Growth Rate Computation: Issue with Logarithmic Transformation of a Negative Variable

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Abstract: Growth rate computation relating to the performance of an economy is usually is based on positive values of a time-dependent variable. There is a constraint to compute growth rate when such a variable is consistently negative. This paper makes an enquiry into the possibility of using a method to overcome this constraint, which is not very popular in India.

Key words: Growth rate, negative growth, logarithmic transformation, time-dependent variable, Current account deficit, reverse of a time-dependent negative variable.

I. INTRODUCTION

The performance of any economy can be evaluated in different ways. One of the popular ways of such an evaluation is the assessment of the growth rate of the economy. Growth rate is a quantified variable in terms of per capita income which is a ratio between national income and size of the population. Normally growth rate represents a variable with positive trend. Sometimes such a variable has negative trends to certain economies or certain sectors of the same economy [1,2].

Somalia, an African Economy, is a classic case of negative growth due to political instability in the 1990s. The case of India in the performance of foreign trade is another context that displays secular negative trend as far as trade balance is concerned [3]. Trade balance shows the difference between exports and imports of goods, services and factor income of a country during a particular year. The trade balance will have been normally a positive value for a developed country like USA [4]. The case is reverse in an emerging country like India. For instance, the trade balance for India has been showing negative values regularly since 1977. Technically, the country has been experiencing a trade deficit since then [3]. The focus of this paper is to consider a practical problem in computing the growth rate of a negative variable showing the macro or sectoral performance of an economy. This is a major issue once the researcher wants to express negative growth rate in logarithmic form.

This paper is organised in three parts along with introduction. The first part deals with the various concepts of growth rate and a suggestive method to circumvent the constraint of computing the growth rate of a negative variable in logarithmic form is shown in the second part. The emerging conclusion is provided in the final part (Section III).

II. GROWTH RATE OF A POSITIVE VARIABLE

Computation of growth rates of different variables is very important in analysing temporal performance of an economy. If \( Y = f(t) \) where ‘t’ stands for time.

The growth of \( Y \) (denoted as \( g_Y \)) w.r.t. time is

\[
g_Y = \frac{dy}{dt} \]

\[
g_Y = \frac{1}{y} \frac{dy}{dt} \]  

The same result will be obtained if \( \ln Y = \ln f(t) \) \( \frac{1}{y} \frac{dy}{dt} \)

Logarithmic function is very useful when ‘\( Y \)’ is a function of some other variable with time dimension [5]. Let us have the following function.

\[
Y = uv \]  

Where \( y \) is nominal GDP, ‘\( u \)’ is the real GDP and ‘\( v \)’ as the price deflator. The three variables in Eqn. (3) vary over time Eqn. (3) can be expressed as

\[
\ln Y = \ln u + \ln v \]

i.e., \( \frac{1}{y} \frac{dy}{dt} = \frac{1}{u} \frac{du}{dt} + \frac{1}{v} \frac{dv}{dt} \) \( \frac{1}{y} \frac{dy}{dt} \)

OR \( g_Y = g_u + g_v \), where \( g \) denotes growth rate.
If \( y = \frac{u}{v} \), (6)

\[
Ie, \quad \frac{1}{y} \frac{dy}{dt} = \frac{1}{u} \frac{du}{dt} - \frac{1}{v} \frac{dv}{dt}
\]

(7)

\[
Ie, \quad g_y = g_u - g_v
\]

Above growth rates can be applied for explaining per capita income \((g_y)\), GDP and population respectively [5].

If \( g_y \) is the result of the two integrated components of an independent variable, the functional form becomes

\[
\ln Y = \ln (u+v), \text{ then}
\]

\[
g_y = \frac{u}{u+v} g_u + \frac{v}{u+v} g_v
\]

(8)

Here the rate of growth of a sum is a weighted average of the rates of growth of its components Eqn. (8) states that the growth of human capital is a weighted average of the growth rates of R and D personnel and manpower engaged in production [2].

The growth rate computation becomes a tedious issue once a variable is available at negative value over a period of time. The data on trade balance in Economic Survey,2012 – 13 shows that the trade balance has been consistently negative for four decades. How can one express this negative trend in growth terms when it is expressed in logarithmic terms? One of the ways of solving this constraint is discussed in the following section, though there are other methods of doing so.

III. GROWTH RATE OF A NEGATIVE VARIABLE

As the Current Account Balance is the difference between exports and imports of four subcomponents, let \( x \) be the exports of country A to country B and \( y \) be the imports of country A from country B, so that the current account balance \((x – y)\) will be negative (current account deficit) as the case of India since 1977 – 78. In order to find the rate of change in current account deficit (CAD) one can follow the following method as the growth rate of a time series negative value cannot be found out in logarithmic form.

Let \( x \) be the exports of country A and \( y \) be the imports of A from the rest of the world(B) and \( x<y; \)

So the trade balance \( x – y \) will be negative

In this case to find the decrease rate of \( x – y \) (growth rate of current account deficit) can be found by taking the growth rate of \( y – x \).

Here the reverse trade balance is \( y – x \), which is positive.

Let \( Y = y – x \); a function of \( t \);

\[
Y = f(t) \quad \text{(say)}
\]

\[
Ie, \quad g_y = \frac{dy}{dt} \quad \text{by log differentiation}
\]

1Current Account trade balance is the sum of balances of goods & services accounts, primary income and secondary income. Balance of goods accounts is the difference between exports and imports of goods while the balance of services/net services is found by subtracting imports of services from its exports. Primary income deals with the income from the manpower and capital. The net of primary income is the difference between the part of wages and salaries, dividend and interest of non-resident Indians received by India and their outflow to the foreigners. Secondary income deals with the outflow and inflow of grants, gifts etc.

2. GDP is the total value of goods and services produced in a country during a specific year. The ratio between GDP in current prices to price index is called the real GDP and the deflator is the relevant price index itself.
Graph – I: Decrease of $x - y$

Here the graph-1 is a smooth continuous curve having a negative slope $-m$ but in graph – 2, there is a smooth continuous curve having a positive slope, $m$. So the decrease (deficit) of $x - y$ is equivalent to the growth of $y - x$.

Hence the decrease rate (deficit) of $x - y$ is same as the growth rate of $y - x$.

IV. CONCLUDING REMARKS

This is an initiation of a deliberation pertaining to the growth rate of a time-dependent negative variable. Here the negative variable is the difference between exports and imports. The logarithmic transformation is made as the reverse of the actual difference. The slope of the second variable is taken as the growth rate. As the theoretical discussion on trade balance is mainly done in developed countries, it is positive in those countries. A secular negative phenomenon of the same occurs in counties like India. As a result, much discussion is not in Indian context. That is why this paper makes an initial attempt to address the constraint in computing the growth rate a time-dependent variable.

REFERENCES