

A study that shows the existence of a simple relationship among square, circle, Golden Ratio and arbelos of Archimedes and from which to identify the real Pi value (Mother Goddess Kaali Maata Unified method)

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Abstract: This study unifies square, circle, Golden Ratio, arbelos of Archimedes and π value. The final result,

in this unification process, the real π value is identified, and is, $\frac{14-\sqrt{2}}{4} = 3.14644660942\dots$

Key words: Arbelos, area, circle, diameter, diagonal, Golden Ratio, Perimeter, π value, side, square

I. Introduction

The geometrical entities and concepts such as circle, square, triangle, Golden Ratio have been studied extensively. "The Golden Ratio is the ratio of two line segments 'a' and 'b' (when $a < b$) such that $\frac{a}{b} = \frac{b}{a+b}$.

The ratio $\phi = \frac{a}{b} = \frac{\sqrt{5}-1}{2} \approx 0.6180339887498948482045868343656$, while the reciprocal $\frac{b}{a} = \frac{\sqrt{5}+1}{2} = 1.6180339887498948482045868343656$. Notice the relationship between the decimals. It suggests that

$\phi + 1 = \frac{1}{\phi}$ ". (A.S. Posamentier and I. Lehmann, 2004, π : A Biography of the World's Most

Mysterious Number, Page 146).

Archimedes (240 BC) of Syracuse, Greece, called the area arbelos that is inside the larger semi circle, but outside the two smaller semi circles of different diameters. By its shape it is also called as "a shoemaker's knife".

The Golden Ratio and the arbelos of **Archimedes** are different concepts. But in this paper by the **grace of God**, it has become possible to see that these two concepts too have an interesting and unexpected inter relationship between each other (one). Further, this relationship has an extended relationship also with the circle (two). It is a well known fact that there exists simple and understable relationship between circle and square (three). As circle, square are related, their **combined** interrelationship has been extended to π value also (four).

There is, thus, a **divine chain of bond (of four interconnecting relations) exists, among square, circle, Golden Ratio, arbelos and π value.** (Here π value means a true/ real/ exact/ line-segment based value. The stress here, on the adjectives to π , has become necessary, because 3.14159265358... of Polygon is attributed or **thrust** on circle. In other words, this number to circle is a **borrowed** number from polygon and its

existence thus can **not** be seen in the **radius** of the circle, naturally. However, the new π value $\frac{14-\sqrt{2}}{4} =$

3.14644660942... (unlike with official π value 3.14159265358...) is inseparable with radius and is, here, humbly submitted to the World of Mathematics:

$$\text{Area of the circle} = r \left(\frac{7r}{2} - \frac{\sqrt{2}r}{4} \right) = \pi r^2$$

$$\text{Circumference of the circle} = 6r + \frac{2r - \sqrt{2}r}{2} = 2\pi r$$

In support of the above formulae, this paper also chooses and confirms that the real π value is $\frac{14-\sqrt{2}}{4} =$

3.14644660942...

II. Procedure

1. Draw a square ABCD. Draw two diagonals. Inscribe a circle with centre 'O' and with radius $\frac{1}{2}$, equal to half of the side AB of the square, whose length is 1.

$$AB = \text{Side} = EN = \text{diameter} = 1$$

$$AC = BD = \text{Diagonal} = \sqrt{2}$$

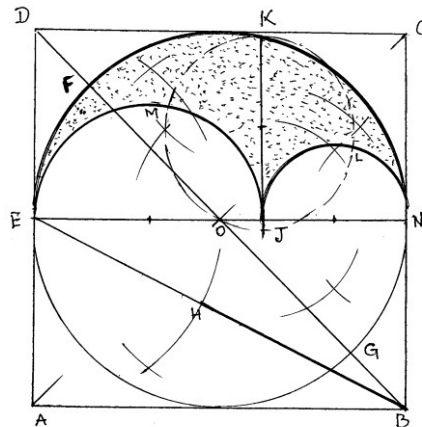
2. E is the mid point of AD

$$AE = \frac{1}{2}, \quad AB = 1$$

$$\text{Triangle EAB, } EB = \text{hypotenuse} = \sqrt{\frac{5}{4}}$$

$$EH = \frac{1}{2}; \quad HB = EB - EH = \sqrt{\frac{5}{4}} - \frac{1}{2} = \frac{\sqrt{5}-1}{2}$$

$$\text{Golden Ratio} = HB = \frac{\sqrt{5}-1}{2}$$



3. EN = Diameter = 1

$$EJ = HB = \frac{\sqrt{5}-1}{2} = 0.61803398874\dots$$

$$JN = EN - EJ = 1 - \left(\frac{\sqrt{5}-1}{2}\right) = \frac{3-\sqrt{5}}{2} = 0.38196601126\dots$$

4. Draw two semicircles on EN. And one semicircle with EJ as its diameter, and second semicircle with JN as its diameter.

5. So, the diameter of the EJ semicircle = Golden Ratio = $\frac{\sqrt{5}-1}{2}$ and

$$\text{the diameter of the JN semicircle} = \frac{3-\sqrt{5}}{2}$$

6. The area present (which is shaded) outside the two semicircles (of EJ and JN) and within the larger EN semi circle, is called **arbelos of Archimedes**.

7. Draw a perpendicular line on EN at J which meets circumference at K.

$$KJ = \sqrt{EJ \times JN} \text{ (this is Altitude Theorem)}$$

$$= \sqrt{\frac{\sqrt{5}-1}{2} \times \frac{3-\sqrt{5}}{2}} = \sqrt{\sqrt{5}-2} = 0.48586827174$$

8. Draw a **full** circle with diameter KJ. It has already been established that this **area** of the full circle is equal to the area of the **shaded region** called arbelos.

9. To calculate the area of the arbelos we have the following formulas.

$$\frac{\pi q(d-q)}{4} \text{ and } \pi \left[\frac{h}{2} \right]^2$$

where h = perpendicular line KJ = $\sqrt{\sqrt{5}-2}$ = diameter of the circle LKM

$\frac{h}{2}$ = radius of LKM circle.

10. Now, let us see the first formula

$$\frac{\pi q(d-q)}{4}$$

$$q = JN = \frac{3-\sqrt{5}}{2}$$

d = EN = diameter = 1

$$= \frac{\pi \left(\frac{3-\sqrt{5}}{2} \right) \left\{ 1 - \left(\frac{3-\sqrt{5}}{2} \right) \right\}}{4} = \left(\frac{\pi}{4} \right) (\sqrt{5}-2)$$

$$= \left(\frac{\pi}{4} \right) \times 0.23606797749$$

11. The conventional formula is πr^2 .

$$KJ = \text{diameter} = h = \sqrt{\sqrt{5}-2}$$

$$\text{Radius} = \frac{\text{diameter}}{2} = \frac{h}{2} = \frac{\sqrt{\sqrt{5}-2}}{2} = 0.24293413587$$

$$\pi \times 0.24293413587 \times 0.24293413587 = \pi \times 0.05901699437$$

Part-II

12. π value is **known** and hence, it is possible to find out the area of the arbelos either from $\frac{\pi q(d-q)}{4}$ or

$$\pi \left(\frac{h}{2} \right)^2$$

13. As there are two π values now $3.14159265358\dots$ and $3.14644660942 = \frac{14-\sqrt{2}}{4}$, the time has come,

to find a way to decide which number actually represents π value.

14. The following formula helps in deciding the real π value. The formula is

Side = a = diameter = d = 1

$$\text{Diagonal} = \sqrt{2} a = \sqrt{2}$$

Perimeter of the square

Half of 7 times of side of square $-\frac{1}{4}$ th of diagonal

$$= \frac{4a}{\frac{7a}{2} - \frac{\sqrt{2}a}{4}} = \frac{4}{\frac{7}{2} - \frac{\sqrt{2}}{4}} = \frac{4}{\frac{14-\sqrt{2}}{4}} = \frac{16}{14-\sqrt{2}}$$

(when a circle is **inscribed** in a square, or when a square is created from the four equidistant **tangents** on a circle, the length of the circumference of the **inscribed** circle can be **demarcated** in the **perimeter** of the superscribed square. It is called **rectification** of the circumference of the circle).

Part III (Area of the arbelos)

Let us calculate now the area of the arbelos with the known two π values, official π value and new one called Gayatri π value.

15. **With official π value**

$$\frac{\pi}{4} \times 0.23606797749 = \frac{3.14159265358}{4} \times 0.23606797749 = \boxed{0.18540735595}$$

(S. No. 10)

$$\pi \times 0.05901699437 = 3.14159265358 \times 0.05901699437 = \boxed{0.18540735594}$$

(S. No. 11)

16. **With Gayatri π value**

$$\frac{\pi}{4} \times 0.23606797749 = \frac{3.14644660942}{4} \times 0.23606797749 = \boxed{0.18569382184}$$

(S.No. 10)

$$\pi \times 0.05901699437 = 3.14644660942 \times 0.05901699437 = \boxed{0.18569382183}$$

(S.No. 11)

17. Finally, we obtain two different values representing same area of the arbelos of **Archimedes**.

Official π value gives: 0.18540735595 and

Gayatri π value gives: 0.18569382184

Which one is the actual value for the area of the arbelos ? The answer can be found in Part IV.

Part IV

18. In the Figure 1 we have Golden Ratio, HB equal to $\frac{\sqrt{5}-1}{2} = 0.61803398874$

19. Let us **divide** area of the arbelos of S.No. 17 with the **Cube of Golden Ratio** = $\left(\frac{\sqrt{5}-1}{2}\right)^3$ and

multiply it with $\frac{16}{14-\sqrt{2}}$ of the formula, derived in the S.No.14, which finally gives the area of the square

ABCD, equal to 1.

The π value that gives the exact area of the square equal to 1 is confirmed as the real π value. Here, the **Golden Ratio decides**, the real π value, by choosing the **correct** area of the arbelos of **Archimedes** of S.No. 17

20. Area of the arbelos obtained with official π value (S.No. 17)

$$\frac{0.18540735595}{\left(\frac{\sqrt{5}-1}{2}\right)^3} \times \frac{16}{14-\sqrt{2}}$$

Let us use simple calculator for the value of cube of Golden Ratio, which gives 0.23606797748

$$= \frac{0.18540735595}{0.23606797748} \times \frac{16}{14-\sqrt{2}}$$

$$= \frac{0.18540735595}{0.23606797748} \times 1.27127534534 = 0.99845732139$$

21. Area of the arbelos obtained with Gayatri π value (S.No. 17). Let us repeat steps of S.No. 20 here:

$$\frac{0.18569382184}{\left(\frac{\sqrt{5}-1}{2}\right)^3} \times \frac{16}{14-\sqrt{2}} = \frac{0.18569382184}{0.23606797748} \times 1.27127534534 = 1$$

As the exact area of the superscribed square is obtained, it is clear, therefore, that, the real π value is $\frac{14-\sqrt{2}}{4}$
= 3.14644660942...

III. Conclusion

It is well known that there exists a simple relationship between circle and square. In the present study, it is clear such simple relation **also** exists between Golden Ratio and arbelos of Archimedes. This paper combines above two kinds of relations and decides the real π value, as $\frac{14-\sqrt{2}}{4} = 3.14644660942\dots$

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