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# Solving n power class (Q) operators using MATLAB

Dr. T. Veluchamy, K.M.Manikanadan, T.Ramesh
*Dept. of Mathematics, Dr.SNS Rajalakshmi college of Arts &Science ,Coimbatore-49, TamilNadu,India*

***Abstract:*** *In this paper we investigate the charactisation of n power class (Q) operators on Hilbert space using MATLAB.*
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***Key words:*** *Normal operator, class (Q) operator, n power class (Q) operator, Hilbert space, Hadamard matrices ,MATLAB R2008a.*

## I. INTRODUCTION

Let H be a Hilbert space and L(H) is the algebra of allbounded linear operators acting on H . An operator T in L(H) is called normal if $T^*T = TT^*$, class(Q)if$T^{*^2}T^2 = (T^*T)^2$, n power class (Q) if $T^{*^2}T^{2n} = (T^*T^n)^2$. In general an power class(Q)operator need not be a normal operator. Existence of Operators on 2 power class (Q ) and 3 power class
(Q) are verified by using MATLAB R2008a version.

**Program no. 1**

Consider the operators $S = \begin{pmatrix} i & 1 \\ 0 & -i \end{pmatrix}$ $T = \begin{pmatrix} i & 0 \\ 1 & -i \end{pmatrix}$These two operators are 2 power class (Q) operators on the complex Hilbert space. S+T is 2 power class (Q)But S+T is not normal.

```
S = [i 1;0 -i]
 T = [i 0;1 -i]
 Sum = S+T
ConjSum = conj(Sum)
ConjSumsq = ConjSum*ConjSum
 Sumpow4 = Sum^4
 L = ConjSumsq*Sumpow4……….(1)
Sumpow2 = Sum^2
 M = ConjSum* Sumpow2………(2)
 N = M*M.………(3)
 k = Sum*ConjSum………(4)
v = ConjSum*Sum
```

On executing this program, equations (1) and (2) are same. From the output we can verify that$(S+T)^{*^2}(S+T)^4 = ((S+T)^*(S+T)^2)^2$. Hence S+ T is 2 power class (Q), Equations (3) and (4) are not equal.. Hence S+T is not normal.

**Definition:**Hadamard matrices are matrices of 1's and -1's whose columns are orthogonal,
H'*H = n*I where [n n]=size(H) and I = eye(n,n).
An n×n Hadamard matrix with n > 2 exists only if rem(n,4) = 0. This function handles only the cases where n, n/12, or n/20 is a power of 2.

**Examples**

The command hadamard (4) produces the 4-by-4 matrix:

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | -1 | 1 | -1 |
| 1 | 1 | -1 | -1 |
| 1 | -1 | -1 | 1 |

**Program No.2**

Consider Hadamard matrix of order 40. Here n = 40 and $\frac{40}{20} = 2 = 2^1$ (power of 2). We verify this matrix of order 40belongs to 40power Class (Q)

```
t = hadamard(40);
tstar = t;
tstarsq= tstar*tstar;
tpow80=t^80;
L = tstarsq*tpow80          …………(1)
tpow40=t^40;
S = tstar*tpow40;
```

V = S*S …………(2)

On executing this program, equations (1)and (2) are same. From the output we can verify that ${T^*}^2T^{80} = (T^*T^{40})^2$. Hence Hardamard matrix of order 40 belongs to 40 power class Q.

**Program no: 03**

Consider Hadamard matrix of order 48. Here n = 48 and $\frac{48}{12} = 4 = 2^2$(power of 2). We verify this matrix of order 48 belongs to 48power Class (Q)

```
t = hadamard(48);
tstar = t;
tstarsq = tstar*tstar;
tpow96=t^96;
K = tstarsq*tpow96      …………(1)
tpow48=t^48;
M = tstar*tpow48;
N = M*M               ……………(2)
```

On executing this program, equations (1) and (2) are same. From the output we can verify that ${T^*}^2T^{96} = (T^*T^{48})^2$. Hence Hardamard matrix of order 40 belongs to 40 power class Q.

**Definition:**

Hilbert matrix is an × n matrix with elements ($h_{ij}$)where $h_{ij} = \frac{1}{i+j-1} 1 \leq i,j \leq n$

**Program No 04**

Let T = $\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix}$ be an operator acting on 3 dimensional Hilbert space. Then T is 2 power class (Q), the matrix of T is a Hilbert matrix of order 3. T is 2 power class (Q) if it satisfies ${T^*}^2T^4 = (T^*T^2)^2$. We can also verify T is 2 normal if $T^*T^2 = T^2T^*$

```
T = [1 1/2 1/3;1/2 1/3 1/4;1/3 1/4 1/5]
ConjT = conj(T)
ConjTsq = ConjT*ConjT
 Tpow4 = T^4
 L = ConjTsq*Tpow4       …………(1)
 Tpow2 = T^2
 M = ConjT*Tpow2
 N = M*M  …………(2)
```

On executing this program, equations (1) and (2) are same. From the output we can verify that ${T^*}^2T^4 = (T^*T^2)^2$

```
// Hilbert matrix of order 3 is 2 normal//
O = ConjT*Tpow2 …………(3)
 P = Tpow2*ConjT…………(4)
```

On executing this program, equations (3) and (4) are same. From the output we can verify that $T^*T^2 = T^2T^*$.

**Program No 05**

Let T = $\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix}$ be an operator acting on 3 dimensional Hilbert space. Then T is 3 power class (Q), the matrix of T is a Hilbert matrix of order 3. T is 3 power class (Q) if it satisfies ${T^*}^2T^6 = (T^*T^3)^2$.

Hilbert matrix of order 3 is 3 power class Q

```
T = [1 1/2 1/3;1/2 1/3 1/4;1/3 1/4 1/5]
ConjT = conj(T)
ConjTsq = ConjT*ConjT
Tpow6 = T^6
 L = ConjTsq*Tpow6        …………..(1)
 Tpow3 = T^3
 M = ConjT*Tpow3
 N = M*M                  ………….(2)
```

On executing this program, equations (1) and (2) are same. From the output we can verify that ${T^*}^2 T^6 = (T^*T^3)^{\ 2}$.

**Definition:**

A bounded operator *T* on a Hilbertspace is said to be nilpotent if $T^n = 0$ for some *n*.

**Program No 06**

Consider the following irreducible nilpotent operators acting on $C_2$.

R = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$, S = $\begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ Then both S and T are normal and ∈2 power class (Q)

```
R = [0 1 0; 0 0 2; 0 0 0]
ConjR = conj(R)
 K = R*ConjR   ………. (1)
 V = ConjR*R ……….(2)
ConjRsq = ConjR*ConjR
 Rpow4=R^4
 L = ConjRsq*Rpow4     ……….. (3)
 Rpow2 = R^2
 M = ConjR*Rpow2
 N =M*M    ………… (4)
```

On executing this program, equations (1) and (2) are same. From the output we can verify that RR* = R* R. Hence R is normal. Similarly, On executing this program, equations (3) and (4) are same. From the output we can verify that ${R^*}^2 R^4 = \ (R^*R^2)^2$The proof for S is similar.

**Program No 7**

Consider the operator $T = \begin{pmatrix} -i & 0 \\ 2 & i \end{pmatrix}$acting on 2 dimensional complex Hilbert space which is 2 *powerclass*($Q$) but not 3 *powerclass*($Q$) .

```
T = [-i 0;2 i]
ConjT = conj(T)
ConjTsq = ConjT*ConjT
 Tpow4 = T^4
 L = ConjTsq*Tpow4            ……….(1)
 Tpow2 = T^2
 M = ConjT*Tpow2
 N = M*M                     ……….(2)
 T = [-i 0;2 i]
ConjT = conj(T)
ConjTsq = ConjT*ConjT
 Tpow6 = T^6
 L = ConjTsq*Tpow6            ……….(3)
 Tpow3 = T^3
 M = ConjT*Tpow3
 N = M*M                      ……….(4)
```

On executing this program, equations (1) and (2) are same. From the output we can verify that. ${T^*}^2 T^4 = (T^*T^2)^2$. Hence T is 2 power class (Q.) . Equations (3) and (4) are not same. From the output we can verify that. ${T^*}^2 T^6 \neq (T^*T^3)^2$. Hence T is not 3 power class (Q).

**Program No 8**

Consider the operators $T = \begin{pmatrix} i & 1 \\ 0 & -i \end{pmatrix} S = \begin{pmatrix} i & 0 \\ 1 & -i \end{pmatrix}$acting on 2 dimensional complex Hilbert space. Then the Product ST of 2power Class Q operators need not be 2 power class Q operator.

```
S = [i,1;0 -i]
 T = [i,0;1,-i]
Product = S*T
ConjProduct = conj(Product)
Conjproductsq = ConjProduct*ConjProduct
 Productpow4 = Product^4
 L = Conjproductsq*Productpow4        ………(1)
 Productpow2 = Product^2
 V = ConjProduct*Productpow2
```

Q = V*V………(2)
On executing this program, equations (1) and (2) are not same. From the output we can verify that.$(ST)^{*^2}(ST)^4 \neq ((ST)^*(ST)^2)^2$

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