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On The Use of Replacement Model to Determine the Appropriate Time to Replace a Deteriorating Industrial Equipment

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Abstract: *Mechanically, all industrial equipments get deteriorating with time and usage and their functions decreased efficiently. In order to manage such equipments, it requires higher operating and maintenance cost, as a result of this increasing repair and maintenance cost, there is a need to replace them. However, in this paper, we attempt to determine the exact time at which the replacement of the given instrument is most economical, The data used in this paper is all about the cost of repairing 250 kva mikano generating plant which was produced by works and services department of The Polytechnic Ibadan, Adeseun Ogundoyin Campus, Eruwa. The replacement model that covers items whose maintenance cost increase with time, ignoring changes in the value of money during the years under study. The result of analysis shows that at 5th year of using the generating plant in institution under study, the minimum average annual cost of 970,221.60 is obtained. Later, average annual cost increased to 999,726.33. This shows that ,it is economically advice able assuming the authority of The Polytechnic Ibadan has removed the generating plant installed at Eruwa Campus of the institution in the year 2008 and replace it with new one in the 2012 (that is after five years of its usage).*

Keywords: *Scrap Value, Depreciation, Average annual cost, Procurement Cost, Replacement cost, Maintenance Cost and Replacement model.*

I. Introduction

In any organization, the persistent increase of repair and maintenance cost necessitates the replacement of the equipments being used in such organization. Ajibola, A.Desu (2012) described asset replacement theory as a management tool to analyze and plan the optimum replacement strategy to be adopted or a firm's asset. A decision as to whether to replace or not should not only be made but also decision as to how often to replace these assets. According to Prem kumar Gupta and D.S Hira (2000), equipment needs replacement not because it no longer performs to the designed standards or specifications, but because more modern equipments performing higher standard are discovered.

In order to take an optimal replacement decision on an asset, the following factors should be highly considered:

(i) The purchase price of the old asset or equipment to be replaced
(ii) Purchase price of the new asset
(iii) Repairs and maintenance cost of the asset to replace.
(iv) Capital cost already consumed on the asset to replace.
(v) Opportunity cost
(vi) Taxation and investment incentives incurred on item to be replaced.

According to P. Rama Murthy (2007), when organizations want to replace machine or equipment, they may come across various alternative choices where they have to compare the various cost elements such as running costs and maintenance costs. In order to select optimal choice, there are various techniques to determine the time to replace deteriorating machines, these includes:

(a) Replacement of items whose maintenance cost increase with tie and value of money remains same during the period.
(b) Replacement of items whose maintenance cost increases with time and value of money also changes with time and
(c) To compare alternative choices or the concept of present value.

In the past, machine replacement problem has been studied by a lot of researchers, but many papers reveal the procedures in modeling machine replacement problem without application. For instance, Allen H.Tai and Wai-kiching (2005) Captured Renewal theory in machine Replacement models which can be interpreted by mathematicians and people from other fields may not understand the information disseminated by their paper.

However, this paper attempts to establish a replacement model and apply the model to determine the appropriate time to replace the equipments whose repair and maintenance cost increase and their efficiency reduce with time. In this paper, we determine the appropriate time to replace the 250 KVA Milan generating plant used for six years in The Polytechnic Ibadan, Adeseun Ogundoyin Campus, Eruwa in Oyo State of Nigeria, through the theoretical frame work of the replacement of items whose maintenance and repair costs increase with time, ignoring changes in the value of money during the period.

Some Useful Terms Used

Scrap Value: The Persistent or constant using of a machine lead to decline in an asset's economic value, this may call for selling out of such machine. The value at which the asset is finally sold off is called its Scrap Value.

Depreciation: Various assets like vehicles, machines, tools, residential structures, generating plants are such that there value goes down with time. This decrease can be because of the wearing out of the asset physically or the asset becoming obsolete. Indeed, we cannot use a machine (e.g. generating plant) forever. Productive capacity of any machine will decline with constant use leading to a fall in its value. On the other hand, some assets become obsolete with introduction of new improved version. Such decreased in value is referred to as depreciation.

Procurement cost: This is the quoted price of inventory minus any discounts allowed plus shipping charges.

Replacement cost: It is the cost at which asset identical to that which is to be replaced, could be currently purchased. In otherwise, it is the current purchased price of an identical asset.

II. Material and Method

The data used in this paper was produced by The Polytechnic Ibadan, Adeseun Ogundoyin Campus, Eruwa. The data covered the six years of repairing the mikano generating plant and its maintenance cost.

The method adopted in this paper is replacement of items whose maintenance and repair costs increase with time, ignoring changes in the value of money during the period under study. This methodology has to do with minimization of the average annual cost of equipment whose maintenance cost is increasing with time and its scrap value is constant. Since the changes in the value of money are not considered, the interest rate is zero and the calculations shall based on average annual cost. If this condition arises, we have two approaches to use, these are:

Case 1: If time't' is a continuous variable.

Lets C= capital cost of the equipment

S= scrap or depreciating value of the equipment.

T=Average annual total cost of the equipment

n=Number of years the equipment has been used.

F (t) =operating and maintenance cost of the equipment at time t.

Hence, Annual cost of the equipment at time t = capital cost – scrap value + maintenance cost at time t.

Now, the total maintenance cost incurred during n years $= \int_0^n \mathrm{f(t).d(t)}$, but

Total cost incurred during *n* years $\Rightarrow TC = C - S + \int_0^n \mathrm{f(t).d(t)}$

So, average annual cost incurred on the equipment i.e $ATC_n = \frac{1}{n}\left[C - S + \int_0^n \mathrm{f(t).d(t)}\right]$

To determine the value of n for which ATC_n is minimum, we shall differentiating ATC_n *w.r.t.n*

i.e. $\frac{d(ATC_n)}{d_n} = -\frac{1}{n^2}(C-S) - \frac{1}{n^2}\int_0^n \mathrm{f(t).d(t)}$

If $\frac{d(ATC_n)}{d_n} = 0$ we have

$$f(n) = \frac{1}{n}\left[C - S + \int_0^n \mathrm{f(t).d(t)}\right] = ATC_n$$

Thus the equipment should be replaced when the average annual cost to data becomes equal to the current maintenance cost. Using this result, one can determine an appropriate time to replace deteriorating industrial or organizational equipment provided an explicit function is given for the maintenance and repair costs.

Case2: Assuming the time't' is a discrete variable, the total cost incurred during n years, is mathematically expressed as:

$TC = C - S + \sum_{t=0}^{n} f(t),$ and average annual cost incurred on the equipment,

$$ATC_n = \frac{1}{n}\left[C - S + \sum_{\mathrm{t=0}}^{\mathrm{n}} \mathrm{f(t)}\right] \text{------(1)}$$

Hence, to estimate the minimum value of n for which ATC_n is minimum, below inequality has to be considered.

$$ATC_{n-1} > ATC_n < ATC_{n+1}.$$

Mathematically, the inequality can be resolved as:

$$ATC_{n-1} - ATC_n > 0 - - - - - -(a) \ \ and$$

$$ATC_{n+1} - ATC_n > 0 - - - - - -(b)$$

From equation (i), if n = n + 1, equ (i) becomes:

$$ATC_{n+1} = \frac{1}{n+1}\left[C - S + \sum_{\mathrm{t=1}}^{\mathrm{n+1}} \mathrm{f(t)}\right]$$

$$= \frac{1}{n+1}\left[C - S + \sum_{\mathrm{t=1}}^{\mathrm{n}} \mathrm{f(t)} + \mathrm{f(n+1)}\right]$$

$$= \frac{n}{n+1}\left[\frac{1}{n}\left[C - S + \sum_{\mathrm{t=1}}^{\mathrm{n}} \mathrm{f(t)}\right]\right] + \frac{f(n+1)}{n+1}$$

$$= \frac{n}{n+1}.ATC_{n+1} + \frac{f(n+1)}{n+1}$$

$$\therefore ATC_{n+1} - ATC_n = \frac{n}{n+1}.ATC_{n+1} + \frac{f(n+1)}{n+1} - ATC_n$$

$$= \frac{f(n+1)}{n+1} + ATC_n\left(\frac{n}{n+1} - 1\right)$$

$$= \frac{f(n+1)}{n+1} - \frac{ATC_n}{n}$$

Since $ATC_{n-1} - ATC_n > 0,$

$$\frac{f(n+1)}{n+1} - \frac{ATC_n}{n+1} > 0 \ \ or$$

$$f(n+1) - ATC_n > 0 \ \ or$$

$$f(n+1) > ATC_n$$

Similarly, $ATC_{n-1} - ATC_n > 0,$ produces

$f(n) > ATC_{n-1}$

From above results, the following replacement policies shall be considered when taking a decision:

a. When the operating and maintenance cost for the next year, $f(n+1)$ is more than the average annual cost of nth year, ATC_n, one has to replace the item at the end of *n* years. That is $f(n+1) > \frac{1}{n}\left[C - S + \sum_{t=0}^{n} f(t)\right]$.

b. If the running expenditure of the present year is less than the previous year's average annual cost, ATC_{n-1}, then, one should not replace the deteriorating item. That is $f(n) < \frac{1}{n-1}\left[C - S + \sum_{t=0}^{n-1} f(t)\right]$.

From above policies it was discovered that n is optimal at the minimum average annual cost.

III. Data Presentation, Analysis and Interpretation of result

The data presented on the table bellow shows the date of repairing and maintenance cost of the generating plant being used at The Polytechnic Ibadan, Adeseun Ogundoyin Campus Eruwa:

Date	**Maintenance/ Repair Cost**
12-01-2008	108,300
21-04-2008	32,000
02-10-2008	65,100
19-01-2009	48,000
25-05-2009	58,000
19-11-2009	40,000
25-03-2010	44,100
08-10-2010	52,300
01-03-2011	83,000
09-06-2011	58,000
18-09-2011	60,000
01-11-2011	62,500
17-04-2012	73,500
19-06-2012	94,500
20-11-2012	80,000
07-01-2013	13,000
11-03-2013	80,750
17-04-2013	187,500
17-05-2013	866,000

Table 1

The data presented above is summarized as thus:

Years (n) of service	2008	2009	2010	2011	2012	2013
Maintenance Cost	97,208	146,000	96,400	263,500	248,000	1,147,250

Table 2

The method earlier mentioned is use to determine the year that has minimum value of average annual cost. The procurement cost (cost price) of the generating plant as at 2008 is ₦5,000,000 and its Scrap Value is ₦ 1,000,000.

Period of service (n)	**Purchase Price – Scrap Value (C - S)**	**Annual Maintenance Cost f(t)**	**Cumulative Maintenance Cost**	**Total Cost (T_c)**	**Average Annual Cost (A_c)**
2008 (1)	4,000,000	97,208	97,208	4,097,208	4,097,208
2009 (2)	4,000,000	146,000	243,208	4,243,208	2,121,604
2010 (3)	4,000,000	96,400	339,608	4,339,608	1,446,536
2011 (4)	4,000,000	263,500	603,103	4,603,108	1,150,777
2012 (5)	4,000,000	248,000	851,108	4,851,108	970,221.60
2012 (6)	4,000,000	1,147,250	1,998,358	5,998,358	999,726.33

Table 3

IV. Findings

The analysis presented on the table 3 shows that the minimum average annual cost is ₦ 970,221.60, this is revealed during the fifth year of using the generating plant. Hence, the generating plant should be replaced after five years of its usage.

V. Conclusion

In this paper, we demonstrate how the replacement model can be applied to determine the appropriate time to replace depreciating equipment, a model that covers item whose maintenance and repairing costs increase with time ignoring changes in the value of money (i.e. either inflation or deflation) during the period under study. From the analysis, it was revealed that at 5^{th} year of using the generating plant, minimum average annual cost (min $[ATM_5]$) is obtained.

This implies that it would have been economically o.k., if the 250KVA Mikano generating plant at The Polytechnic Ibadan, Adeseun Ogundoyin Campus Eruwa has been removed and replaced after five years of its usage.

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