Intuitionistic Fuzzy R-Ideals of BCI-Algebras with Interval Valued Membership & Non Membership Functions

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Abstract: The purpose of this paper is to define the notion of an interval valued Intuitionistic Fuzzy R-ideal (briefly, an i-v IF R-ideal) of a BCI – algebra. Necessary and sufficient conditions for an i-v Intuitionistic Fuzzy R-ideal are stated. Cartesian product of i-v Fuzzy ideals are discussed.

I. Introduction:

The notion of BCK-algebras was proposed by Imai and Iseki in 1996. In the same year, Iseki [6] introduced the notion of a BCI-algebra which is a generalization of a BCK-algebra. Since then numerous mathematical papers have been written investigating the algebraic properties of the BCK/BCI-algebras and their relationship with other universal structures including lattices and Boolean algebras. Fuzzy sets were initiated by Zadeh [10]. In [9], Zadeh made an extension of the concept of a Fuzzy set by an interval-valued fuzzy set. This interval-valued fuzzy set is referred to as an i-v fuzzy set. InZadeh also constructed a method of approximate inference using his i-v fuzzy sets. In Birwa’s defined interval valued fuzzy subgroups of Rosenfeld’s nature, and investigated some elementary properties. The idea of “intuitionistic fuzzy set” was first published by Atanassov as a generalization of notion of fuzzy sets. After that many researchers considers the Fuzzifications of ideal and sub algebras in BCK/BCI-algebras. In this paper, using the notion of interval valued fuzzy set, we introduce the concept of an interval-valued intuitionistic fuzzy R-ideal of a BCI-algebra, and study some of their properties. Using an i-v level set of i-v intuitionistic fuzzy set, we state a characterization of an intuitionistic fuzzy R-ideal of BCI-algebra. We prove that every intuitionistic fuzzy R-ideal of a BCI-algebra X can be realized as an i-v level R-ideal of an i-v intuitionistic fuzzy R-ideal of X. In connection with the notion of homomorphism, we study how the images and inverse images of i-v intuitionistic fuzzy R-ideal become i-v intuitionistic fuzzy R-ideal.

II. Preliminaries:

Let us recall that an algebra (X,*,0) of type (2,0) is called a BCK-algebra if it satisfies the following conditions:1. (x*y)*(x*z)=(x*z)*(y*x), 2. x*z≤y*z and  z*y ≤z*x, 3. 0*(x*y)=0*(0*y), 4. 0*x)*(0*y), 5. 0*(x*y) = (0*x) *(0*y), 6. 0*(0*(x*y))=0*(y*x), 7. x*(y*z)≤x*y

An intuitionistic fuzzy set A in a non-empty set X is an object having the form A= {<x,µA(x),υA(x)>/xEX}, Where the functions µA : X→[0,1] and υA: X→[0,1] denote the degree of the membership and the degree of non membership of each element xe X to the set A respectively, and 0≤ µA(x) +υA(x) ≤1 for all xe X.Such defined objects are studied by many authors and have many interesting applications not only in the mathematics. For the sake of simplicity, we shall use the symbol A=[µA, υA] for the intuitionistic fuzzy set A={µ(x),υ(x)/ xEX}.

Definition 2.1: A non empty subset I of X is called an ideal of X if it satisfies: 1. 0∈I, 2. x*y∈I and y∈I ⇒ x*y∈I.

Definition 2.2: A fuzzy subset µ of a BCI-algebra X is called an intuitionistic fuzzy ideal of X if it satisfies:
1. µ(0)≥µ(x), 2. µ(0)≥min {µ(x*y),µ(y)}, for all x,y∈X

Definition 2.3: A non empty subset I of X is called an R-ideal of X if it satisfies:
1. 0∈I, 2. x*(y*z)∈I and y∈I imply x*z∈I.Putting z=0 in (2) then we see that every R-ideal is an ideal.

Definition 2.4: A fuzzy set µ in a BCI-algebra X is called an intuitionistic fuzzyR-ideal of X if
1. µ(0)≥µ(x), 2. µ(0)≥min {µ ((x*z)*(y*z)),µ(y)}.

Definition 2.5: Let A and B be two fuzzy ideal of BCI algebra X. The fuzzy set A∩B membership function µA∩B is defined by µA∩B(x)=min{µA(x),µB(x)}, x∈X.

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Definition 2.6: Let A and B be two fuzzy ideal of BCI algebra X. The fuzzy set $A \cup B$ with membership function $\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$, $\forall x \in X$.

Definition 2.7: Let A and B be two fuzzy ideal of BCI algebra X with membership function respectively. A is contained in B if $\mu_A(x) \leq \mu_B(x)$. $\forall x \in X$.

Definition 2.8: Let A be a fuzzy ideal of BCI algebra X. The fuzzy set $A^m$ with membership function $\mu_{A^m}(x)$ is defined by $\mu_{A^m}(x) = \min\{\mu_A(x)\}$, $\forall x \in X$.

Definition 2.9: An IFS $A = \langle X, \mu_A, v_A \rangle$ in a BCI-algebra X is called an intuitionistic fuzzy ideal of X if it satisfies:(F1) $\mu_A(0) \geq \mu_A(x)$ and $v_A(0) \leq v_A(x)$, (F2) $\mu_A(x) \geq \mu_A(x+y)$, $\forall x \in X$.

Definition 2.10: An intuitionistic fuzzy ideal A of a BCI-algebra X is called an intuitionistic fuzzy ideal if it satisfies (F1) and (F2) $\mu_A(x) \geq \mu_A(x+y)$, $\forall x \in X$.

Definition 2.11: An intuitionistic fuzzy ideal A of a BCI-algebra X is called an intuitionistic fuzzy ideal if it satisfies (F1) and (F2) $\mu_A(x) \geq \mu_A(x+y)$, $\forall x \in X$.

Lemma 3.3: An i-v intuitionistic fuzzy ideal of an algebra X is called an interval-valued intuitionistic fuzzy ideal of X if it satisfies (F1) $\mu_A(0) \geq \mu_A(x)$ and $v_A(0) \leq v_A(x)$, (F2) $\mu_A(x) \geq \mu_A(x+y)$, $\forall x \in X$.

Theorem 3.2: Let A be an i-v intuitionistic fuzzy ideal of X if it satisfies (F1) and (F2) $\mu_A(x) \geq \mu_A(x+y)$, $\forall x \in X$.

III. Interval-valued Intuitionistic Fuzzy R-ideals of BCI-algebras

Definition 3.1: An interval-valued intuitionistic fuzzy ideal A of a BCI-algebra X is called an interval-valued intuitionistic fuzzy ideal of X if it satisfies (F1) $\mu_A(0) \geq \mu_A(x)$ and $v_A(0) \leq v_A(x)$, (F2) $\mu_A(x) \geq \mu_A(x+y)$, $\forall x \in X$.

Lemma 3.3: An i-v intuitionistic fuzzy ideal A of a BCI-algebra X is called an interval-valued intuitionistic fuzzy ideal of X if it satisfies (F1) $\mu_A(0) \geq \mu_A(x)$ and $v_A(0) \leq v_A(x)$, (F2) $\mu_A(x) \geq \mu_A(x+y)$, $\forall x \in X$.

Corollary: Assume that A is an i-v intuitionistic fuzzy ideal of X. For any $x, y \in X$, we have $[\mu_A(x), \mu_A(y)] = \bar{A}(x, y)$.

\[\mu_A(x) = \min\{\mu_A(x), \mu_A(y)\}\]

\[\mu_A(x) = \max\{\mu_A(x), \mu_A(y)\}\]

And $[v_A(x), v_A(y)] = \bar{A}(x, y)$.

\[\mu_A(x) = \max\{\mu_A(x), \mu_A(y)\}\]

\[\mu_A(x) = \min\{\mu_A(x), \mu_A(y)\}\]
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It follows that $\mu^L_A(x) \geq \min \{\mu^L_A(x,y), \mu^L_A(y)\}, \mu^U_A(x) \leq \max \{\mu^U_A(x,y), \mu^U_A(y)\}$

And $\mu^L_A(x) \geq \min \{\mu^L_A(x,y), \mu^L_A(y)\}, \mu^U_A(x) \leq \max \{\mu^U_A(x,y), \mu^U_A(y)\}$

Hence $(\mu^L_A, \mu^U_A)$ and $(\nu^L_A, \nu^U_A)$ are intuitionistic fuzzy ideals of X.

**Theorem 3.4:** Every i-v intuitionistic fuzzy R-ideal of a BCI-algebra X is an i-v intuitionistic fuzzy ideal.

**Proof:** Let $A = \{(\mu^L_A, \mu^U_A), (\nu^L_A, \nu^U_A)\}$ be an i-v intuitionistic fuzzy R-ideal of X. Assume $(\mu^L_A, \mu^U_A)$ and $(\nu^L_A, \nu^U_A)$ are intuitionistic fuzzy R-ideals of X. Hence by lemma 3.3, A is an i-v intuitionistic fuzzy ideal of X.

**Definition 3.5:** An i-v intuitionistic fuzzy set $A$ in X is called an interval-valued intuitionistic fuzzy BCI-sub algebra of X if $\vec{A}(x,y) \geq \min \{\mu_A(x), \mu_A(y)\}$ and $\overline{A}(x,y) \leq \min \{\overline{A}(x), \overline{A}(y)\}$ for all $x,y \in X$.

**Theorem 3.6:** Every i-v intuitionistic fuzzy R-ideal of a BCI-algebra X is an i-v intuitionistic fuzzy sub algebra of X.

**Proof:** Let $A = \{(\mu^L_A, \mu^U_A), (\nu^L_A, \nu^U_A)\}$ be an i-v intuitionistic fuzzy R-ideal of X, where $(\mu^L_A, \mu^U_A)$ and $(\nu^L_A, \nu^U_A)$ are intuitionistic fuzzy R-ideals of X. Then $(\mu^L_A, \mu^U_A)$ and $(\nu^L_A, \nu^U_A)$ are intuitionistic fuzzy subalgebras of X. Hence, A is an i-v intuitionistic fuzzy subalgebra of X.

4. Cartesian product of i-v intuitionistic fuzzy R-ideals

**Definition 4.1:** An intuitionistic fuzzy relation $\rho$ on any set A is an intuitionistic fuzzy subset A with a membership function $\mu_A : \rho \times A \times \rho \rightarrow [0, 1]$ and non membership function $\nu_A : \rho \times A \times \rho \rightarrow [0, 1]$. Let $\mu_A$ and $\overline{A}$ be two membership functions and $\overline{A}$ and $\overline{B}$ be two non membership functions of each x e X to the i-v subsets A and B, respectively. Then $\mu_A \times \mu_B$ is membership function and $\nu_B \times \nu_A$ is non membership function of each element (x,y) e X to the set A x B and defined by $A \times B = \{(x,y) \} | Page

**Lemma 4.2:** Let $\overline{A}$ and $\overline{B}$ be two membership functions and $\overline{A}$ and $\overline{B}$ be two non membership functions of each x e X to the i-v subsets A and B, respectively. Then $\mu_A \times \mu_B$ is membership function and $\nu_B \times \nu_A$ is non membership function of each element (x,y) e X to the set A x B and defined by $A \times B = \{(x,y) \} | Page
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\[\begin{align*}
= \max \{ & \max \{ \nu^L_A((x \ast z) \ast (z \ast y)), \nu^L_B((x \ast z) \ast (z \ast y)) \}, \max \{ \nu^U_A(y), \nu^U_B(y) \} \} , \\
& \max \{ \nu^L_A((x \ast z) \ast (z \ast y)), \nu^L_B((x \ast z) \ast (z \ast y)) \}, \max \{ \nu^U_A(y), \nu^U_B(y) \} \} , \\
& r \max \{ (\{x \ast 1\} \ast (z \ast 1)), (x \ast (z \ast 1)), (\{z \ast 1\} \ast (z \ast 1)), (z \ast (z \ast 1)) \} \\
& = r \max \{ (\{x \ast 1\} \ast (z \ast 1)), (x \ast (z \ast 1)), (\{z \ast 1\} \ast (z \ast 1)), (z \ast (z \ast 1)) \} \\
& \text{Hence } A \times B \text{ is an i-v intuitionistic fuzzy R-ideal of } X \times X.
\end{align*}\]

Definition 4.5. Let \( \mu_{A \times B}, \nu_{A \times B} \) respectively, be an i-v membership and non membership function of each element \((x, y)\) to the set \( B \). Then stronger i-v intuitionistic fuzzy set relation on \( X \times X \), that is a membership function relation \( \mu_{A \times B} \) and non membership function relation \( \nu_{A \times B} \) whose i-v membership and non membership function, of each element \((x, y) \in X \times X \) and defined by \( \mu_{A \times B}((x, y), (z, y')) = \min \{ \mu_A(x, y), \mu_B(z, y') \} \) and \( \nu_{A \times B}((x, y), (z, y')) = r \max \{ \nu_A(x, y), \nu_B(z, y') \} \)

Theorem 4.7. Let \( B = [\{ \mu^L_B, \mu^U_B \}, \{ \nu^L_B, \nu^U_B \}] \) be an i-v subset in a set \( X \), then the strongest i-v intuitionistic fuzzy relation on \( X \) that is a i-v A on \( B \) and defined by \( A_{B} = [\{ \mu^L_{A \times B}, \mu^U_{A \times B} \}, \{ \nu^L_{A \times B}, \nu^U_{A \times B} \}] \) be an i-v subset in a set \( X \) and \( A_{B} = [\{ \mu^L_{A \times B}, \mu^U_{A \times B} \}, \{ \nu^L_{A \times B}, \nu^U_{A \times B} \}] \) be the strongest i-v intuitionistic fuzzy relation on \( X \). Then \( B \) is an i-v intuitionistic R-ideal of \( X \) if and only if \( A_{B} \) is an i-v intuitionistic fuzzy R-ideal of \( X \times X \).

Proof: Let \( B \) (any i-v intuitionistic fuzzy a-ideal of \( X \). Then \( \mu_{A \times B}(0, 0) = \min \{ \mu_A(0), \mu_B(0) \} \) \( \geq \min \{ \mu_A(x, y), \mu_B(x, y) \} \) and \( \nu_{A \times B}(0, 0) = r \max \{ \nu_A(x, y), \nu_B(x, y) \} \).

Theorem 4.8. If \( \mu_{A} \) is a i-v intuitionistic fuzzy a-ideal of BCI-algebra \( X \), then \( \mu'_{A} \) is also i-v intuitionistic fuzzy R-ideal of BCI-algebra \( X \).

Proof: For all \( x, y \in X \),

\[\begin{align*}
1. \mu_{A}(0) & \geq \mu_{A}(x), \nu_{A}(0) \leq \nu_{A}(x), [\mu_{A}(0)]^m \geq [\mu_{A}(x)]^m, [\nu_{A}(0)]^m \leq [\nu_{A}(x)]^m \\
\mu_{A}(0)^m & \geq \mu_{A}(x)^m, \nu_{A}(0)^m \leq \nu_{A}(x)^m, \mu_{A'}(0) \geq \nu_{A'}(x), \nu_{A'}(0) \leq \nu_{A'}(x) \text{ } \forall x \in X.
\end{align*}\]
2. $\overline{\mu}_A(x) \geq \min \{ \overline{\mu}_A((x \ast z)^{(z \ast y)}), \overline{\mu}_A(y) \}$. $[\overline{\mu}_A(x)]^m = \overline{\mu}_A(x) \geq \min \{ \overline{\mu}_A((x \ast z)^{(z \ast y)}), \overline{\mu}_A(y) \}$

$\overline{\mu}_{A^c}(x) \geq \min \{ \overline{\mu}_{A^c}((x \ast z)^{(z \ast y)}), \overline{\mu}_{A^c}(y) \}$

$\overline{\mu}_{A^c}(y) \geq \min \{ \overline{\mu}_{A^c}((x \ast z)^{(z \ast y)}), \overline{\mu}_{A^c}(y) \}$

3. $\overline{\nu}_A(x) \leq \max \{ \overline{\nu}_A((x \ast z)^{(z \ast y)}), \overline{\nu}_A(y) \}$. $[\overline{\nu}_A(x)]^m \leq \max \{ \overline{\nu}_A((x \ast z)^{(z \ast y)}), \overline{\nu}_A(y) \}$

$\overline{\nu}_{A^c}(x) \leq \max \{ \overline{\nu}_{A^c}((x \ast z)^{(z \ast y)}), \overline{\nu}_{A^c}(y) \}$

Theorem 4.9: If $\mu$ is a i-v intuitionistic fuzzy R-ideal of BCI-algebra X, then $\overline{\mu}_{A \ast B}$ is also a i-v intuitionistic fuzzy R-ideal of BCI-algebra X.

Proof: For all $x, y, z \in X$

1. $\overline{\mu}_A(0) \geq \overline{\mu}_A(x)$, $\overline{\nu}_A(0) \leq \overline{\nu}_A(x)$ and $\overline{\mu}_B(0) \geq \overline{\mu}_B(x)$, $\overline{\nu}_B(0) \leq \overline{\nu}_B(x)$

$\overline{\mu}_{A \ast B}(0) \geq \overline{\mu}_{A \ast B}(x)$, $\overline{\nu}_{A \ast B}(0) \leq \overline{\nu}_{A \ast B}(x)$

2. $\overline{\mu}_A(x) \geq \min \{ \overline{\mu}_A((x \ast z)^{(z \ast y)}), \overline{\nu}_A(y) \}$, $\overline{\nu}_A(y) \geq \min \{ \overline{\mu}_A((x \ast z)^{(z \ast y)}), \overline{\nu}_A(y) \}$

$\overline{\nu}_A(y) \geq \min \{ \overline{\mu}_A((x \ast z)^{(z \ast y)}), \overline{\nu}_A(y) \}$

$\overline{\mu}_{A \ast B}(x) \geq \min \{ \overline{\mu}_{A \ast B}((x \ast z)^{(z \ast y)}), \overline{\nu}_{A \ast B}(y) \}$

3. $\overline{\nu}_A(x) \leq \max \{ \overline{\nu}_A((x \ast z)^{(z \ast y)}), \overline{\nu}_A(y) \}$, $\overline{\nu}_B(x) \leq \max \{ \overline{\nu}_B((x \ast z)^{(z \ast y)}), \overline{\nu}_B(y) \}$

$\overline{\nu}_B(y) \leq \max \{ \overline{\nu}_B((x \ast z)^{(z \ast y)}), \overline{\nu}_B(y) \}$

$\overline{\nu}_{A \ast B}(x) \leq \max \{ \overline{\nu}_{A \ast B}((x \ast z)^{(z \ast y)}), \overline{\nu}_{A \ast B}(y) \}$

$\overline{\nu}_{A \ast B}(y) \leq \max \{ \overline{\nu}_{A \ast B}((x \ast z)^{(z \ast y)}), \overline{\nu}_{A \ast B}(y) \}$

Theorem 4.10: If $\mu$ is a i-v intuitionistic fuzzy R-ideal of BCI-algebra X, then $\overline{\mu}_{A \ast B}$ is also a i-v intuitionistic fuzzy R-ideal of BCI-algebra X.

Proof: For all $x, y, z \in X$

1. $\overline{\mu}_A(0) \geq \overline{\mu}_A(x)$, $\overline{\nu}_A(0) \leq \overline{\nu}_A(x)$ and $\overline{\mu}_B(0) \geq \overline{\mu}_B(x)$, $\overline{\nu}_B(0) \leq \overline{\nu}_B(x)$

$\overline{\mu}_{A \ast B}(0) \geq \overline{\mu}_{A \ast B}(x)$, $\overline{\nu}_{A \ast B}(0) \leq \overline{\nu}_{A \ast B}(x)$

2. $\overline{\mu}_A(x) \geq \min \{ \overline{\mu}_A((x \ast z)^{(z \ast y)}), \overline{\nu}_A(y) \}$, $\overline{\nu}_A(y) \geq \min \{ \overline{\mu}_A((x \ast z)^{(z \ast y)}), \overline{\nu}_A(y) \}$

$\overline{\nu}_A(y) \geq \min \{ \overline{\mu}_A((x \ast z)^{(z \ast y)}), \overline{\nu}_A(y) \}$

$\overline{\mu}_{A \ast B}(x) \geq \min \{ \overline{\mu}_{A \ast B}((x \ast z)^{(z \ast y)}), \overline{\nu}_{A \ast B}(y) \}$

3. $\overline{\nu}_A(x) \leq \max \{ \overline{\nu}_A((x \ast z)^{(z \ast y)}), \overline{\nu}_A(y) \}$, $\overline{\nu}_B(x) \leq \max \{ \overline{\nu}_B((x \ast z)^{(z \ast y)}), \overline{\nu}_B(y) \}$

$\overline{\nu}_B(y) \leq \max \{ \overline{\nu}_B((x \ast z)^{(z \ast y)}), \overline{\nu}_B(y) \}$

$\overline{\nu}_{A \ast B}(x) \leq \max \{ \overline{\nu}_{A \ast B}((x \ast z)^{(z \ast y)}), \overline{\nu}_{A \ast B}(y) \}$

$\overline{\nu}_{A \ast B}(y) \leq \max \{ \overline{\nu}_{A \ast B}((x \ast z)^{(z \ast y)}), \overline{\nu}_{A \ast B}(y) \}$
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