source: MELIÈS, WIKIMEDIA, 2025a.

In the 1960s and 1970s, films such as Stanley Kubrick's 2001: A Space Odyssey (Neves *et al*, 2000) and Andrei Tarkowski's Solaris opened up possibilities for a deeper understanding of both science and the cinematic possibilities surrounding the nature of human knowledge. In the 21st century, Christopher Nolan's Interstellar (Ghizoni, Neves, 2018) created an icon of contemporary cinema that combines the science of cosmology with science fiction in doses that recall both 2001 and Solaris.

More recently, based on the work of Cixin Liu and amplified by a successful series produced by Netflix, science fiction has once again risen to the category of *grand cinéma* by exploring a story rooted in the physics and cosmology of Isaac Newton, with his *systeme du monde*, that is, the three-body problem.

This paper will divide the history of the three-body problem into three parts: Cixin Liu's book and the Netflix series; the physical question of the three bodies from Isaac Newton's perspective; and the question of chaos and chance in Ekeland's perspective. All parts have as a transversal theme: the question of the pendulum as a physical object and as an instrument of transdisciplinary reflection, involving science, art and science fiction.

II. The Three-Body Problem By Cixin Liu

Some years ago we wrote a paper entitled *Science fiction in physics teaching: improvement of science education and History of Science via informal strategies of teaching* (Neves *et al*, 2000).

In this paper we explore the potential of using science fiction, whether in literature or cinema, to explore issues such as conceptual errors, the history of science and epistemology, as well as, of course, the issue of the art-science relationship. A year ago we saw in Netflix channel the series *The Three-Body.* The film was born based upon the book with the same title, *The Three-Body Problem*, by writer Cixin Liu, originally published in parts in the Chinese magazine 科幻世界 - Science Fiction World (Fig. 2), throughout 2006, and collected in book form in 2008. The main theme of the book concerns a contact established between the Earth civilization and an alien civilization. This contact occurred when astrophysicist Ye Wenjie managed to send an electromagnetic signal to the Sun, which in turn re-sent it throughout space. According to her theory, our star worked as an antenna amplifying radio waves by about a hundred million times (LIU, 2016, p. 213 and p. 217). Since there was not enough time to switch the electrical systems connected to the radar antenna from transmitting mode to monitoring mode, immediately after sending the signal to the Sun, Wenjie asked a base station employee to tune the channel of a conventional military radio so that he could detect the wave. Unfortunately, nothing was recorded by this low-sensitivity device, which was usually only used by the base to communicate with the outside world.

Fig. 2. Cover of one of the issues of the magazine 科幻世界 - Science Fiction World, in which the work The Three-Body Problem was published, in 2006.

Source: LIU, 2016.

Wenjie lost hope that some extraterrestrial civilization would one day be able to read the message she had sent, the final part of which was as follows:

> [...] Driven by the best intentions, we wish to establish contact with other civilized societies in the universe. We wish to work together with you to develop a better life in this vast universe (LIU, 2016, p. 141 and p. 280).

To her surprise, however, about nine years after sending the above message, Ye Wenjie noticed something strange about the shape of the waves received by the signal monitoring system at the Red Coast Base, the research institute where she worked and where the radar antenna she had used to send the signal in 1971 was installed. By decoding those unexpected signals, Ye had the privilege of being the first person to read a message from space, which read:

> Do not respond!
> Do not respond!!
> Do not respond!!!
> This world has received your message.
> I am a pacifist. Your civilization is fortunate that I was the first to receive your message in this world. I am warning you: Do not respond! Do not respond!! Do not respond!!!
> There are tens of millions of stars heading your way. If you do not respond, this world will not be able to determine the source of your transmission.
> However, if you do respond, the source will be located immediately. Your planet will be invaded. Your world will be conquered!
> Do not respond! Do not respond!! Do not respond!!! (LIU, 2016, p. 222)

Even though she was stunned by the unusual event, Wenjie was calm enough to realize that the source of that transmission could only be about 4.5 light years away and, therefore, must have come from some planet connected to Alpha Centauri, the closest star to the Sun.

The transmission lasted four hours and was enough for Wenjie to learn about the existence of Trisolaris and the fact that the civilization on that planet was extinguished and reborn several times at irregular and unforeseen intervals of time. This occurred because the planet they lived on was subject to the gravitational force of three suns, which made the trajectory described by that planet around them unpredictable. In addition, she learned that, because of this peculiar inconsistency, the Trisolarans had decided to migrate to the stars in search of stable places to settle. They had already concluded that the planet itself was at serious risk of being torn apart or even swallowed by one of the suns.

However, the message sent by Wenjie had been picked up by a Trisolaran working at one of the thousands of listening posts designed to detect possible signs of intelligent life in the universe. For about two hours, he read on his signal-receiving screen that there was a planet Earth ruled by a single Sun and learned of the existence of a human civilization living in a paradise with an eternally stable climate. Thus, he realized that he was faced with a unique opportunity to do something that could bring some brightness to his humble life: to send a message that could save an entire civilization! His reasoning was that he was an ordinary being who lived at the lowest level of society, that no one paid attention to him, that he spent his life alone, without money, without status, without love, without hope. If he could save a beautiful and distant world that he had fallen in love with, then his life would not have been in vain. Hence the emphasis on not responding to his message.

Although the Trisolaran listener begged for no response, Wenjie believed that it was impossible to expect a spontaneous moral awakening from humanity, that human civilization no longer had the capacity to improve itself on its own. So she adjusted the system's transmission frequency and, without hesitation, once again sent a signal to the Sun to amplify and rebroadcast into space. This time, the message read:

> Come! I will help you conquer this world. Our civilization can no longer solve its own problems.
>
> We need your strength to intervene (LIU, 2016, p. 224).

After sending this message, Wenjie remained at the Red Coast Base for another two years, until she received a letter inviting her to teach at the university where she had previously taught. Six months after returning to Tsinghua University, Wenjie began to develop a plan for a large radio astronomy observatory.

While looking for a suitable location for the observatory, Wenjie met Mike Evans, the son of the president of a multinational oil company who had inherited four and a half billion dollars from his father. Having developed pan-species communism, the basic belief that all species on Earth were created equal, Evans moved to China to try to save a species of bird from extinction. He believed that the seed of pan-species communism had already germinated some time ago in the ancient East due to Buddhist philosophy, which gave importance to all that had life. However, he soon found that the Chinese forests were devastated like everywhere else in the world and ended up becoming completely disillusioned with humanity. It was then that Ye Wenjie revealed to Mike Evans the entire story she had experienced at the Red Coast Base, including her contact with an extraterrestrial civilization.

Three years later, Wenjie and Evans met again and he revealed that he had adapted an oil tanker to send and receive radio signals through a huge parabolic antenna installed on its deck, thus confirming all the information she had told him the last time they were together. Evans informed Wenjie that the Trisolaran Interstellar Fleet was already on its way to the Solar System, with an estimated arrival time of four hundred and fifty years, and he also introduced the Trisolaran Earth Organization (TEO), which had about two thousand people at that time. Finally, he invited her to be the commander-in-chief of the TEO, saying that the intention of its members was to ask the Trisolaran civilization to reform human civilization and suppress the madness and evil of humanity, so that the Earth could become a harmonious, prosperous and pure world. With the intention of recruiting people who had lost faith in Earth civilization, who hated their own species, who were willing to betray it and who even adopted the extinction of the human race as their ultimate ideal, OTT invested an immense amount of resources to develop a powerful software for a video game called Three-Body. The game was accessed through a V-suit, a piece of equipment widely used by the gaming community, consisting of a panoramic visor helmet and a sensory response jacket that simulated the environment of the alien planet. To achieve its objective, Three-Body served as the main channel for the dissemination of Trisolaran culture among players:

> The game explained Trisolaran culture and history through a facade that drew on elements of Earth history and human society, making it easier for beginners to understand. Once the player had advanced to a certain level and begun to admire Trisolaran civilization, the OTT would contact the player, examine the player's inclinations, and eventually recruit those who were approved to join the organization (LIU, 2016, p. 258).

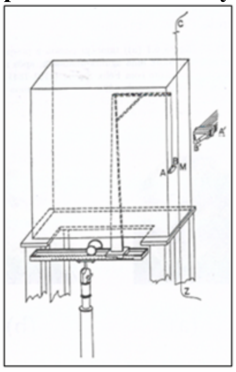
And it is through this extremely original literary resource that Cixin Liu will pay great homage to the history of the development of science. The author will take us to the virtual environment of the game in the company of Wang Miao, a physicist who worked at the Nanotechnology Research Center, more than forty years after Ye Wenjie had sent his first electromagnetic signal towards the Sun.

In fact, Wang had entered the game out of curiosity, after learning that Shen Yufei, one of the members of the organization Frontiers of Science, also played Three-Body. This organization had gradually formed, based on the various congresses and university meetings held throughout the year 2005, chosen by UNESCO as the World Year of Physics. This international group of famous and highly influential academics had as its central objective to use scientific methods to try to discover whether there is a limit to the development of science, since since the second half of the 20th century, physics had gradually lost the conciseness and simplicity of classical theories, making modern theoretical models and experimental verifications increasingly complex, vague and imprecise.

In addition to belonging to the Frontiers of Science, Shen Yufei was also part of the Earth-Trisolaris Organization, but deep down he believed in a less radical solution than the simple extinction of humanity. Perhaps he thought that the ideal solution would be to find a way to keep the Trisolarans on their home planet and prevent the invasion of Earth. Perhaps he naively believed that it would be possible to make the arrival of aliens unnecessary if he could find the general solution to the three-body problem, thus saving both Trisolaris and Earth. After all, that was what she and her boyfriend Wei Cheng were looking for, that is, to create a mathematical model that would allow the prediction of all the movements of the three bodies, whatever the initial configuration of the system.

After this long introduction, there are Wang Miao complex incursions in the game. In the first incursion appears the problem of the pendulum. As for pendulums, it is worth remembering that these devices were very useful for the development of modern science. Galileo Galilei (1564-1642) studied them extensively to investigate the free fall of bodies (GALILEI, 1988, p. 87-98) and Christiaan Huygens (1629-1695) built precise clocks based on the properties of pendulums and wrote an entire work on the subject (HUYGENS, 1986).

Fig. 3. Experimental apparatus built by Biot and Savart to measure the physical interaction between a small magnetized needle AB suspended by an insulating wire, close to an electrically conducting wire CZ, positioned vertically



Source: MATTHEWS, 2006.

With the help of pendulums, Isaac Newton (1642-1727) established the laws of collision and conservation of Mechanics, and was also able to demonstrate that celestial bodies were subject to the same laws as terrestrial bodies, thus unifying the Aristotelian supralunar and sublunar worlds (NEWTON, 1990; 2008; 2018). One of Newton's main biographers, the historian of science Richard S. Westfall, correctly stated that “without the pendulum, there would have been no Principia” (WESTFALL, 1990, p. 82 apud MATTHEWS, 2006, p. 2). The pendulum continued to be used throughout the subsequent centuries to develop various branches of Physics, such as Mechanics, Hydrodynamics, Electrostatics and Electromagnetism. The torsion and oscillation balances built by Charles Augustin de Coulomb (1736-1806), for example, helped him conclude, in 1785, that the electrostatic force was inversely proportional to the square of the distance between two electrified particles interacting with each other (ASSIS, 2022).

In 1798, Henry Cavendish (1731-1810) also used a torsion balance to determine the density of the Earth, an important problem since Newton had determined that the densities of the Sun, Jupiter and Saturn could be expressed in terms of the Earth's density and suggested that this value was five or six times greater than the density of the Earth if it were formed entirely of water (NEWTON, 2008, p. 206-208; CAVENDISH, 1921; CLOTFELTER, 1987, p. 210-211). In 1820, Jean-Baptiste Biot (1774-1862) and Félix Savart (1791-1841) observed the equilibrium position of a small magnetized needle, suspended by an insulating wire, placed next to another electrically conducting wire positioned vertically (Fig. 3). Then, they measured the periods of the small oscillations of the needle around this equilibrium position, for different distances between the suspended needle and the conducting wire. They then concluded from the experiment that the intensity of the magnetic force exerted by the wire on the magnetic molecules (north pole and south pole) of the needle was inversely proportional to the distance between the wire and these molecules and perpendicular to the plane defined by the straight line connecting each molecule to the wire and the axis of the wire. In other words, they used an oscillating pendulum to determine the physical interaction between a wire carrying an electric current and the needle of a compass (ASSIS, CHAIB, 2006).

The work on the movement of pendulums in fluids undertaken by George Gabriel Stokes (1819-1903), and published in 1850, led him to discover the law of viscosity, known today as Stokes' law (STOKES, 1850; 1901). A year after Stokes published his work, Jean Bernard Léon Foucault (1819-1868) invited the French scientific community "to see the Earth rotate in the Southern Hall of the Paris Observatory", by oscillating a 28 kg lead sphere, suspended by a 67 meter rope fixed to the ceiling of the hall (TOBIN, 2003; ACZEL, 2003; ACZEL, 2005 apud MATTHEWS, 2006, p. 2). However, the guests saw nothing more than the precessional movement performed by the pendulum, simply because it is impossible for anyone to observe the Earth's rotation while standing on its surface! In 1883, Ernst Mach (1838-1916) was perhaps the first to dispute that the precession of Foucault's pendulum could provide proof of the Earth's rotation, arguing that this interpretation presupposed an analysis from the point of view of a reference system other than the Earth itself. From the Earth's reference point, it is the pendulum that moves, therefore it is necessary to admit the existence of fictitious forces acting on it laterally, so that we can understand the reason for its precession movement (MACH, 1893, p. 231-232; ASSIS, 2013, p. 148-152, p. 157-159 and p. 173-177; GARDELLI, 2010, p. 590-597).

Regardless of the interpretation given to Foucault's experiment, what we intend to emphasize at this point is that once again scientists used the pendulum device to try to understand a specific physical phenomenon. In order not to dwell too much on the subject in question, we would like to present as a final example the case of the Hungarian scientist Loránd Eötvös (1848-1919). In his case, pendulums were used to establish the proportionality between inertial mass and gravitational mass, which later became better known with the formulation of the Equivalence Principle, proposed by Albert Einstein (1879-1955) in 1907 (HENTSCHEL, 2005, p. 167). Examples like these make us suspect that it was no coincidence that Cixin Liu chose the pendulum to spark the imagination of his readers with descriptions of such grandiose and unforgettable scenes. By using this experimental device in his first foray into the game (but also in others, as we will see later), Cixin Liu paid great homage to Science. As for the mention of a mythological figure, as we pointed out above, Cixin Liu refers to Fu Xi as an advisor to King Zhou. He is the proponent of using pendulums to appease the behavior of an animistic sun, whose moods are unpredictable. Nevertheless, neither Fu Xi nor King Wen were able to correctly predict the arrival of Stable Eras.

In the history of civilizations, it is known that the Aztecs also considered the Sun to be an animistic being, as they believed that the purpose of human beings was to feed it with their own blood, since, without this fluid, the Sun would run out and come to an end (SÉJOURNÉ, 1964, p. 20-21). It can be said that the human condition of constituting meaning for everyday events and experiences emerged from the development of beliefs by ancient peoples, initially expressed through magic, rituals and myths, whose study of their origins is capable of outlining a path through which human thought evolved (GOMES, 2004, p. 36). And it is precisely this path that Cixin Liu will follow when presenting us with the explanations that will be given for the problem of the three bodies throughout Wang Miao's incursions into the game.

There are more Wang Miao five incursions in the game.

According the text till now, it can be said that Netflix series treats the Three-Body Game as entertainment for viewers and not as a tribute to Science. All the richness provided by the mention of pendulums and kings belonging to Chinese history is wasted, preventing a deeper analysis of how the game evolves with each level that is overcome. It is no longer possible to appreciate all the human effort to understand nature through scientific thought. Therefore, it is possible to say that Cixin Liu's book is superior to the Netflix series, because in addition to being admirable and unforgettable, it manages to intertwine political, philosophical, religious and historical aspects. It is a story that was born a classic. Indeed, as the back cover of the book states, "The Three-Body Problem is a chronicle of the human march towards the ends of the universe".

III. Newton And Beyond

The study of gravity is part of the high school curriculum. For better or worse, Kepler's Three Laws and Newton's Law of Universal Gravitation are discussed (MACEDO, 2023). Based on this premise, our challenge is to explain to the reader, who only had contact with Physics in high school, why the three-body problem is only simple in appearance, and is considered one of the most complex problems in Physics (MOULTON, 1914; GOLDSTEIN, POOLE, SAFKO, 2001; VALTONEN, KARTTUNEN, 2005).

To do this, it is important to revisit some fundamental concepts of these theories. Let's start by recalling what Kepler's Three Laws say:

1. Law of Elliptical Orbits: This law tells us that the trajectory of each planet around the Sun describes an ellipse, with the Sun positioned at one of its foci. This implies that the distance between the planet and the Sun is not constant, but varies throughout its orbit.
2. Law of Areas: According to this law, an imaginary line connecting a planet to the Sun covers equal areas in equal intervals of time. This means that the speed at which a planet orbits the Sun is not constant: it decreases as the planet moves away from the Sun and increases as it approaches.
3. Harmonic Law: This law establishes a relationship between the orbital period of a planet (the time it takes to complete one revolution around the Sun) and its average distance from the Sun, indicating that the square of the

orbital period is proportional to the cube of the average distance from the Sun. This tells us that the further a planet is from the Sun, the slower it moves in its orbit.

Kepler (1571-1630) was unable to explain convincingly why planets have elliptical orbits, covering equal areas in equal times. Nor did he tell us why the particular distance-period relationship he discovered is valid. Lucie (1977, p. 113-114, emphasis added), reflecting on the reasons for Kepler's failure, makes the following statement:

> It is also true that Kepler did not have the strength to go from the three laws to the general theory that contains them. There are several reasons for his failure: firstly, he "geometrized" the wrong theory, that of regular polyhedra. Secondly, he could not have gone from the laws to the theory without a Physics of motion. And Kepler's physics, with its finite Universe, its erroneous concept of forces as producers of velocity and not of accelerations, was still Aristotelian; and finally, assuming that these obstacles were overcome, Kepler did not have the mathematical resourcefulness necessary to achieve that goal. So Kepler could not have arrived at the Law of Universal Gravitation. On the other hand, we can ask ourselves – gratuitously – whether Newton, without Kepler's Laws, would have discovered it.

At the end of the quote, the author asks whether Newton (1642-1727) could have discovered the Law of Universal Gravitation without Kepler's Laws. This is a controversy that we cannot go into here in this article. For more details on the possible paths that Newton took until the final presentation of his Theory of Universal Gravitation in his most famous book, *Philosophiae naturalis principia mathematica* (*Mathematical Principles of Natural Philosophy*, generally referred to as *Principia*, first published on July 5, 1687), we recommend reading Teixeira, Peduzzi and Freire Jr (2010). The fact is that, when examining Kepler's Laws in Principia, Newton (2012, 2016) showed that a body, initially following an inertial motion, when subjected to a central force, adopts a behavior that respects the Law of Areas. Consequently, a body moving in a curve obeying the law of areas is subject to a central force. In this same work, the English physicist also showed that, in order to satisfy Kepler's Harmonic Law, this central force must be centripetal and inversely proportional to the square of the distance between the body and the focus to which this force is directed. In the case of the Law of Elliptical Orbits, he demonstrated that if a body is subject to this central force, centripetal and inversely proportional to the square of the distance between the body and the focus to which this force is directed, the orbit will not only be elliptical (in particular, circular), but depending on the initial conditions, it may also be parabolic or hyperbolic. Furthermore:

> [...] by Law III [known as Newton's Third Law, or, Law of Action and Reaction], attractions are exerted in the direction of bodies, and the actions of the attracted bodies are always reciprocal and equal, so that if there are two bodies, neither the attracted nor the attracting body is really at rest, but both (by Corollary IV of the Laws of Motion), considering that they are mutually attracted, revolve around a common center of gravity. And if there are more bodies which are attracted by a body, which is again attracted by them, or which attract each other, these bodies will so move relative to each other, that their common center of gravity will be either at rest or moving uniformly in a straight line [...] (NEWTON, 2016, p. 223).

In this way, each planet describes an elliptical orbit not around the Sun, but around the common center of mass of both. The Sun, in turn, does not remain fixed at the focus of this ellipse; it also rotates around this same center of mass. Evidently, since the mass of the Sun is much greater than the mass of any other planet in our system, its movement is almost imperceptible (NEWTON, 2012).

According to Cohen and Westfall (2002), Newton's greatest contribution was to demonstrate that this central force is present in both celestial and terrestrial bodies, and is therefore universal. Furthermore, it is only attractive, directly proportional to the masses of the bodies that interact with each other and inversely proportional to the square of the distance between their centers. Today we know it as the Universal Gravitational Force, whose magnitude can be found by the following equation:

where F is the magnitude of the Universal Gravitation Force; m_1 and m_2 are the masses; r is the distance; and G is the constant of proportionality, often called the universal gravitational constant.

Also in the *Principia*, after presenting his *Theory of Universal Gravitation*, Newton (2012) used it to explain several phenomena that were already known (with the exception of the flattening of the Earth's poles), but which still did not have a theoretical explanation consistent with the observed data, such as:

- Variations in planetary orbits: although the planetary orbits in our solar system are approximately elliptical (almost circular), they are not perfect and unchangeable. Newton explained that gravitational interactions between the planets cause disturbances in their orbits, resulting in variations in the shape, size, and orientation of the orbital ellipses over time. One of the most notable examples of orbital variations is the interaction between Jupiter and Saturn. Newton showed that the mutual gravitational forces between these two planets cause variations in their orbits around the Sun, resulting in periodic changes in the eccentricity and inclination of their trajectories;

- Flattening of the Earth's poles: Newton, in a pioneering move, predicted and theoretically calculated the flattening of the Earth's poles, due to the Earth's rotation. The Earth rotates around its axis, which generates a centrifugal force that acts perpendicular to the axis of rotation. This force is maximum at the equator and zero at the poles. At the same time, the gravitational force, which attracts all points on the Earth towards its center, acts to counterbalance the centrifugal force. The interaction of these two forces results in a flattening at the poles and a bulging at the equator;

- Precession of the Equinoxes: the precession of the equinoxes is a gradual movement in the orientation of the Earth's axis of rotation, which alters the position of the stars in relation to the equinoxes and solstices over time. This phenomenon was observed and documented by ancient astronomers, but its explanation was only provided with Newton's gravitational theory. The Earth is not a perfect sphere, but rather an oblate spheroid due to its rotation. This bulge at the equator is susceptible to the gravitational forces exerted by the Moon and the Sun. The gravitational pull that these celestial bodies exert on the Earth's equatorial bulge generates a torque, which tends to "pull" the Earth's axis of rotation. The torque resulting from the gravitational interaction with the equatorial bulge is not capable of significantly altering the tilt of the Earth's axis relative to the orbital plane; instead, it causes the Earth's axis to trace a cone in space, a motion known as precession. This precession motion means that the point on the celestial sphere toward which the Earth's axis points (currently near the North Star) changes slowly over time. The complete cycle of the precession of the equinoxes takes approximately 26,000 years to complete. This means that the orientation of the Earth's axis relative to the fixed stars gradually changes over this period, affecting the apparent position of the stars, the constellations, and the time of year at which the equinoxes and solstices occur;

- Irregularities in the Moon's orbit around the Earth: Newton showed that the Sun's gravitational force exerts a significant influence on the Moon's orbit, causing various irregularities, such as the precession of this orbit and variations in the Moon's speed and position throughout its orbit;

- Explanation of the occurrence of tides: Newton was the first to provide a convincing scientific explanation for tides, he showed that tides are caused primarily by the Moon's gravitational force, with an additional contribution from the Sun. The Moon's gravitational force exerts a pull on the Earth and its oceans. This force varies with distance, being strongest on the side of the Earth closest to the Moon and weakest on the side opposite. Although the Moon has the greatest influence on tides, the Sun also exerts a significant gravitational force. Newton explained that the Sun's contribution to tides is about half that of the Moon. The interaction between the gravitational forces of the Moon and the Sun creates different types of tides;

- The motion of comets: Newton demonstrated that comets follow conical orbits around the Sun, which can be elliptical, parabolic, or hyperbolic. He also considered that gravitational interactions between comets and other celestial bodies, such as planets and the Sun, can alter the orbits of comets. For example, Jupiter, with its great mass, can capture comets in elliptical orbits or eject them from the Solar System. Newton's approach to explaining the motion of comets was validated by later astronomical observations. The accurate prediction of the orbits of comets, such as that of Halley's Comet, was a confirmation of the universal applicability of Newton's laws.

Despite the triumphs of Newton's universal gravitation, it remained a plausible hypothesis for about a century. During this period, the theory faced a sea of anomalies. According to Cindra (1996, p. 54):

> […] due to the insistence of the Newtonians, the difficulties were eventually overcome. Euler and Lagrange managed to demonstrate that the variations observed in the orbits of Jupiter and Saturn were not secular. They were, in fact, periodic, but with long periods of duration. And Laplace, using the theory of perturbations (1787) managed to present a complete mathematical explanation for the anomalous acceleration of the Moon. In the mid-19th century, the Newtonian theory of gravitation convincingly demonstrated its effectiveness, especially after the work of Le Verrier and Adams, leading to the discovery of the planet Neptune (1845-1846).

After this preamble, we can delve deeper into the analysis of the three-body problem, which can be defined, without much rigor, as follows:

> *Let us consider a system that has three point masses subject only to mutually exerted gravitational actions, according to Newton's Law of Gravitation. Given the initial positions and velocities of each body, is it possible to calculate, exactly and analytically, these values at any given time (past or future)?*

The great beauty of this problem is its apparent simplicity. After all, through Newton's analysis of Kepler's Laws, it is clear that the answer to this question is affirmative when the system is composed of only two bodies (ANTUNES, 1975; SYMON, 1982). However, for situations involving more bodies, for example, the Sun-Moon-Earth system, he did not arrive at an analytical solution, much less an accurate one. For these cases, based on his Theory of Gravitation, Newton offers only some reflections and calculations to estimate the effects of the disturbances caused by a third body in the orbit of two bodies that attract each other, for example, in Proposition XIII. In Theorem XII of Book III, he does this to show the degree of disturbance of Jupiter in the orbit of Saturn, around the Sun. In his words:

> [..] there arises, therefore, a disturbance of Saturn's orbit at every conjunction of this planet with Jupiter, so perceptible that astronomers are puzzled by it. As the planet is differently located in these conjunctions, its eccentricity is sometimes increased, sometimes decreased, its aphelion is sometimes carried forward, sometimes backward, and its mean motion is alternately accelerated and retarded [...] (NEWTON, 2020, p. 210).

From these considerations, anyone who encounters the three-body problem for the first time might be led to believe that the difficulty in solving it lies in the gravitational disturbances external to the system, as happens in the two-body problem (Sun-Saturn, for example). However, this is not the case, since the problem statement assumes that the only interactions to be considered are the gravitational actions mutually exerted by the three bodies. According to Laplace (1825, p. 304, our translation, emphasis added):

> It is in Euler's first work, on the movements of Jupiter and Saturn, that we must refer to the first research on the disturbances of planetary movements. This work, awarded a prize by the Academy of Sciences in 1748, was delivered to the secretariat of this Academy on July 27, 1747, a few months before Clairaut and d'Alembert communicated to the Academy the similar research they had carried out on the three-body problem, which they called so because they applied their solutions to the motion of the Moon attracted by the Sun and the Earth. However, the differences in their methods compared to those of Euler prove that they had not borrowed anything from his work. It was printed in 1749, the year in which d'Alembert's work on the precession of the equinoxes was also published, which makes this year notable in the history of celestial mechanics.

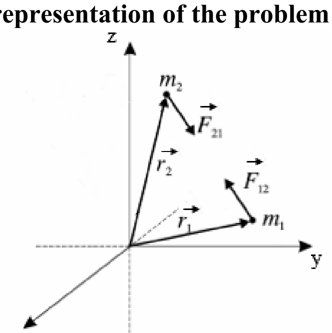
In other words, the first attempts to find more precise solutions to the three-body problem are attributed to Euler, but the people who coined this term were Clairaut (1713-1765) and d'Alembert (1717-1783). Since then, other great physicists and mathematicians have dedicated themselves to this task.

Returning to our previous reflection, if the difficulty in solving the three-body problem does not lie in the disturbances external to the system, which are disregarded, where does it lie then? To answer this question, we will explain, in general terms, how the solution to the two-body problem (also known as Kepler's problem) is obtained. Next, we will present what changes with the introduction of a third body into the system. We will pose the two-body problem as follows:

> *Two bodies with point masses move due only to mutual gravitational interaction. Given the initial positions and velocities of each body, obtain the positions and velocities of each body at any other instant in time.*

As a first step in solving the problem, we will represent these bodies in an inertial reference system xyz (fig. 4):

Fig. 4. General representation of the problem of two bodies.



Source: Authors

To obtain a general solution to the problem, we must represent it vectorially. In this case, we have:
- $\boldsymbol{r_1}$ is the position vector of the body with mass m_1;
- $\boldsymbol{r_2}$ is the position vector of the body with mass m_2;
- $\boldsymbol{F_{12}}$ is the vector of the gravitational force that mass m_2 exerts on m_1;
- $\boldsymbol{F_{21}}$ is the vector of the gravitational force that mass m_1 exerts on m_2.

The equations of motion of the bodies with masses m_1 and m_2 are obtained by applying Newton's Second Law to each body, remembering that, by Newton's Third Law, the gravitational force that mass m_2 exerts on m_1 is equal, in magnitude, to the gravitational force that mass m_1 exerts on m_2.

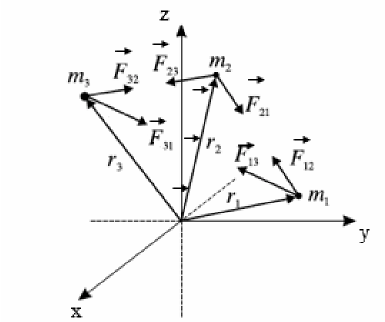
In applying Newton's Second Law, we will deal with the instantaneous acceleration of each body in the x, y and z directions. This acceleration is obtained from the temporal variation of the instantaneous velocity of each body in the x, y and z directions, which, in turn, is calculated from the instantaneous temporal variation of the x, y and z coordinates of each body.

In high school, we do not have the necessary mathematical tools, such as derivatives, integrals and differential equations, to solve this type of situation. Therefore, without the possibility of demonstrating here, in order to obtain the six coordinates (three for each body) and the six velocity components (three for each body) as functions of time, we need to find the solution for a system that has more unknown variables than the conditions necessary to solve the equations obtained. However, there are some physical principles that are used to reduce the number of variables, making the system solvable. They are:

- Conservation of total mechanical energy, total linear momentum and total angular momentum: this occurs because we are dealing with an isolated system subject only to conservative forces;
- Movement of the center of mass: the center of mass of the system moves with a rectilinear and uniform velocity or is at rest in relation to the inertial reference frame.

Using these principles and considering the artifice of changing the origin of the system to one of the masses or to the center of mass, it is possible to find a general and exact solution to the problem. Let us now see what happens when we introduce just one more body into this system (fig. 5).

Fig. 5. General representation of the three-body problem



Source: Authors

By inserting a third body, we add three more coordinates and three velocity components as functions of time to be calculated. Consequently, we will have to apply Newton's Second Law once more. However, this increase in the number of unknown variables is not accompanied by an increase in the conditions necessary to solve the equations, since there are no new physical principles to be used, other than those already mentioned in the two-body problem, that can reduce the number of these variables. Therefore, even using the trick of changing the origin of the system to one of the masses or to the center of mass of the system, it is not possible to find a general and exact solution to the problem.

This intriguing mathematical complexity, of a problem that is simple to state, has kept many mathematicians and physicists awake at night, from Newton to the present day. The search for its solution was even the prize of a competition announced in the famous *Acta Mathematic Journal* in 1885 by the King of Sweden and Norway, Oscar II (1829–1907). The central question was expanded, requesting the resolution of the n-body problem, where n is a number equal to or greater than 3, and is thus stated:

> Given an arbitrary system of several point masses that attract each other, according to Newton's Laws, try to find, under the assumption of no collisions, a representation of the coordinates of each point as a series in a variable that is some known function of time, and for all values the series must converge uniformly (VALTONEN et al., 2005, p. 134, our translation).

The prize was won by the Frenchman Henri Poincaré (1854-1912). According to Valtonen *et al.* (2005) and Barrow-Green (1997), he adopted a simpler form of the three-body problem[1], considering three masses: the first very large, the second small compared to the first, and the third negligible. The two largest masses described a circular motion around their center of mass, while the third moved in the same plane as the other two, its motion being disturbed by the larger ones. However, the presence of the third mass did not

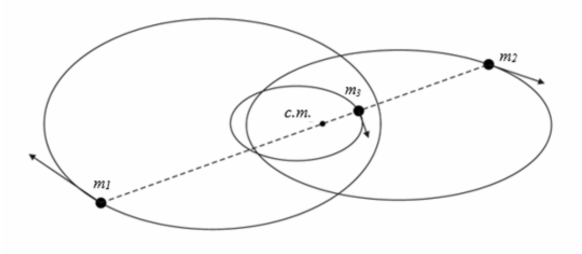
[1] Any simpler form of the three-body problem is known as a restricted three-body problem. In this case, the problem is not approached in its general form, but rather with the adoption of some initial restriction, such as, for example, the three bodies orbiting in the same plane or one of the masses being significantly smaller than the other two (MARCHAL, 1990).

influence the movements of the two largest. Thus, the movements of the larger masses were known, through the two-body problem, remaining to determine the coordinates of the position and velocities at each instant of the body of negligible mass. Despite the simplification given to the problem, Poincaré demonstrated that it was not possible to find these data, exactly, for the third body. If the answer to this restricted case were discovered, then he could try to generalize the calculation procedures used to find the solution to the more general three-body problem. Thus, Poincaré won the prize not for solving the problem, but for showing that there is no complete solution to it.

In the work that Poincaré submitted to the competition, he used some mathematical procedures that showed that small variations in the initial configuration of the three bodies led to orbits with large variations in behavior, which implies unpredictability, even in deterministic systems. In this way, he showed that deterministic systems do not necessarily imply predictable and regular behavior. This phenomenon is now known as sensitivity to initial conditions, which is one of the ways of characterizing a chaotic system. For this reason, this study by Poincaré is considered the initial milestone of Chaos Theory (BARROW-GREEN, 1997).

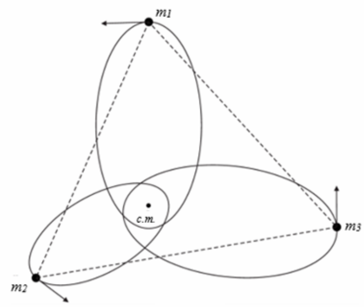
Regarding analytical solutions, they were found for some specific cases of the restricted three-body problem, in which certain particular conditions are imposed in each situation. Among the solutions obtained, those classified as homographic solutions stand out. According to Fernandes and Mello (2016), a homographic solution is one in which the configuration of the bodies at an instant t (in relation to the center of mass of the system) remains similar to itself when t varies. In other words, the ratios of the mutual distances between the bodies remain constant. In 1767, Euler (1707-1783) found three families of periodic solutions in which the three masses are collinear at each instant (Fig. 6). In 1772, Lagrange (1736-1813) found a family of solutions in which the three masses form an equilateral triangle at each instant (Fig. 7). These solutions are valid for any mass ratios and are classic examples of homographic solutions.

Fig. 6. Homographic solutions due to Euler. Each body describes an ellipse with a focus at the center of mass c.m. and, at each instant, the three bodies are on the same line



Source: Adapted from Fernandes, Mello (2016)

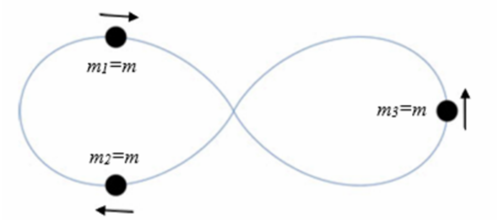
Fig. 7. Homographic solutions due to Lagrange. Each body describes an ellipse with a focus at the center of mass c.m. and, at each instant, the three bodies are on the vertices of the same equilateral triangle



Source: Adapted from Fernandes, Mello (2016)

According to Valtonen *et al* (2005), for a long time these were the only known complete solutions to a restricted three-body problem. However, in 1993, Moore (1993) discovered, through computational numerical calculations, that three equal masses, in the same plane, can chase each other around the same figure-of-eight curve (Fig. 8). In 2000, Montgomery and Chenciner (2000) found an exact solution to the equations of motion of these three bodies.

Fig. 8. Three equal bodies chasing each other in a stable figure-of-eight orbit



Source: Adapted from Valtonen *et al* (2005)

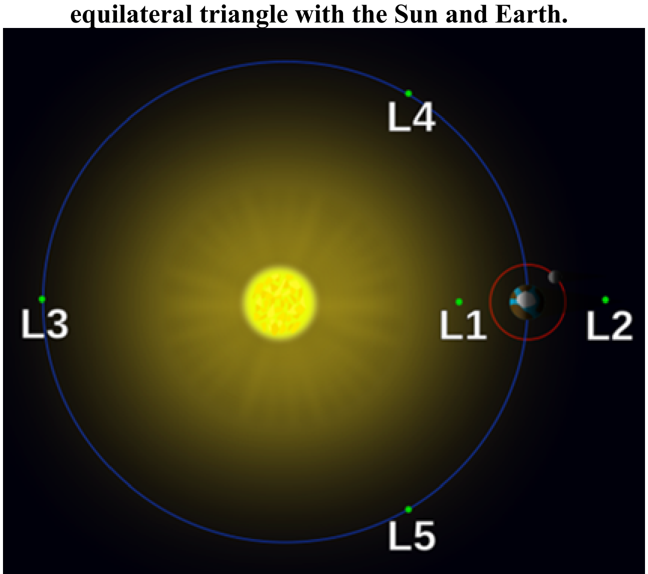
This case is part of a class of solutions often called choreographic, which are distinguished by the specific characteristic that the bodies follow the same trajectory, in a plane, and are equally spaced in time (CHENCINER, 2007; MARCHESIN, PRATES, LEITE, 2018; QUARESMA, RODRIGUES, 2019; VALTONEN *et al,* 2005). According to Marchesin, Prates and Leite (2018), hundreds of choreographies have already been discovered. Furthermore, the number of "distinct" choreographies increases rapidly with N, the number of bodies involved in the problem. However, the existence of many of these solutions still requires analytical proof. Another crucial question is the stability of these solutions, so that they have astronomical relevance.

Another class of restricted three-body problem that has been widely studied is the so-called restricted circular three-body problem, which consists of assuming that:

> [...] the mass m of a body is much smaller than the masses M1 and M2 of the other two; that is: M1 ~ M2 >> m, so that the gravitational effects of m on M1 and M2 can be neglected. Thus, the motion of the two bodies with larger masses is not significantly influenced by the third, but certainly the motion of m is influenced by the others. As examples of systems in which this approximation is reasonable, we can mention the Sun-Earth-Moon systems [...], and Earth-Moon-artificial satellite [...]. In this approach, the original problem is divided into two simpler ones. Initially, the motions of M1 and M2 are determined analytically, as in the two-body problem, which allows finding a resulting gravitational field due to these two bodies at each point in space. Then, the motion of m subject to this resulting gravitational field is calculated. A new simplification can be made by imposing that the movement of m is restricted to the plane of the trajectories of M1 and M2 [...] (MARTINS, ZANOTELLO, 2018, p. 2).

In 1772, Lagrange examined the restricted circular three-body problem, considering the scenario in which M2 is larger than M1 and both describe circular orbits around the center of mass of the system. He identified the existence of five equilibrium positions, known as Lagrange points. Each pair of celestial bodies (Sun-Earth; Earth-Moon; Saturn-Sun; Jupiter-Saturn; etc.) has its own (and different) set of Lagrange points. Figure 6 shows the Lagrange points for the Sun-Earth pair (figure 8).

Fig. 9. Five Lagrange points, L1, L2, and L3 lie along the Sun-Earth axis. Points L4 and L5 form an equilateral triangle with the Sun and Earth.



Source: Adapted from Onody (2024)

Lagrange points are of significant importance in celestial mechanics and astrophysics. The L1, L2, and L3 points are unstable equilibrium points, meaning that spacecraft or satellites positioned at these locations will need to perform constant maneuvers to maintain the same position. According to Onody (2024, unpaginated):

> [...] The L4 and L5 points are stable as long as the ratio between the masses of the two massive bodies is greater than 24.96 (which is the case in the Earth-Sun system). Due to this stability, it is at the L4 and L5 points that we find Trojan asteroids. They accompany a planet (any planet), following the same orbit as it. Some go ahead, others follow behind the planet. Almost all the planets in the solar system have their Trojan asteroids. Earth has 2 known Trojan asteroids: 2010TK7, which has a diameter of 300 meters, and 2020XL5, whose diameter is 1.2 km. Both follow ahead of Earth close to the L4 point.

In summary, the study of the three-body problem has contributed significantly to the advancement of science, especially in the understanding of planetary systems, the dynamics of celestial bodies, and the development of chaos theory. The search for approximate solutions and advanced numerical methods continues to be an active field of research, with implications that extend from celestial mechanics to theoretical physics. Understanding the limitations imposed by the three-body problem is essential for elementary physics teachers. In addition to being a topic rich in physical concepts, it also provides an opportunity to discuss the nature of science and the limits of determinism. By presenting this problem to students, teachers can show that even the most well-established theories, such as Newtonian mechanics, have limits, and that the search for answers often leads us to new questions and challenges.

IV. Epistemological Implications

Ivar Ekeland, discussing about the orbits' disturbances, states that:

> Lalande and Clairaut calculated that the disturbances caused by Jupiter and Saturn would delay the return of Halley's Comet by one year and eight months, whose appearance they announced for mid-April 1989, getting it right to within a month; the comet appeared as they had predicted and passed the indicated point on 1759 March. Adams, in 1845, and Le Verrier, in 1846, explain the inequalities observed in the trajectory of Uranus since its discovery in 1781, attributing them to the presence of an unknown planet, and calculate the elements of its trajectory. On September 18th, 1846, Le Verrier wrote to a Berlin astronomer, Galle, to communicate the coordinates of the planet. Upon receiving the letter, Galle focused his telescope on the constellation Aquarius, at the indicated location, and on the day 25 he replied to Le Verrier: "Sir, the planet whose position you have indicated to us really exists." All these successes had immense resonances in their time, as those that ours, space exploration has taken place, from the first Sputnik in 1957to the Apollo mission in 1969, which made it possible to realize one of humanity's oldest dreams. These are therefore experimental verifications of Newton's law, and it can be said that few laws of physics have been so well confirmed today: it and only it regulates the movements of the solar system. The progress of science has forced it to be slightly modified to take into account the theory of general relativity (Einstein, 1915), but this relativistic correction is very slight: its most important effect is an inequality of 42 seconds per century in the movement of Mercury.
>
> Will it be enough to have understood the mechanism to predict the behavior? The history of astronomy seems to answer affirmatively. Today we have at our disposal ephemerides spanning 44 centuries, that is, we know precisely the positions of the planets and the Moon for more than four thousand years; these can be used, for example, to identify eclipses and astronomical phenomena provided by antiquity, and thus to specify certain historical dates. It would be difficult to find irregularities in the solar system or discover that chance intervenes in the movement of the planets. If we are to predict the future, one of the few things we can say without fear of being wrong is that the Sun will rise tomorrow, that is, that the Earth will follow its path on its orbit, discovered by Kepler.

And follows:

Nothing of the sort: all that is nothing more than an illusion, not optical, but of scale. **We now know that the solar system is chaotic,** but its characteristic time—remember, it is the time at the end of which a small disturbance is multiplied tenfold—is of the order of ten million years. This is very long on our scale. After all, ten million years ago we were still in the Tertiary era, and there were no human beings on Earth. However, it is short on the scale of the solar system, which was formed about five billion years ago, or 500 times the characteristic time, and probably has as many more to go. What humanity has been able to perceive during its brief existence is an instant of this long period of time, and not its development; a photo, and not a film. Let us consider that since its discovery by Le Verrier 150 years ago, the planet Neptune has not had time even to complete one revolution around the Sun!

The chaotic character of the solar system is linked to the names of two of the greatest mathematicians of this century: the Frenchman Henri Poincaré (1854-1912) and the Russian André Kolmogorov (1903-1987). One after the other, they devoted themselves to the equations of celestial mechanics, and the resulting work forms the basis of the modern theory of dynamical systems. Poincaré discovered certain situations that necessarily lead to chaotic motion. Kolmogorov identifies others that lead to the opposite, a stable and perfectly predictable motion: trajectories oscillate indefinitely around certain mean positions, and small perturbations do not amplify in the course of the motion, which allows predictions to be made over the very long term. The question then is whether the truth lies in the situation described by Poincaré or in the one proposed by Kolmogorov.

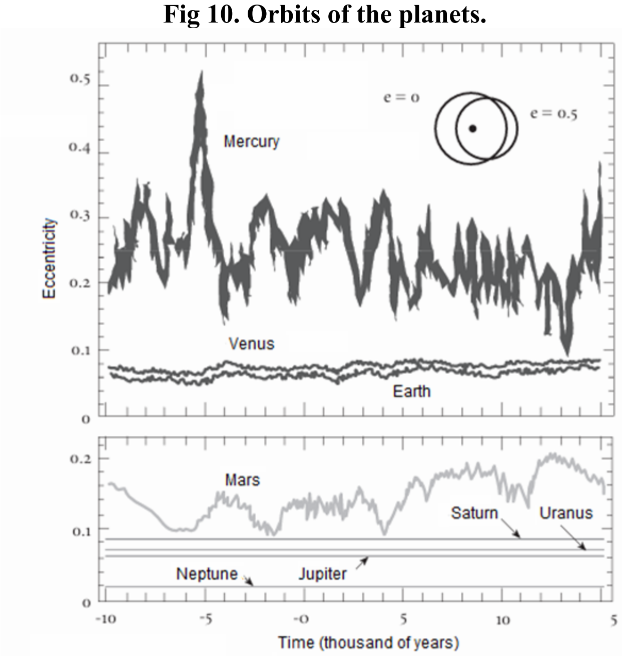
The answer lies in an extremely long and delicate calculation, beyond the reach of artisanal means such as paper, pencil, logarithm tables and mechanical calculators. For Laplace and for Le Verrier, with all their genius and their great patience, the extreme limit of predictability is of the order of a million years. Moreover, one can speak of a fairly vulgar predictability, referring to the main elements of the orbits: if it is a question of calculating the precise positions of the planets, the horizon is much closer, since the best ephemerides available today only cover 44 centuries. However, we now know that instabilities only appear on a much longer time scale, of the order of a hundred million years. It was necessary to wait for recent advances in numerical techniques and computer equipment to finally be able to follow the solar system with a certain precision in time intervals of this order.

To understand the results of these simulations, it is good to keep in mind the structure of the solar system. According to our time scale, the planets describe ellipses, in which the Sun occupies one of the foci. These ellipses are located more or less in the same plane, and the planets describe their orbits in the same direction. There are nine of them: the inner planets (Mercury, Venus, Earth, Mars) and the outer ones (Jupiter, Saturn, Uranus, Neptune, Pluto). [fig. 10].

Pluto plays a particular role because it has a very low mass, which probably makes it an old satellite of Neptune, and it has no influence on the rest of the system. Jupiter is by far the largest planet, followed by the other outer planets (with the exception of Pluto). Between Mars and Jupiter, there are no planets but an asteroid belt, made up of a myriad of fragments of all sizes that orbit the Sun. In 1988, G. J. Sussman and J. Wisdom of the Massachusetts Institute of Technology (MIT) followed the outer planets numerically for 875 years using a supercomputer designed for this purpose, and demonstrated that Pluto's motion is chaotic, with a characteristic time of the order of 50 million years. In 1989, Jacques Laskar of the Bureau des Longitudes followed the solar system for 200 million years, and showed that the motion of the inner planets (which includes our own) is chaotic, with a characteristic time on the order of ten million years. An inaccuracy of one centimeter in the position of these planets becomes an inaccuracy of a million kilometers at the end of 200 million years, which obviously does not allow any prediction on this time scale. Chance comes in here because our measurements cannot distinguish initial positions or velocities, and, moreover, with very different trajectories.

Deformation of orbits. These figures show how the orbits of the planets deform over time. The absciss axis (bottom) represents time, from ten billion years in the past to fifteen billion in the future; the ordinate axis (on the left) represents eccentricity: the closer one is to 0, the closer the orbit is to a perfect circle. At the top right, the extreme cases are represented, e = 0 (perfect circle) and e = 0.5 (ellipse). It can be seen that the orbits of the outer planets (except Pluto) are almost circular, and that the orbits of Mars, and especially Mercury, undergo significant variations.

Fig 10. Orbits of the planets.



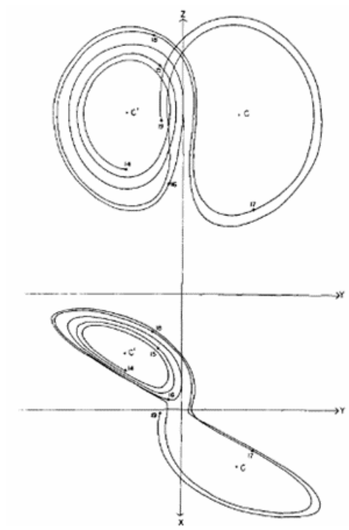
Source: EKELAND, 2001.

Although the word chaos directly refers to the idea of disorder, in Physics the concept of chaos is well specified and quantified. There are numerical measurements and mathematical proofs, depending on the system being studied, that characterize such systems as being chaotic.

Chaotic systems have unpredictability as one of their characteristics. The root of this unpredictability lies in the sensitive dependence on the initial conditions, that is, an experiment cannot be faithfully repeated to its first execution, because even with the greatest efforts in trying to provide the same initial situation as the previous experiment, an imperceptible difference will cause the result to be significantly different.

The expression "butterfly effect" is often used to refer to this characteristic of chaotic systems and has become widely known, but its meaning is not so clear to most people. The origin of the expression is unknown. It may have originated due to the "discovery" of sensitivity to initial conditions made in a weather forecasting context by Edward Lorenz in the 1970s: "the flapping of a butterfly's wings in Asia could, in theory, cause a hurricane in Europe" (CAMARGO, NEVES, 2009). It may also be a reference to the Lorentz attractor (1963) which refers to a graph in the shape of a butterfly's wings (fig. 11). We would also venture to say, in a reference to Art-Science, that the name would be based on the short story by the American sci-fi writer Ray Bradbury, "A Sound of Thunder" (BRADBURY, 1993), where a small butterfly plays a fundamental role in the course of History (CAMARGO, NEVES, 2007).

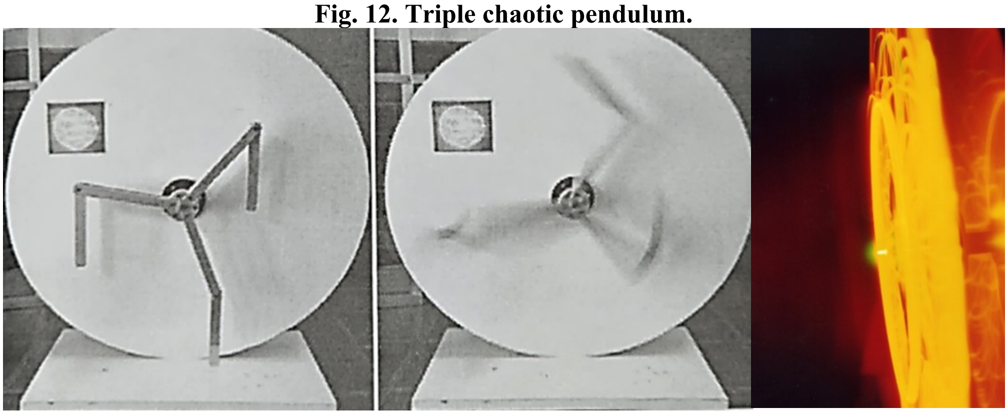
Fig. 11. Numerical solution of convection equations



Source: LORENZ, 1963.

We decided, as a way of producing an analogy, to build a triple pendulum, as illustrated in fig. 12. We installed LED lamps at the tip of each pendulum (including one in the center). The result is what can be seen in the series of photographs in fig. 15.

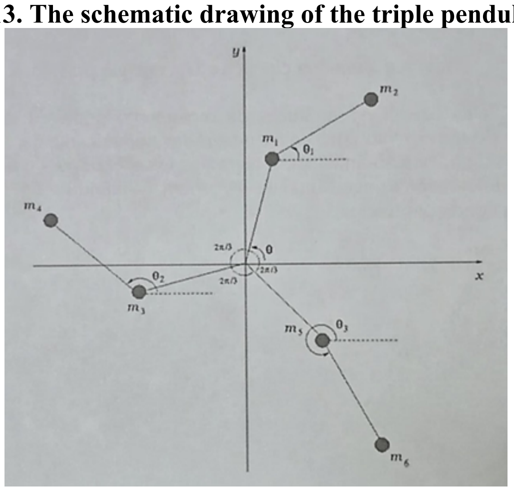
Fig. 12. Triple chaotic pendulum.



Source: Authors.

The triple chaotic pendulum is therefore composed of a three-armed helix (with an angle of 120° between the arms, figs. 12, 13), fixed to a wooden support and with additional mobile arms at each end of the helix.

Fig 13. The schematic drawing of the triple pendulum.



Source: Authors.

To simulate the trajectories of the triple pendulum, we write the coordinates of each arm of the pendulum, according to the fig. 13:

$$x_1 = l \cos(\theta)$$
$$y_1 = l \operatorname{sen}(\theta)$$
$$x_2 = x_1 + l \cos(\theta_1)$$
$$y_2 = y_1 + l \operatorname{sen}(\theta_1)$$
$$x_3 = l \cos(\theta + 2\pi/3)$$
$$y_3 = l \operatorname{sen}(\theta + 2\pi/3)$$
$$x_4 = y_3 + l \cos(\theta_2)$$
$$y_4 = y_3 + l \operatorname{sen}(\theta_2)$$
$$x_5 = l \cos(\theta + 4\pi/3)$$
$$y_5 = l \operatorname{sen}(\theta + 4\pi/3)$$
$$x_6 = x_5 + l \cos(\theta_2)$$
$$y_6 = y_5 + l \operatorname{sen}(\theta_2)$$

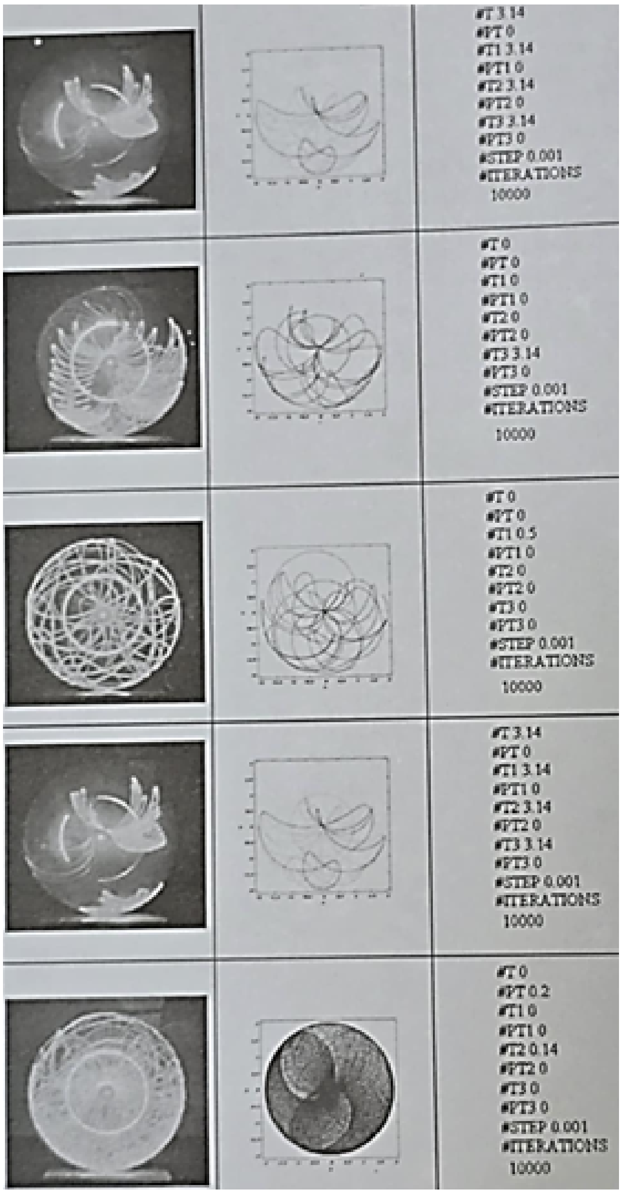
To determine the equations of motion, we calculate the Lagrangian L:

$$L = K - U$$
$$p_{\theta 1} = \partial L / \partial \dot{\theta}_1$$
$$\dot{p}_{\theta 1} = \partial L / \partial \theta_1$$

where K is the cinetic energy and U the potential energy.

Using the canonical equations of motion, we can determine the equations of motion and then integrate them numerically, since there is no analytical solution. We integrate these equations numerically with a 4th order Runge-Kutta integrator , using different initial conditions, and the result can be seen in fig. 14.

Fig. 14. equations numerically with a 4th order Runge-Kutta integrator



Source: CAMARGO & NEVES, 2009, p. 41.

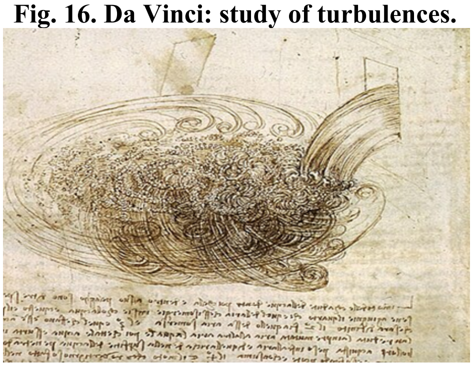
We can see that the numerical simulation reproduces in an analogous way what we can appreciate in reality in the experiment carried out, as shown in the photographs in fig. 15 (CAMARGO, NEVES, 2009).

Fig 15. The dynamics of the triple chaotic pendulum as a function of the time.

Source: Authors

The image of three pendulums interacting in a dynamic where determinism does not prevail, but simply chance, also resembles, in an Art-Science reading, Leonardo da Vinci's conceptions when describing atmospheric phenomena such as winds and typhoons (fig. 16).

Fig. 16. Da Vinci: study of turbulences.



Source: DA VINCI, WIKIMEDIA, 2005.

Probably, in addition to numerical methods, it is necessary to think of a "new mechanics", as André Assis does, with his relational mechanics.The **three-body problem** and **relational mechanics** are linked through their focus on the dynamics of systems involving multiple interacting bodies.

In the three-body problem, the motion of three celestial bodies is influenced by their mutual gravitational forces. This results in complex, often chaotic trajectories that are difficult to predict analytically. The problem highlights the importance of understanding the relationships between the bodies and their relative positions and velocities.

Relational mechanics, developed by Prof. André Koch Torres Assis (1999, 2013, 2014, 2022), takes this idea further by emphasizing the relationships between objects rather than their absolute positions. Assis' theory is based on Mach's principle (ASSIS, 1989), which suggests that the inertia of a body is influenced by the distribution of all other masses in the universe. This approach provides a new perspective on the dynamics of systems, including the three-body problem, by focusing on the relative distances and velocities between bodies.

In essence, relational mechanics offers a framework for understanding the complex interactions in the three-body problem by considering the relationships between the bodies rather than their individual motions.

Assis wrote:

> The main point of this paper was the introduction of the postulate which asserts that the sum of all forces of any kind acting on a body is zero, together with the use of a Weber force law for electric and gravitational interactions. In this model we have found that all inertial forces are in fact gravitational forces due to the interactions of any body with the isotropic distribution of matter around it. As this is a relational theory, Mach's ideas have

> been implemented and the role of inertial frames of reference have been clarified and identified with frames which are nonaccelerated relative to the "fixed stars." The greatest limitation of this model is that it is based on an action-at a distance theory (...). As a result, it is not a definitive or final theory but should be valid in systems with slowly varying motions in which time retardation is not a serious factor. A theory which involves the generation and propagation of gravitational waves will be presented in a future paper, but it can be mentioned here that the structure of equations (....) strongly suggests that the velocity of gravitational waves will be the velocity of light or will be equal to c/(x)½. These equations also suggest that the generation. propagation, and detection of gravitational waves should be similar to the case of electromagnetic waves, though it should be noted that in general we only have gravitational masses of the same kind, whereas in electromagnetism we readily obtain positive and negative charges. So, there should be no dipole radiation of gravitational waves. A very interesting proposal to extend Weber's law to include electromagnetic radiation, through the introduction of time retardation, was made recently by Wesley [...]. An earlier proposal in this direction was made by Parry Moon and Spencer [...]. An alternative way of obtaining time delays in an action-at-a-distance theory was given in Ref.[...]. The main idea is to obtain time delays by many-body interactions via a law of induction. All these ideas are important and should be further investigated. For a review of possible sources and methods of detection of gravitational waves see Ref.[...]. An important topic which we want to treat in a future work is the interaction of electromagnetic waves with matter. In particular, we want to deal with the deflection of light in a gravitational field and with the gravitational redshift. Due to the relevance of these topics we will treat them in a separate work. In this paper, we have also shown how one can obtain exactly the same value for the advance of the perihelion as given by general relativity but on the basis of quite different concepts. Moreover, we have obtained an expression for the gravitational "constant" G which is dependent on the distribution of mass in the universe and on time. As a consequence, we disagree with Dirac's position [...], according to which a theory that does not satify the strong equivalence principle cannot explain the advance of the perihelion of Mercury. This shows the connection of this work to deep questions of cosmology. (ASSIS, 1989).

The three-body problem is a phenomenon that is deeply rooted in the understanding of natural phenomena and is difficult to understand. It is because of this difficulty that we use this paper to understand the case and chance using Literature, Art and Science, in an inseparability that Leonardo Da Vinci and Galileo Galilei already thought about (SILVA, NEVES, 2015).

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