Regular Weakly Contra Open Mappings in Intuitionistic Fuzzy Topological Spaces

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Abstract: In this paper we introduce intuitionistic fuzzy contra regular weakly generalized open mappings and intuitionistic fuzzy contra regular weakly generalized closed mappings. We investigate some of their properties.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy regular weakly generalized closed mappings and intuitionistic fuzzy contra regular weakly generalized open mappings.

I. Introduction

After the introduction of Fuzzy set (FS) by Zadeh [15] in 1965 and fuzzy topology by Chang [2] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov [1] in 1983 as a generalization of fuzzy sets. In 1997 Coker [3] introduced the concept of intuitionistic fuzzy topological space. In this paper we introduce the notion of intuitionistic fuzzy contra regular weakly generalized closed mappings and intuitionistic fuzzy contra regular weakly generalized open mappings and study some of their properties.

II. Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form \( A = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in X \} \) where the functions \( \mu_A(x) : X \to [0, 1] \) and \( \nu_A(x) : X \to [0, 1] \) denote the degree of membership (namely \( \mu_A(x) \)) and the degree of non-membership (namely \( \nu_A(x) \)) of each element \( x \in X \) to the set A, respectively, and \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \) for each \( x \in X \). Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form \( A = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in X \} \) and \( B = \{ (x, \mu_B(x), \nu_B(x)) \mid x \in X \} \). Then

(a) \( A \subseteq B \) if and only if \( \mu_A(x) \leq \mu_B(x) \) and \( \nu_A(x) \geq \nu_B(x) \) for all \( x \in X \)
(b) \( A = B \) if and only if \( A \subseteq B \) and \( B \subseteq A \)
(c) \( A^c = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in X \} \)
(d) \( A \cap B = \{ (x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x)) \mid x \in X \} \)
(e) \( A \cup B = \{ (x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x)) \mid x \in X \} \).

For the sake of simplicity, we shall use the notation \( A = \{ (x, \mu_A, \nu_A) \} \) instead of \( A = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in X \} \). Also for the sake of simplicity, we shall use the notation \( A = \{ (x, \mu_A, \nu_A, \nu_B) \} \) instead of \( A = \{ (x, (\mu_A, \mu_B), (\nu_A, \nu_B)) \} \).

The intuitionistic fuzzy sets \( 0_\tau = \{ (x, 0, 0) \mid x \in X \} \) and \( 1_\tau = \{ (x, 1, 1) \mid x \in X \} \) are the empty set and the whole set of X, respectively.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on a non empty set X is a family \( \tau \) of IFSs in X satisfying the following axioms:

(a) \( 0_\tau, 1_\tau \in \tau \)
(b) \( G_1 \cap G_2 \in \tau \) for any \( G_1, G_2 \in \tau \)
(c) \( (\forall G_i \in \tau \) for any arbitrary family \( \{ G_i \mid i \in J \} \subseteq \tau \)

In this case the pair \( (X, \tau) \) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in \( \tau \) is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement \( A^c \) of an IFOS A in an IFTS \( (X, \tau) \) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [3] Let \( (X, \tau) \) be an IFTS and \( A = \{ (x, \mu_A, \nu_A) \} \) be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

\[ \text{int}(A) = \bigcup \{ G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A \} \]

\[ \text{cl}(A) = A \cup \{ A \in \tau \} \]
cl(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}.

Note that for any IFS A in (X, \tau), we have cl(A') = (int(A))' and int(A') = (cl(A))' [14].

**Definition 2.5:** [5] An IFS A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \} in an IFTS (X, \tau) is said to be
(a) [4] intuitionistic fuzzy semi closed set (IFSCS in short) if int(cl(A)) \subseteq A
(b) [4] intuitionistic fuzzy \alpha-closed set (IF\alphaCS in short) if cl(int(cl(A))) \subseteq A
(c) [4] intuitionistic fuzzy pre-closed set (IFPCS in short) if cl(int(A)) \subseteq A
(d) [4] intuitionistic fuzzy regular closed set (IFRCS in short) if cl(int(A)) = A
(e) [13] intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) \subseteq U whenever A \subseteq U and U is an IFSOS.
(f) [10] intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if
\text{scl}(A) \subseteq U , whenever A \subseteq U and U is an IFSOS.
(g) [8] intuitionistic fuzzy \alpha generalized closed set (IF\alphaGCS in short) if
\text{acl}(A) \subseteq U , whenever A \subseteq U and U is an IFSOS.

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy \alpha-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy \alpha generalized open set (IFSOS, IF\alphaOS, IFPOS, IFROS, IFGOS, IFGPOS and IFGSCS respectively) if the complement A' is an IFCS, IF\alphaCS, IFPCS, IFRCS, IFGCS, IFGCS and IF\alphaGCS respectively.

**Definition 2.6:** [5] An IFS A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \} in an IFTS (X, \tau) is said to be an intuitionistic fuzzy regular weakly generalized open set (IFRWGOS in short) if cl(int(A)) \subseteq U whenever A \subseteq U and U is an IFROS in X.

The family of all IFRWGOSs of an IFTS (X, \tau) is denoted by IFRWGO(X).

**Definition 2.7:** [5] An IFS A is said to be an intuitionistic fuzzy regular weakly generalized open set (IFRWGOS in short) in (X, \tau) if the complement A' is an IFRWGOS in X.

The family of all IFRWGOSs of an IFTS (X, \tau) is denoted by IFRWGO(X).

**Result 2.8:** [5] Every IFCS, IF\alphaCS, IFGCS, IFRCS, IFPCS, IF\alphaGCS is an IFRWGOS but the converses need not be true in general.

**Definition 2.9:** [6] Let (X, \tau) be an IFTS and A = \{ x, \mu_A, \nu_A \} be an IFS in X. Then the intuitionistic fuzzy regular weakly generalized interior and an intuitionistic fuzzy regular weakly generalized closure are defined by
rwgint(A) = \cup \{ G / G \text{ is an IFRWGOS in } X \text{ and } G \subseteq A \}
kwgcl(A) = \cap \{ K / K \text{ is an IFRWGOS in } X \text{ and } A \subseteq K \}.

**Definition 2.10:** [3] Let f be a mapping from an IFTS (X, \tau) to an IFTS (Y, \sigma). If B = \{ (y, \mu_B(y), \nu_B(y)) / y \in Y \} is an IFS in Y, then the pre-image of B under f denoted by f^{-1}(B), is the IFS in X defined by f^{-1}(B) = \{ (x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))) / x \in X \}.

If A = \{ (x, \lambda_A(x), \omega_A(x)) / x \in X \} is an IFS in X, then the image of A under f denoted by f(A) is the IFS in Y defined by f(A) = \{ (y, f(\lambda_A(y)), f(\omega_A(y))) / y \in Y \} where f_-(\nu_A) = 1-f(1-\nu_A).

**Definition 2.11:** [7] A mapping f: (X, \tau) \rightarrow (Y, \sigma) from an IFTS (X, \tau) into an IFTS (Y, \sigma) is called an intuitionistic fuzzy regular weakly generalized continuous (IFRWG continuous in short) if f^{-1}(B) is an IFRWGCS in (X, \tau) for every IFS B of (Y, \sigma).

**Definition 2.12:** [6] A mapping f: (X, \tau) \rightarrow (Y, \sigma) from an IFTS (X, \tau) into an IFTS (Y, \sigma) is called an intuitionistic fuzzy regular weakly generalized irresolute (IFRWG irresolute in short) if f^{-1}(B) is an IFRWGCS in (X, \tau) for every IFRWGCS B of (Y, \sigma).

**Definition 2.13:** A mapping f: (X, \tau) \rightarrow (Y, \sigma) from an IFTS (X, \tau) into an IFTS (Y, \sigma) is said to be
(a) [11] intuitionistic fuzzy closed mapping (IFCM for short) if f(A) is an IFCS in Y for every IFCS A in X.
(b) [4] intuitionistic fuzzy semi closed mapping (IFSCM for short) if f(A) is an IFCS in Y for every IFCS A in X.
(c) [4] intuitionistic fuzzy pre-closed mapping (IFPCM for short) if f(A) is an IFCS in Y for every IFCS A in X.
Definition 2.14: [5] An IFTS (X, τ) is said to be an intuitionistic fuzzy \( rv T_{1/2} \) (IF \( rv T_{1/2} \) in short) space if every IFRWGCS in X is an IFCS in X.

Definition 2.15: [5] An IFTS (X, τ) is said to be an intuitionistic fuzzy \( rw T_{1/2} \) (IF \( rw T_{1/2} \) in short) space if every IFRWGCS in X is an IFPCS in X.

III. Intuitionistic Fuzzy Contra Regular Weakly Generalized Open Mappings

In this section we introduce intuitionistic fuzzy contra regular weakly generalized open mappings. We investigate some of their properties.

Definition 3.1: A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) from an IFTS (X, τ) into an IFTS (Y, σ) is called an intuitionistic fuzzy contra regular weakly generalized open mapping (IFcRWGOM in short) if f(A) is an IFRWGCS in Y for every IFOS A in X.

Example 3.2: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \{x, (0.2, 0.3), (0.8, 0.7)\} \), \( G_2 = \{y, (0.4, 0.5), (0.6, 0.5)\} \). Then \( \tau = \{0., G_1, 1.\} \) and \( \sigma = \{0., G_2, 1.\} \) are IFTs on X and Y respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFcRWGOM.

Theorem 3.3: Every IFcOM is an IFcRWGOM but not conversely.
Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IFcOM. Let A be an IFOS in X. Then \( f(A) \) is an IFCS in Y. Since every IFCS is an IFRWGCS, \( f(A) \) is an IFRWGCS in Y. Hence \( f \) is an IFcRWGOM.

Example 3.4: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \{x, (0.8, 0.7), (0.2, 0.3)\} \), \( G_2 = \{y, (0.4, 0.5), (0.6, 0.5)\} \). Then \( \tau = \{0., G_1, 1.\} \) and \( \sigma = \{0., G_2, 1.\} \) are IFTs on X and Y respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFcRWGOM.

Theorem 3.5: Every IFcαOM is an IFcRWGOM but not conversely.
Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IFcαOM. Let A be an IFOS in X. Then \( f(A) \) is an IFCS in Y. Since every IFCS is an IFRWGCS, \( f(A) \) is an IFRWGCS in Y. Hence \( f \) is an IFcRWGOM.

Example 3.6: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \{x, (0.6, 0.8), (0.4, 0.2)\} \), \( G_2 = \{y, (0.5, 0.3), (0.5, 0.7)\} \). Then \( \tau = \{0., G_1, 1.\} \) and \( \sigma = \{0., G_2, 1.\} \) are IFTs on X and Y respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFcRWGOM but not an IFcOM since \( IFS A = \{x, (0.8, 0.7), (0.2, 0.3)\} \) is an IFOS in Y but \( f(A) = \{y, (0.8, 0.7), (0.2, 0.3)\} \) is not an IFCS in Y, since \( cl(f(A)) = 1 \neq f(A) \).

Theorem 3.7: Every IFcPOM is an IFcRWGOM but not conversely.
Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IFcPOM. Let A be an IFOS in X. Then \( f(A) \) is an IFPCS in Y. Since every IFPCS is an IFRWGCS, \( f(A) \) is an IFRWGCS in Y. Hence \( f \) is an IFcRWGOM.

Example 3.8: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \{x, (0.1, 0.6), (0.9, 0.3)\} \), \( G_2 = \{y, (0.5, 0.3), (0.5, 0.7)\} \). Then \( \tau = \{0., G_1, 1.\} \) and \( \sigma = \{0., G_2, 1.\} \) are IFTs on X and Y respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFcRWGOM but not an IFPCM since \( IFS A = \{x, (0.1, 0.6), (0.9, 0.3)\} \) is an IFOS in Y but \( f(A) = \{y, (0.1, 0.6), (0.9, 0.3)\} \) is not an IFCS in Y, since \( cl(f(A)) = 0 \neq f(A) \).

Theorem 3.9: Every IFcαGOM is an IFcRWGOM but not conversely.
Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IFcαGOM. Let A be an IFOS in X. Then \( f(A) \) is an IFGCS in Y. Since every IFGCS is an IFRWGCS, \( f(A) \) is an IFRWGCS in Y. Hence \( f \) is an IFcRWGOM.

Example 3.10: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \{x, (0.6, 0.5), (0.3, 0.5)\} \), \( G_2 = \{y, (0.7, 0.6), (0.3, 0.4)\} \). Then \( \tau = \{0., G+ 1.\} \) and \( \sigma = \{0., G_2, 1.\} \) are IFTs on X and Y respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFcRWGOM but not an IFcOM since \( IFS A = \{x, (0.6, 0.5), (0.3, 0.5)\} \) is an IFOS in Y but \( f(A) = \{y, (0.1, 0.6), (0.9, 0.3)\} \) is not an IFCS in Y, since \( cl(f(A)) = 0 \neq f(A) \).
Relation between various types of intuitionistic contra fuzzy open mappings.

\[ \text{IFcOM} \rightarrow \text{IFcGWGOM} \rightarrow \text{IFcGWGCM} \rightarrow \text{FcRWGOM} \]

In this diagram by “A \rightarrow B” we mean A implies B but not conversely.

IV. Intuitionist Fuzzy Contra Regular Weakly Generalized Closed Mappings

In this section we introduce intuitionistic fuzzy contra regular weakly generalized closed mappings and investigate some of their properties.

Definition 4.1: A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\) is called an intuitionistic fuzzy contra regular weakly generalized closed mapping (IFcRWGCM in short) if \( f(A) \) is an IFcRWGOS in Y for every IFCS A in X.

Example 4.2: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = (x, (0.8, 0.7), (0.2, 0.3), (0.0, 0.5)) \) and \( G_2 = (y, (0.5, 0.3), (0.5, 0.7)) \). Then \( \tau = \{0, 1\} \) and \( \sigma = \{0, 1\} \) are IFTs on X and Y respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFcRWGCM.

Theorem 4.3: Every IFcCM is an IFcRWGCM but not conversely.

Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IFcCM. Let A be an IFCS in X. Then \( f(A) \) is an IFOS in Y. Implies \((f(A))\) is IFcCM since IFS A = \( (x, (0.2, 0.3), (0.8, 0.7)) \) is an IFCS in X but \( f(A) = (y, (0.0, 0.5), (0.5, 0.7)) \) is not an IFCS in Y, since \( \text{cl}(f(A)) = G_2 \upuparrows f(A) \).

Example 4.3: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IFcCM. Let A be an IFCS in X. Then \( f(A) \) is an IFOS in Y. This implies \((f(A))\) is an IFcCS in Y. Since every IFcCS is an IFcRWGCS, \((f(A))\) is an IFcRWGOS in Y. Hence \( f(A) \) is an IFcGWGCM.

Theorem 4.5: Every IFcGWGCM is an IFcRWGOS but not conversely.

Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IFcGWGOM. Let A be an IFCS in X. Then \( f(A) \) is an IFcGWGOS in Y. Since every IFcGWGOS is an IFcRWGOS, \((f(A))\) is an IFcRWGOS in Y. i.e \( f(A) \) is an IFcRWGOS in Y. Hence \( f(A) \) is an IFcRWGCM.

Example 4.4: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IFcGWGOM. Let A be an IFCS in X. Then \( f(A) \) is an IFcGWGOM. \((f(A))\) is an IFcGWGOM in Y. i.e \( f(A) \) is an IFcGWGOM.

Theorem 4.7: Every IFcGWGOM is an IFcRWGCM but not conversely.

Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IFcGWGOM. Let A be an IFCS in X. Then f(A) is an IFcGWGOM. \((f(A))\) is an IFcGWGOM in Y. i.e \( f(A) \) is an IFcGWGOM.
Theorem 4.9: Every IFcαGCM is an IFcRWGCM but not conversely.
Proof: Let f: (X, τ) → (Y, σ) be an IFcαGCM. Let A be an IFCS in X. Then f(A) is an IFαGOS in Y. This implies (f(A)) is an IFOgCS in Y. Since every IFαGOS is an IFRWGCS, (f(A)) is an IFRWGCS in Y. Hence f(A) is an IFcRWGCM.

Example 4.10: Let X = {a, b}, Y = {u, v} and G1 = (x, (0.6, 0.5), (0.3, 0.5)), G2 = (y, (0.5, 0.6), (0.5, 0.4)). Then τ = {0., G1, 1.} and σ = {0., G2, 1.} are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is IFcRWGCM but not an IFαGCM since IFS A = (x, (0.3, 0.5), (0.6, 0.5)) is an IFCS in X but f(A) = (y, (0.3, 0.5), (0.6, 0.5)) is not an IFαGCS in Y, since αcl(f(A)) = 1. ⊆ G2.

Theorem 4.14: Let f : (X, τ) → (Y, σ) be an IFcRWGCM. Then for every IFS A of X, f(cl(A)) is an IFcRWGCS in Y.
Proof: Let f be any IFS in X. Then cl(A) is an IFCS in X. By hypothesis, f(cl(A)) is an IFRWGOS in Y. Hence f(cl(A)) is an IFcRWGCS in Y.

Theorem 4.15: Let f : (X, τ) → (Y, σ) be an IFcRWGCM where Y is an IF rwT 1/2 space. Then f is an IFOM.
Proof: Let f be an IFcRWGCM. Then for every IFCS A of X, f(A) is an IFRWGOS in Y. This implies (f(A)) is an IFRWGCS in Y. Since Y is an IF rwT 1/2 space, (f(A)) is an IFCS in Y. i.e f(A) is an IFOS in Y. Hence f is an IFOM.

Relation between various types of intuitionistic contra fuzzy closedness.

IFcCM → IFcαGCM → IFcPCM → IFcRWGCM

In this diagram by “A → B” we mean A implies B but not conversely.

V. Conclusion
In this paper we have introduced Intuitionistic Fuzzy Contra Regular Weakly Generalized Open Mappings and Intuitionistic Fuzzy Contra Regular Weakly Generalized Closed Mappings and studied some of its basic properties. Also we have studied the relationship between Fuzzy Contra Regular Weakly Generalized Closed Mappings and some of the intuitionistic fuzzy mappings already exist.

References
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