IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 1 Ver. 1 (Jan - Feb. 2015), PP 19-22
www.iosrjournals.org

Model of Mathematics Teaching: A Fuzzy Set Approach

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Abstract: *The quality of learning and teaching Mathematics has been one of the major challenges and concern of the educators. Mathematics is often considered as a subject that a student either understand or doesn't with little in between. Now a days modeling is one of the central ideas in mathematics education. The concept of teaching is fundamental for the study of student cognitive action. In this paper we have developed a model for mathematics teaching and analyses the skill of teachers using fuzzy logic.*
Keywords: *Fuzzy sets and logic, Possibility, Probability, Uncertainty*

I. Introduction

There are often situations in real life in which definitions do not have clear boundaries, for example this happens when we speak about the 'high temperature' of a place, about 'young people' of a town, about the 'tall pupils' of a school, etc. The fuzzy set theory was created in response to the need to have a mathematical representation of such kind of situations.

Let U denote the universal set of the discourse. Then a fuzzy set A in U, initiated by Zadeh [1], is defined by means of the membership function m_A , which assigns to each element of U a real value from the interval [0,1]. More specifically

$A = \{(x, m_A(x)), x \in U\}$, where $m_A : U \to [0,1]$

The value $m_A(x)$, called the membership degree of x in A, express the degree to which x verifies the characteristic property of A. Thus, the nearer the value $m_A(x)$ to 1, the higher the membership degree of x in A.

It is always said that the formal mathematical teaching comes from the very exactness' of the science of Mathematics and therefore there is no possible space in Mathematics for any lack of definition, or vagueness. But, the Mathematics teaching skill that teachers have about the various Mathematics teaching methodology concepts is usually imperfect, characterized by a different degree of depth. This suggests that the application of fuzzy set theory in Mathematics teaching could be a very promising tool in order to represent such situations and get useful conclusions.

II. The Model Of Mathematics Teaching

Consider by Sutton [2] learning and teaching Mathematics are both complex active process. Teachers are constantly making decisions as they facilitate a daily learning environment in which they work with their students as active learners. They must also undertake long-term planning to connect daily efforts into the total education of each student. At the same time, teachers share more responsibility for their student's successes with the other factors in educational community, including their colleagues, their institutions and the policies of the educational system.

From [3], Based on review of literature in Mathematics education, the philosophy of Mathematics, the philosophy of education and research on Teaching and Learning, Kuhs and Ball (1986) identified at least four dominant and distinctive views of how Mathematics should be taught.

- Learner focused
  Focuses on learner's personal construction of Mathematical knowledge (Manouchehri & Enderson 2003)
- Content focused with emphasis on conceptual understanding
  Driven by content that is focused on conceptual understanding (Thompson, 1992).
- Content focused with emphasis on performance
  It deals with Student performance, mastery of Mathematical rules and procedures, combined with use of exact, rigorous, Mathematical language (leung 1995)
  It is an instrumentalist view (ernest 1989). It is based on knowledge of Mathematics is demonstrated by correctly answering and solving problems using learned rules.
- Class room focused
  Class room focused with Mathematical teaching based on knowledge about effective class rooms. Class room activity must be well structure and efficiently organized according to effective teacher behaviors identified in process-product studies on teaching effectiveness(Thompson 1992)

Teacher's cognition utilizes in general concepts that are inherently graded and therefore fuzzy.

Finally, concerning the stages of Mathematics Teaching presented above, notice that the learner focused is an introductory stage. Next, we shall consider content focused with emphasis on understanding and performance as a single stage of the whole process. The final stage is class room focused. This hypothesis, without changing the substance of thing at all will make technically easier the development of fuzzy framework for model of Mathematics Teaching.

III. A Fuzzy Set Representation Of The Models Of Mathematics Teaching

For special facts on fuzzy sets and uncertainty theory, we refer freely to [6,9].

Let us consider the group of n teachers $n \geq 2$, during the Mathematics Teaching process in the class room.

We denote $A_i, i = 1,2,3,$ the states of learner focused, content focused with emphasis on understanding and performance, class room focused respectively and by a,b,c,d and e are the linguistic labels of ordinary, inspired, practical, innovative and excellent teaching respectively in each of the A_i's set.

$$U = \{a, b, c, d, e\}$$

We represent the A_i's as fuzzy sets in U. For this $n_{ia}, n_{ib}, n_{ic}, n_{id}$ and n_{ie} respectively denote the number of teachers whose teaching is ordinary, inspired, practical, innovative and excellent at the state $A_i, i = 1,2,3$.

We define the membership function m_{Ai}, in terms of the frequencies, (ie) by

$$m_{Ai}(x) = \frac{n_{ix}}{n}$$

for each x in U. Thus we can write

$$A_i = \left\{\left(x, \frac{n_{ix}}{n}\right) : x \in U\right\}, i = 1,2,3$$

In [4,5,8] we have to represent all possible teacher's profiles (over all states) during the mathematics teaching process we consider a fuzzy relation, say R in U^3 of the form

$$R = \{(s, m_R(s)) : s = (x, y, z) \in U^3\}$$

To determine properly the membership function m_R we give the following definition.

A triple (x,y,z) is said to be well ordered if x corresponds to a degree of success equal or greater than y and y corresponds to a degree of success equal or greater than z.

For example, the profile (c,c,a) is well ordered, while (b,a,c) is not. We define now the membership degree of s to be

$$m_R(s) = m_{A1}(x). m_{A2}(y). m_{A3}(z)$$

If s is a well ordered profile, and zero otherwise. In fact, if for example (b,a,c) possessed a non zero membership degree, given that the degree of success at the state of classroom focused is negligible.

In order to simply our notation we shall write m_s instead of $m_R(s)$. Then the possibility r_s of the profile s is given by

$$r_s = \frac{m_s}{\max\{m_s\}}$$

where $\max\{m_s\}$ denotes the maximal value of m_s, for all s in U^3. In other words r_s is the 'relative membership degree' of s with respect to the other profiles.

In [7] it is further to described how the above model can be used in studying through the calculation of the pseudo –frequencies f(s) $= \sum_{t=1}^{k} m_s(t)$ and the corresponding possibilities $r(s) = \frac{f(s)}{\max\{f(s)\}}$ and the corresponding probability of teachers' profile is $p(s) = \frac{m_s}{\sum_{s \in U^3} m_s}$ the combined results of the performance of two or more groups during the mathematics teaching of the same real situation , or alternatively the performance of the same group during the mathematics teaching process of different situations.

IV. An Application Of Fuzzy Set Theory To The Model Of Mathematics Teaching: A Classroom Experiment

Consider the set of students from the school of Thiruvarur District.They are trained by group of 12 teachers to improve their mathematical skill. Our characterizations of teachers performance at each state of the model of mathematics teaching process involved

- Ordinary teacher, the mathematical skill of the student is not satisfactory.
- Inspired teacher, the mathematical skill of the student is not bad.
- Practical teacher, the students get more marks but less mathematical skill.
- Innovative teacher, the mathematical skill of the student is good.
- Excellent teacher, the mathematical skill of the student is best.

After examine the students' profile , group discussion and feedback, we found that 6,2 and 4 teachers taught practically ,innovative and excellent respectively at the states of learner focused.

Therefore we obtained $n_{ia} = 0, n_{ib} = 0, n_{ic} = 6, n_{id} = 2, n_{ie} = 4$. Thus the stage learner focused was represented as a fuzzy set in U in the form

$$A_1 = \{(a,0),(b,0),(c,\tfrac{6}{12}),\left(d,\tfrac{2}{12}\right),(e,\tfrac{4}{12})\}$$

In the same way we represented the stage content focused with emphasis on understanding and performance and classroom focused as fuzzy set in U by

$$A_2 = \{\left(a,\tfrac{3}{12}\right),\left(b,\tfrac{1}{12}\right),(c,\tfrac{5}{12}),\left(d,\tfrac{3}{12}\right),(e,0)\}$$

$$A_3 = \{\left(a,\tfrac{3}{12}\right),\left(b,\tfrac{2}{12}\right),(c,\tfrac{7}{12}),(d,0),(e,0)\} \text{ respectively.}$$

Using the given definition we calculated the membership degree of the 5^3 in total (ordered samples of 3 objects taken from 5) possible teachers' profiles (see column $m_s(1)$ in table 1 below)

For example , for s = (c,b,a) one finds that

$$m_s = m_{A1}(c).m_{A2}(b).m_{A3}(a)$$
$$= \frac{6}{12}.\frac{1}{12}.\frac{3}{12} = 0.0104$$

It turned out that (c,c,c) was the profile of maximal membership degree 0.1215. Therefore the possibility of each s in U^3 is given by $r_s = \frac{m_s}{0.1215}$

For example . the possibility of (c,b,a) is $\frac{0.0104}{0.1215} \simeq 0.0856$,while the possibility of (c,c,c) is of course 1.

A few days later the same set of students again trained by the another group of 15 teachers to improve their skill. Working as before we found that

$$A_1 = \{(a,0),\left(b,\tfrac{3}{15}\right),(c,\tfrac{7}{15}),\left(d,\tfrac{5}{15}\right),(e,0)\}$$

$$A_2 = \{\left(a,\tfrac{4}{15}\right),\left(b,\tfrac{5}{15}\right),(c,\tfrac{6}{15}),(d,0),(e,0)\}$$

$$A_3 = \{\left(a,\tfrac{2}{15}\right),\left(b,\tfrac{5}{15}\right),(c,\tfrac{8}{15}),(d,0),(e,0)\}$$

Then we calculated the membership degrees of all possible profiles of the teachers group (see columnm$_s(2)$ in table 1). It turned out that (c,c,c) was the profile possessing the maximal membership degree 0.0995 and therefore the possibility of each s is given by

$$r_s = \frac{m_s}{0.0995}.$$

Calculating the possibilities of all profiles for the two groups (see columns $r_s(1)$ and $r_s(2)$ of table 1 below).

The probability of a teachers' profile is defined by

$$p(s) = \frac{m_s}{\sum_{s \in U^3} m_s} = \frac{f(s)}{\sum f(s)}$$

For example , the probability of (c,b,a) is $\frac{0.0311}{1.135} \simeq 0.024$ while the probability of (c,c,c) is 0.1947.

We obtained a qualitative view of teachers' performance during the mathematics teaching process expressed in mathematical terms. Finally the combined results of performance of the two groups were studied by calculating the pseudo-frequencies f(s) and the corresponding possibilities r(s) and probabilities p(s) of all teachers profiles (see table 1).

Table 1: Probabilities and possibilities of teacher's profile with non zero pseudo-frequencies

A_1	A_2	A_3	$m_s(1)$	$r_s(1)$	$m_s(2)$	$r_s(2)$	f(s)	r(s)	p(s)
b	b	b	0	0	0.0222	0.2231	0.0222	0.1005	0.0196
b	b	b	0	0	0.0088	0.0888	0.0088	0.0398	0.0078
c	c	c	0.1215	1	0.0995	1	0.221	1	0.1947
c	c	a	0.0521	0.4288	0.0248	0.2492	0.0769	0.3479	0.0678
c	c	b	0.0347	0.2856	0.0622	0.6251	0.0969	0.4385	0.0854
c	b	b	0.0069	0.0568	0.0518	0.5206	0.0587	0.2656	0.0517
c	b	a	0.0104	0.0856	0.0207	0.208	0.0311	0.1407	0.0274
c	a	a	0.0312	0.2568	0.0166	0.1668	0.0478	0.2163	0.0421
d	d	a	0.0104	0.0856	0	0	0.0104	0.047	0.0092
d	d	b	0.0069	0.0568	0	0	0.0069	0.0312	0.0061
d	d	c	0.0243	0.2	0	0	0.243	0.1099	0.0214
d	a	a	0.0104	0.0856	0.0118	0.1185	0.0222	0.1005	0.196
d	b	a	0.0034	0.027	0.0148	0.1487	0.0182	0.0824	0.16
d	b	b	0.0023	0.0189	0.037	0.3718	0.0393	0.1778	0.0346
d	c	a	0.0174	0.1432	0.0178	0.1789	0.0352	0.1593	0.0310
d	c	b	0.0116	0.0954	0.0444	0.4462	0.056	0.2534	0.4393
d	c	c	0.0405	0.3333	0.0711	0.7145	0.1116	0.5049	0.0983
e	a	a	0.0208	0.1712	0	0	0.0208	0.0941	0.0183

e	b	b	0.0046	0.0379	0	0	0.0046	0.0208	0.0041
e	c	a	0.0347	0.2856	0	0	0.0347	0.157	0.0306
e	c	b	0.0231	0.1901	0	0	0.0231	0.1045	0.0204
e	c	c	0.0810	0.666	0	0	0.0810	0.3665	0.0714
e	d	a	0.0208	0.1712	0	0	0.0208	0.0941	0.0183
e	d	b	0.0139	0.1144	0	0	0.0139	0.0628	0.0122
e	d	c	0.0486	0.4	0	0	0.0486	0.2199	0.0428

Note : $\mathrm{f}(s) = \mathrm{m_s}(1) + \mathrm{m_s}(2) : \sum \mathrm{f}(\mathrm{s}) = 1.135$

V. Conclusions

From the experiment of the previous sections it becomes the evident that use of fuzzy set theory for the mathematical representation of the process of teaching, apart from its theoretical interest, leads to useful quantitative conclusions which can give an effective help the teacher of mathematics to improve their teaching methodology.

It becomes also evident that the same method, with the proper each time modifications, could be used in the classroom during the teaching process not only of mathematics but of many other cognitive subjects.

References

[1]. Zadeh, L.A. Information and Control,8,338-353,1965.
[2]. Sutton, Jhon, Ed. Krueger, Alice Ed, ED Thoughts: What we know about mathematics teaching and learning.
[3]. Models of Mathematics teaching – CCSU. www.math.ccsu.edu/.../models of Mathematics Tea.
[4]. Michael Gr. Voskoglou, Fuzzy Measure For Students' Mathematical Modeling Skills, international journal of fuzzy logic system Vol. 2 No.2, 13-26, April 2012.
[5]. Michael Gr. Voskoglou, The Process Of Learning Mathematics: A Fuzzy Set Approach Http://Www.Ipv.Pt./Millenium/17_Ect4.Htm, March 2014. Page 1-7
[6]. Klir. G.J & Folger,T.A. (1988), Fuzzy Sets And Uncertainty And Information, Prentice Hall London.
[7]. Michael Gr. Voskoglou, (2011), Stochastic and fuzzy models in Mathematics Education, Artificial Intelligence and Management, Lambert Academic Publishing, Saarbrucken, Germany.
[8]. Michael Gr. Voskoglou, The process of learning mathematics: A fuzzy set approach, Heuristics and Didactics of exact sciences, 10, 9-13.
[9]. Klir. G.J/Bo Yuan, Fuzzy sets and fuzzy logic, Theory and applications, Prentice Hall London.(2010)