

Computational Financing Techniques and Fundamental Challenges in Portfolio Optimization

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Abstract: *It is very fundamental to note that scientific computational financing is a rapid growing field, especially in the area of investment management. Considering the opinions of various researchers in this field, it is still very obvious that there are many unresolved challenges. These challenges had constituted limitations to the financial securities portfolio managers over the years in optimizing a particular objective function. It is on this foundation, that this paper reviewed relevance of computational financing in portfolio management, especially as it affects decisions on investment diversification, price option, risk management, investment viability and acceptability. The paper also identified some complex challenges of computational financing in optimizing portfolio such as capital allocation among alternative opportunities, portfolio constraints, taxes computation and investor's portfolio preference. It was discovered that the conventional assumption of Markowitz does not hold which consequently leads to inefficient portfolio optimization performance. This paper, therefore suggested that the mean and variance of expected return should first be predicted with Functional Link Artificial Neural Network Model. The value of mean and variance generated will further serve as input in multi objective swarm intelligence techniques for better performance of portfolio optimization.*

Keywords: *Computational Finance, Relevance, Challenges, Portfolio Optimization*

I. Introduction

One of the areas of economic activities that required scientific computing is the financial sector of the economy. Therefore, literatures revealed that this industry had experienced accelerated changes in what is regarded as computational financing. Computational finance can be described as a cross-disciplinary field which has its route from mathematical finance, numerical method, computational intelligence and computer simulations to make trading, hedging and investment decisions as well as assisting the management to make good management decision. Computational finance is a branch of applied computer science that deals with problems of practical interest in finance. A close area of discipline is the study of data and algorithms currently used in finance and the mathematics of computer programs that realize financial models or systems. Computational finance emphasizes practical on numerical methods rather than mathematical proofs and focuses on techniques that apply directly to economic analyses. It is an interdisciplinary field between mathematical finance and numerical methods. The two major areas are efficient and accurate in computation of fair values of financial securities and the modeling of stochastic price series. Various methods of computation had been adopted in the past by the practitioners of computational financing with the main focus on how to determine the financial risks created by certain financial instruments.

Generally, individual who performs activities of computational financing are regarded as 'Quants' which symbolizes the quantitative skills required to perform the professional assignment. Specifically, the knowledge of C++ programming language, as well as that of mathematics such as stochastic calculus, multivariate calculus, linear algebra, differential equations, probability theory and statistical inference are often the prerequisite to the position. In recent time, the methods of computational financing have advanced as neural network and evolutionary computation have opened new method for the computation.

In about three decades ago, the term computational finance was not in existence for common usage, but today this area has become a distinct and enormous popular academic discipline. The field has become more popular as it attracts interest of professionals from mathematics, computer science, physics and economics. It is fundamental to note that the relevance of the discipline to the banking industry and investment analysis has contributed significantly to its growth trend. Also, the drawing of these various disciplines to computational financing has created formidable intellectual challenges that are intrinsic to financial markets, but it is worth noting that many of these basic challenges in financial analysis are still unresolved.

Therefore, this paper sees it as a necessity to identify various relevance of computational financing especially as it affects decisions on diversification, price option, risk, viability and acceptability of investments. Also, in this paper, we focus on discussing challenges by describing a relatively simple problems that all investors are facing especially in managing a portfolio of financial securities over time to optimize a particular objective function and revealing how complex it is to solve real world problems. The remaining part of this

paper is arranged thus: statement of the problem; objective of the paper; historical trend of computational financing technique; portfolio optimization challenges; relevance of computational finance; policy recommendation and finally conclusion.

1.2 Statement of the Problem

The primary target of every portfolio manager is to make accurate decision on investment so as to optimize his portfolio objective. Despite the degree of popularity and long time application of computational finance technique in achieving this fundamental objective, portfolio managers still fail to perform significantly especially in area of capital allocation among alternative opportunities, portfolio constraints, taxes computation and investor's portfolio preference. For instance there are much empirical regularities of investors' preferences that can induce path dependence in the value function, and for each of these situations, the computational demands quickly become intractable. Also, it is obvious that the computations are considerably more involved when there is tax than when tax is not involved.

1.3 Objectives of the paper

The main objective of this paper is to review relevance of computational financing in managing portfolio efficiently, especially in area of decisions on investment diversification, price option, risk management, investment viability and acceptability. In the same effort, the paper further intends to identify some complex challenges associated with management of portfolio optimisation and recommend advance technique capable of eliminating the existing challenges.

1.4 Historical Trend of Computational Finance

The emergence of computational finance can be traced back to 1930s when some astute investors started adopting mathematics formulae to price stocks and bonds. In 1950s Harry Markowitz introduced computational finance by way of mean-variance optimization to solve portfolio selection problem. The major problem of the analysts at that time was the shortage of computers for the analysis. Therefore, mathematical finance started with the same conceive, but diverged by making simplifying assumptions to establish relationship in closed forms that did not require sophisticated computer science analysis.

Ed Thorp and Michael Goodkin, as hedge fund managers pioneered the use of computers in arbitrage trading in 1960s. During this period, Eugene has developed Efficient Markets Hypothesis (EMH) which required analysis of a large amount of financial data. In 1970s, the main target of computational finance shifted to option pricing and analysis of mortgage securitizations. The introduction of personal computer in the late 1970s and early 1980s brought about an exploration of wide range of computational finance applications. Although, many of the new techniques emerged from signal processing and speech recognition, rather than traditional field of computational economics e.g optimization and time series analysis. As a result of the winding cold war by the end of 1980s, a large number of physicians and applied mathematicians moved into the study of finance. These groups of people are popularly known as financial engineer and quantitative portfolio managers. As at this time, computational finance became a popular and distinct academic subfield. In 1994, the first degree programme was offered by Carnegie Mellon University. Recently, the expansion of this discipline became wider as it has expanded into visually every area of finance and the demand for the professionals has increased drastically.

1.5 Portfolio Optimization Challenges

1.5.1 Portfolio Objective Optimization Problem

One of the basic problems of portfolio management is the optimization of portfolio objective which has called for attention of several authors in the recent time. The major portfolio optimization problem involve the decision of an individual investors to allocate resources among various investment opportunities within a specified period of time in order to maximized some established objective functions. In achieving this cardinal objective, portfolio managers are confronted with certain constraints such as tax liabilities, income payment, loan repayment provisions and other flows of income that determine total budget of an investor. Main while, for these constraints to be efficiently and effectively managed, they have to be measured and computational finance has to come in at this junction in calculating accurate values for the expected constraints variables. In considering a simple framework of portfolio optimization, it is obvious, in practical sense, that constraints such as taxes, investor preference, etc can create difficult computational challenges.

Hypothetically, let C_t represents the consumption expenditures of the investor at date t and let W_t denote the investor's wealth just prior to date t consumption. This example assumes that the investor has a lifetime utility function $U(C_0, C_1, C_2)$ defined over each consumption path (C_0, C_1, C_2) that summarizes how much he values the entire path of consumption expenditures. Then, assuming absent market frictions and that

the investor's utility function is time-additive and time-homogeneous. This is mathematically expressed as follow:

$$U(C_0, C_1, C_2) = u(C_0) + u(C_1) + u(C_2) \quad (1)$$

the investor's dynamic portfolio optimization problem at t=0 is given by:

$$V_0(W_0) = \text{Max}_{C_0, C_1, C_2} E_0[u(C_0) + u(C_1) + u(C_2)] \quad (2)$$

subject to:

$$W_t - C_t = x_t S_t + y_t B_t, \quad t = 0, 1, 2 \quad (3)$$

$$W_{t+1} = x_t S_{t+1} + y_t B_{t+1}, \quad t = 0, 1 \quad (4)$$

$$C_t \geq 0, \quad t = 0, 1, 2 \quad (5)$$

$$x_t, y_t \in Z^+, \quad t = 0, 1, 2 \quad (6)$$

$$x_2 = y_2 = 0$$

where, Z^+ indicates the non-negative integers, and x_t and y_t are the number of shares of stocks and bonds, respectively, that the investor holds in his portfolio immediately after date t. The requirement that x_t and y_t are non-negative means that borrowing and short sales are not allowed and that is the constraint that many investors face. That x_t and y_t are required to be non-negative integers simply reflects the fact that it is not possible to purchase a fractional number of stocks or bonds. The constraint (4) states that W_{t+1} is equal to W_t multiplied by the gross return on the portfolio between dates t and t+1. This problem is easily solved numerically using the standard technique of stochastic dynamic programming. In particular, since $V_2(W_2) = u(W_2)$, we can then compute $V_1(W_1)$ using the Bellman equation so that can have the below equation.

$$V_1(W_1) = \text{Max}_{C_1} [u(C_1) + E_1[V_2(W_2)]] \quad (7)$$

subject to the constraints in (7). An aspect of (4) that makes it particularly easy to solve is the fact that the value function V_1 depends solely on one state variable, W_1 . This enables us to solve (7) numerically without too many computations.

1.5.2 Taxation and Computation Problem

Another area of problem to a portfolio manager is the significant impact that taxes can have on investment portfolio performance. Bertsimas (1998) opines that taxes have a significant role to play in portfolio optimization objective. Most seasoned investors are conscious of the significant impact that taxes can impose on the performance of their investment portfolio, hence taxes has a major influence most portfolio optimization problems. Let practically understand how taxes can increase the computational complexity of such optimization problems. Assuming trading profits in the stock is subject to a capital gains tax in the simple model above (7). Since this model has only two future periods, short-term and long-term capital gains are not distinguished, and for expositional simplicity we equally assumed that capital losses from one period cannot be used to offset gains from a later period. Even though, these assumptions seems do make the problem easier to solve, it is obvious that the computations are considerably more involved when there is tax than when tax is not involved.

To solve the dynamic portfolio optimization problem with taxes, we use dynamic programming. However, the value function is now no longer only a function of wealth, but also depends on past stock prices and the number of shares of stock purchased at each of those prices. In other words, the value function is now path-dependent. If we use $N_{s,t}$ to denote the number of shares of stock that was purchased at date $s \leq t$ and still in the investor's portfolio immediately after trading at date t, then the portfolio optimization problem may be expressed as:

$$V_0(W_0) = \text{Max}_{C_0, C_1, C_2} E_0 [u(C_0) + u(C_1) + u(C_2)] \quad (8)$$

subject to

$$W_t - C_t - \text{Max}_{\sum_{s=0}^{t-1} \gamma (S_t - S_s) (N_{s,t-1} - N_{s,t})} = x_t S_t + y_t B_t, \quad t = 0, 1, 2 \quad (9)$$

$$W_{t+1} = x_t S_{t+1} + y_t B_{t+1}, \quad t = 0, 1 \quad (10)$$

$$x_t = \sum_{s=t}^t N_{s,t}, \quad t = 0, 1, 2 \quad (11)$$

$$C_t \geq 0, \quad t = 0, 1, 2 \quad - \quad (12)$$

$$N_{0,0} \geq N_{0,1} \geq N_{0,2} \geq 0 \quad - \quad (13)$$

$$N_{1,1} \geq N_{1,2} \geq 0 \quad - \quad (14)$$

$$X_t, y_t \in Z^+, \quad t = 0, 1, 2 \quad - \quad (15)$$

$$x_2 = y_2 = 0 \quad - \quad (16)$$

where, γ is the capital gains tax rate. When $t=2$, the value function depends on $(W_2, S_1, S_2, N_{0,1}, N_{1,1})$ and we arrive at the below equation:

$$V_2(W_2, S_1, S_2, N_{0,1}, N_{1,1}) = u[W_2 - \text{Max}(0, \sum_{s=0}^1 \gamma (S_2 - S_s) N_{s,1})] \quad (17)$$

This makes date-2 consumption simply W_2 less any capital gains taxes that must be paid. At $t=1$, the value function depends on $(W_1, S_1, N_{0,0})$ and we can write the Bellman equation as follow:

$$V_1(W_1, S_1, N_{0,0}) = \text{Max}_{C_1} [u(C_1) + E_1[V_2(W_2, S_1, S_2, N_{0,1}, N_{1,1})]] \quad (18)$$

If we compare (18) with (7), it is obvious that the presence of taxes has made the dynamic portfolio optimization problem considerably more difficult.

1.5.3 Portfolio Preferences problem

Another important aspect of portfolio optimization challenge is the objective function that represents the investor's preferences. Conventionally, these preferences have been represented by time-additive and time-homogeneous utility functions, which yields important computational advantages because it implies that the value function at date t does not depend on the investor's consumption choices prior to date t . Unfortunately, the assumptions of time-additivity and time-homogeneity seem to be inconsistent with the empirical evidence on the consumption and portfolio choices of investors.

For instance, it is well known that every rational individual tends to shift relatively towards his consumption level over a period of given time. According to Kahneman, Slovic and Tversky (1990), this assumption implies that preferences depend not only on today's consumption level, but also on the previous consumption level. This is popularly recognized as "habit formation", such preferences imply that the value function $V_t(\cdot)$ is a function of $(C_0, C_1, \dots, C_{t-1})$ in addition to any other relevant stated variables. As in the case with taxes, these problems quickly become intractable as the number of time periods increases. According to Kahneman, Slovic, and Tversky (1982), it is often accepted that there are a few highly parametrized models of habit formation in which closed-form solutions are available, but generally, these models must be solved numerically as in Heaton (1995). There are many other empirical regularities of investors' preferences that can induce path dependence in the value function, and for each of these cases, the computational demands quickly become intractable.

1.5.4 Portfolio Constraints

One of the challenges that called for attention is the portfolio constraint challenge. Some unconstrained problems that admit closed-form solutions create computational complexity once constraints are added. In practice, however, closed-form solutions are rarely available for realistic portfolio optimization problems, with or without portfolio constraints. Such problems must be solved numerically, in which case, imposing constraints can sometimes reduce the number of computations since they limit the feasible region over which the value function must be evaluated. A practical example of this is the impact of the constraints in (3) on the basic portfolio optimization problem described above. In that scenario, we introduced constraint that x_t and y_t are non-negative integers, eliminating the possibility of borrowing or short selling. This implies that only a small number of values for W_1 are possible, and consequently, the number of computations needed to evaluate $V_1(W_1)$ drastically falls.

In another perspective, some elements of constraints can proportionally increase the number of computations required, despite the fact that they limit the feasible set. This typically occurs when the constraints increase the problem dimension. For instance, in the basic portfolio optimization problem described in this paper, consider imposing the additional constraint that the cumulative number of shares transacted (both purchased and sold) up to date t is bounded by some function, $f(t)$. In practice, these types of constraints are

often imposed on investment funds so as to reduce transactions costs and the risk of “churning”. When such a constraint is imposed, the value function is no longer a function of only W_t , but also of the cumulative number of shares transacted up to date t . This path dependence created in the value function makes constraints to increase the computational complexity of even the simplest portfolio optimization problems substantially.

1.6 Portfolio Optimisation and Relevance of Computational finance

Presently, all full service institutional finance firms employ computational finance professionals in their banking and finance operations, while there are many other boutique firms’ employees that specialize in quantitative trading alone.

The usage of computational finance in area of investment and banking is very essential. It became important due to the huge amount of money involved in this type of economic activities. Computational finance plays a significant role as one of the instruments adopted to evaluate every potential investment, whether it is a new business to be executed or business already established. For instance, an investor who has a huge capital to be invested in different securities such as stocks, bonds etc, with random returns needs the employment of computational finance to optimize his objective. Assuming each security denoted $i = 1, \dots, n$, estimate of its expected return is μ_i and variance is σ_i^2 are known. Also, for any two securities denoted i and j , their correlation coefficient ρ_{ij} is given. Therefore, if we represent the proportion of the total capital invested in security i by x_i , one can compute the expected return and the variance of the resulting portfolio $x = x_1, \dots, x_n$ as follows:

$$E[x] = x_1\mu_1 + \dots + x_n\mu_n = \mu^T x, \tag{19}$$

and

$$Var[x] = \sum_{i,j} \rho_{ij} \sigma_i \sigma_j x_i x_j = x^T Q x \tag{20}$$

where $\rho_{ii} \equiv 1$, $Q_{ij} = \rho_{ij} \sigma_i \sigma_j$, and $\mu = (\mu_1, \dots, \mu_n)$ (21)

The portfolio vector x must satisfy $\sum_i x_i = 1$ and there may or may not be additional feasibility constraints. Feasibility portfolio x is called efficient, if it has the maximal expected return among all portfolios with same variance, or alternatively, if it has the minimum variance among all portfolios that have at least a certain expected return. The collection of efficient portfolios forms the efficient frontier of the portfolio universe. Also, Markowitz formulated a portfolio optimization problem called mean-variance optimization (MVO) problem. This model can be formulated in three different dimensions, but in equivalent ways. One formulation results in the problem of finding a minimum variance portfolio of the securities 1 to n that yields at least a target value of expected return. In a mathematical form, this formulation produces a convex quadratic programming problem:

$$\begin{aligned} \text{Min}_x \quad & x^T Q x \\ & e^T x = 1 \\ & \mu^T x \geq R \\ & x \geq 0 \end{aligned} \tag{22}$$

Where e is an n -dimensional vector, all of which components are equal to 1. The first constant shows that the proportions x_i should sum 1. While the second constant shows that the expected return is not less than the target value and the objective function corresponds to the total variance of the portfolio.

The presence of risk in every economic activity especially financing activity, became inherent as the environment which they operate is full of uncertainty. Since the presence of risk cannot be eliminated totally, it has to be managed. Failure to properly manage risk leads to failure of business outfits. The technical approach to manage risk required quantitative risk measurement, which will adequately reveal the vulnerabilities of the investment. One of the approaches of doing this is the portfolio variance as in the Markowitz model, the Value-at-Risk (VaR). The mathematical expression is as follow:

$$\begin{aligned} \mu^T x \\ RM(x) \leq \gamma \\ e^T x = 1 \\ x \geq 0 \end{aligned} \tag{23}$$

According to Markowitz model, x_i is the proportion of the total capital invested in security, μ is the expected return vector for the different securities. $RM(x)$ indicates the value of a particular risk measure for portfolio x and γ . Adopting alternative approach, we can minimize the risk measure, while constraining the expected return of the portfolio to achieve or exceed a given target value R which is similar to equation (22)

Assets rationalization problem have the same mathematical structure as portfolio selection problem. Considering this problem scenario, the focus is not to choose a portfolio of stocks or other securities, but to

identify the optimal investment among a set of asset classes. For instance, asset classes can be categorized as large capitalization stocks, government bonds, corporate bonds etc. One can therefore economically invest in these classes of assets by purchasing the relevant mutual fund. Investment companies can construct index funds in numerous ways. One of the ways is to solve a clustering problem where similar stocks have one representative in the index fund. This consequently leads to formulation of integer programming. Computational financing is very relevant in area of capital rationing. The availability and adoptability of the instrument enhance potential investors the ability to make rational decision on where and how to allocate the scarce resources. It assists to prevent putting large sum of fund in an investment that simply does not appear viable. The tool has capacity to foresee the future status of an investment. In economic theory, it is clear that one of the major investment (economics) problems is that resources are limited and the question of how to allocate the scarce resources became a topical issue. Hence, computational finance contributes significantly in solving this fundamental economic problem.

Again, it is worthwhile to recognize that computational finance enhance the ability of financial manager to manage the world financial risk. The timely adoption of financial computation principles assists speculators and stockholders to manage both individual and corporate portfolio. Portfolio management comprises of investment activities that involve combination of many projects by individual or corporate in order to hedge against risks. One of the paramount roles of computational finance is to identify and analyse risks. Analysis of individual risk assists portfolio manager to identify businesses that can be economically combined as portfolio.

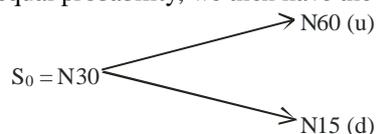
Unfortunately, the model capturing asset allocation problem failed to address assets/liability management problem. Therefore, stochastic programming approach is adopted to capture optimization under uncertainty. For better understanding, let assume that L_t represents the liability of a company in period t for $t = 1, \dots, T$. We equally assumed that the liability L_t is random with known distributions. It therefore, became reasonable that to solve the asset/liability management problem, we need to determine which assets and what quantities the company should hold at a particular point in time to maximize its expected wealth at the end of period T . let further assume asset classes that a company can be choose having random returns again with known distributions is symbolized by R_{it} for asset class i in period t . The period problem of asset/liability management can be formulated as a stochastic program since the holding decision to be made for each period is a function of asset returns and liabilities in the previous periods. Hence, the mathematical function is expressed as below.

$$\begin{aligned} & \text{Max}_x \quad E(\sum_i x_i, T) \\ & \sum_i (1 - R_{it}) x_{i, t-1} - \sum_i x_{i, t} = L_t, \quad t=1, \dots, T \\ & \quad \quad \quad x_{i, t} \geq 0 \quad \forall_{i,t} \end{aligned}$$

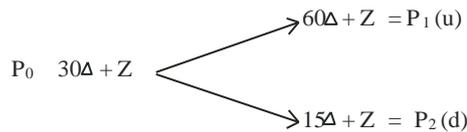
In the mathematical expression above, the objective function represents the expected total wealth at the end of the last period. The stated constraints show that the surplus left after liability L_t is covered, will be invested in asset class i .

In 1952, Harry Markowitz's adapted mathematics concepts to finance. He calculated mean return and covariance for common stocks and this enables him to quantify the concept of diversification in a market. He revealed how to compute the mean return and variance for a given portfolio and argued that investors should hold only those portfolios whose variance is minima among all portfolio with a given mean return.

In order to find a fair value of an option, there is need to solve a pricing problem. The option pricing problem can be solved using sophisticated mathematical approach. There are various ways of solving option pricing problems. In solving the option pricing problems, we attempt to establish a portfolio of assets with assumed prices, with expectation that if updated properly, through time, it will give the same payoff as a given financial instrument is called a replicating portfolio or hedge for that instrument. For instance, if we assume that one share of stock ABC is currently valued at N30, assuming the price of ABC will either double or halve as the case may be with equal probability, we then have the below hypothetical scenario.



Assuming we purchase one share of ABC stock for N40 a month from today, the fair price of the option will be as follow: It is assume that no interest is paid on borrowing or lending and no commission is paid on buy and selling. Remember that the option paid off will be N20 if the price of the stock goes up and N10 if it goes down. Assuming this portfolio has Δ shares of ABC and Z cash, the portfolio would now be $N30 + Z$ today. Hence, the following month payoffs for ABC portfolio will be:



Let choose Δ and Z in a way that

$$60\Delta + Z = 20$$

$$15\Delta + Z = 0$$

In this form, portfolio replicates the payoff of the option at the expiration date. It therefore, gives $\Delta^{1/2}$ and $Z = -7.5$, which indicates the hedge we are finding. Then, this portfolio worth is $P_0 = 30\Delta + Z = -N7.5$ today and the fair price of the option must also be N7.50.

Another area that computational finance plays significant role is the area of corporate strategic planning. For example, recognizing the operating structure of a business so as to ensure profit maximization may appear very facilitating at first glance, but more detail and otherwise results may emerge when computational finance instrument is applied. Compilation and running of investment data through a process of computational finance may in fact expose some drawbacks to the existing plan that already hiding before the application of the instrument. The complete exposition of expected expenses associated with a business as shown by computational finance may prove that such a business is costlier than anticipated. Therefore, computational finance assists in providing some realistic information on what could happen before any corporate plan is executed.

Merton introduced stochastic calculus into the study of finance. He attempted to understand how prices are fixed in financial market, which is the classical economics question of equilibrium. In the same vain, Fischer Black, and Myron Schoes were developing their option pricing formula. This provided a solution for a practical problem, finding a fair price for a European call option, that is, the right to buy one share of a given stock of a specified price and time. Such options are frequently purchased by investors as a risk-hedging device and computational finance instrument has helped to reveal how to price numerous other derivative securities.

1.7 Policy Recommendations

The review of portfolio optimization challenges and techniques revealed that the adoption of Harry Markowitz conventional mean-variance model of portfolio optimization which assumed that the expected return is considered as the mean of the past return does not hold again. This inadequacy has led to continuous inefficiency performance of portfolio optimization. Therefore, to witness efficiency and high performance of portfolio optimization, a multi objective optimization model is recommended which suggests that the mean and variance of expected return are first predicted with a low complexity functional link artificial neural network model (FLANN). The mean and variance value predicted are further considered in multi-objective swarm intelligence techniques. This technique is a Non-dominated Sorting Generic Algorithm II (NSGA-II) and Multi Objective Particle Swarm Optimization (MOPSO). A multi objective is the technique of computing a vector of decision variables that satisfies the constraints and optimize a vector function whose components symbolize objective function. The performance of the suggested technique is found to be more efficient.

II. Concussion

This paper has succeeded in discussing portfolio optimization challenges and relevant computational financial techniques in achieving optimization function. The persistent non-performance of portfolio optimization revealed that Markowitz technique is inadequate. The introduction of new technique FLANN and NSGA-II as suggested by this paper removes the wrong assumption of Markowitz. It is therefore submitted by this paper that the adoption of this technique will impact significantly in addressing optimization challenges such as measurement of taxes, investment preference, capital rationing, investment diversification, price option and risk management.

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