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Rainy and Dry Days as a Stochastic Process (Albaha City)

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Abstract: *In last years, the rain in Albaha is differ from year to year, this leads to the standard that the rain falls in this city. In this study we try to find a mathematical model represent this observations. Then we try to contact the plain of this study; the problem, goals, the methodology in analysis also the concepts that related to Markov-Chain that applied in the study. Markov-Chain helps for obtaining the model that represent rainy, dry days for the years (1431-1434) for AlBaha (Rain falls). Lastly we test the model by obtaining the matrix for rainy and dry days. by comparing the case in each day and the previous by four days.*
Keywords: *Markov Chains - Rainy Days - Dry Days - Stochastic Process- Rain falls.*

I. The problem:

Knowing of rainy and dry days helps for forecasting, that leads for obtaining a mathematical model represent these days.

II. The Goal:

To build a mathematical model represents rainy and dry days for E-Baha and to find probability distribution for the first recurrence time for each day and the average and also to determine the duration of weather.

III. The Hypothesis:

Rainy and dry days can be represented by Markov-Chain.

IV. The Methodology:

We use the analytical methodology and using Markov-Chain for representing the data.

Previous Studies:

1- Gambil (2003), Analyze the behavior of rain for the period (1975-2001) in Kuala Lumpur Using Markov-Chain.

2- Mirato (2000) Applied Markov-Chain for forecasting the duration of dry days in Asmara (June – September).

3- Abdo El-Malik Applied Markov-Chain for rainy and dry days in Juba (South Sudan).

Markov-Chains:

There is a Markov property and it means (the probability of transition is the process from the case (n) to case (n+1) depends only in the case and not depends on cases before.

If in Markov-Chain the time space and case space is discrete then we call the process Markov-Chain for random variable $x_1, x_2, ...$

Then here the conditional probability depends on x_n that means

$$p(x_{n+1} = k_{n+1} \mid x_1 = k_1, x_2 = k_2 ..., x_n = k_n) = P(x_{n+1} = k_{n+1} = k_n)$$

That mean the probability that the process in time $n+1$ in case k_{n+1} depends only in case that in time n is k_n .

We can write $p(x_{n+1} = j \mid x_n = 1) = p_{ij}$

The transition probability p_{ij} did not depends on time (n) .

That means Markov-Chain is homogeneous

(Tow State Markov-Chain).

Here we suppose we have two cases (0), (1) and the process can be seen in time $0,1,2,3,...$ if the process in case (0) in sometime, then the probability of staying in case (0) for coming time is $p_{00} = (1-\alpha)$ and the transition probability for case (1) is $p_{01} = \alpha$

If in case (1) this transition probability to case (0) in the coming time is

$p_{10} = \beta$

and the probability that will be in case (1) is $p_{11} = (1-\beta)$ one step

The above is called one slap transition probability, it can be presented by a matrix called transition probability matrix as below

$$p = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}$$

p_1^n: The probability process in case (1) in time n.

p_0^n: The probability process in case (0) in time n.

or in a vector as

$$p_n = \begin{bmatrix} p_0^n & p_1^n \end{bmatrix}$$

If vector of initial probability distribution when n=0 the

$$p(0) = \begin{bmatrix} p_0^{(0)} & p_1^{(0)} \end{bmatrix}$$

Then we can rewrite the transition probability matrix as

$$p(n) = \begin{bmatrix} p_{00}^{(n)} & p_{01}^{(n)} \\ p_{10}^{(n)} & p_{11}^{(n)} \end{bmatrix}$$

That means as an example element(ij) in matrix p_n gives the probability for process in case (j) in time n .

V. Probability distribution at equilibrium:

After a period the Markov-Chain will be stationary, (probability will be stationary) and if the process in two steps we can write as a vector:

$$\pi = \begin{bmatrix} \pi_0 & \pi_1 \end{bmatrix}$$

and when stationary

$p(n) = p(n-1) = \pi$ where

$\pi = \pi_p$

And this equation can be solved

$\pi_1 = \dfrac{\alpha}{\alpha+\beta}$ where

$$\pi_0 = 1-\pi_1 = 1-\frac{\alpha}{\alpha+\beta} = \frac{\alpha+\beta-\alpha}{\alpha+\beta}$$

$$\therefore \pi_0 = \frac{\beta}{\alpha+\beta}$$

If the distribution in equilibrium the π is the initial distribution

$$p^{(n)} = p^{(0)} p^n$$

$$p^{(1)} = \pi p = \begin{bmatrix} \dfrac{\beta}{\alpha+\beta} & \dfrac{\alpha}{\alpha+\beta} \end{bmatrix} \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\beta(1-\alpha)}{\alpha+\beta} + \dfrac{\alpha\beta}{\alpha+\beta} & \dfrac{\alpha\beta}{\alpha+\beta} + \dfrac{\alpha(1-\beta)}{\alpha+\beta} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\beta-\alpha\beta+\alpha\beta}{\alpha+\beta} & \dfrac{\alpha\beta+\alpha-\alpha\beta}{\alpha+\beta} \end{bmatrix}$$

$$= \left[\frac{\beta}{\alpha+\beta} \quad \frac{\alpha}{\alpha+\beta} \right] = [\pi_0 \qquad \pi_1] = \pi$$

And by the same method

$$p^{(n)} = \pi$$

That mean $p^{(1)}, p^{(2)}, p^{(3)}, ... = \pi$

And not depends on (n)

VI. Distribution of first recurrence time:

Supposing the process in time (0) and case (0) and it return to case (1) in time (1) then we say T (as a random variable) is a time for that. So the probability distribution for t $(f_{00}^{(1)}, f_{00}^{(2)}, f_{00}^{(3)}, ...$

Where

$f_{00}^{(i)}$ is the probability of returing to case (0) for first time in time (i) and that means

$$p(T = i) = f_{00}^{(i)}$$

Say R represent the sequence till of rainy days in the process in time (1) then the probability of continuous rain till day (n) then it will be dry in day (n+1) is the

$$p(R = n) = (x_1 = 1, x_2 = 1..., x_n = n+1 = 0 / 1 x_1 = 1) = (1-\beta)^{n-1}\beta$$

and this is a geometric distribution by mean $1/\beta$ so the probability distribution for dry sequence days is

$p = (D = n) = (1-\alpha)_{\alpha}^{n-1}$ with mean $1/\alpha$

The application of the model:

	Rainy		Dry
Rainy	288	37	325
Dry	37	6	43

The data represent the amount of rain by day for rainy months and it represents the condition of day and the coming day.

This table represents the sequence of rainy and dry days.

The dry (0), rainy by (1) then the transition probability is

$$p = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}$$

Then we can calculate the transition probability by

$$p = \begin{bmatrix} \frac{288}{325} & \frac{37}{325} \\ \frac{37}{43} & \frac{6}{43} \end{bmatrix} = \begin{bmatrix} 0.886 & 0.114 \\ 0.860 & 0.140 \end{bmatrix}$$

This means if the day is dry, then the probability of it will be for the coming day rainy is 0.114

From matrix p we can obtain the n step matrix

$$p^2 = \begin{bmatrix} 0.886 & 0.114 \\ 0.860 & 0.140 \end{bmatrix} \begin{bmatrix} 0.886 & 0.114 \\ 0.860 & 0.140 \end{bmatrix} = \begin{bmatrix} 0.8836 & 0.1169 \\ 0.88236 & 0.11764 \end{bmatrix}$$

This means if the day is dry, then the probability after two days will be rainy is 0.1169 From above we can calculate 4 steps matrix or 8 steps matrix, at n steps transition matrix then the system will be stationary

$$p^8 = p^4 \times p^4 = \begin{bmatrix} 0.882957 & 0.117043 \\ 0.892957 & 0.117043 \end{bmatrix}$$

Then we can obtain the probability of the process in case (0), (1) as (π_1, π_0)

$$\pi_0 = \frac{\beta}{\alpha+\beta} = \frac{0.860}{0.974} = 0.883$$

$$\pi_1 = \frac{\alpha}{\alpha+\beta} = \frac{0.114}{0.974} = 0.117$$

Then

$$\pi = [\pi_0 \quad \pi_1] = [0.883 \quad 0.117]$$

This assume that the ratio of dry days in the season of rain for the long run is 88.3
Then the distribution of first recurrence time for case (1) is

$$f_{00}^{n} = p(T=n) = \alpha(1-\beta)^{n-2}\beta, n = 2,3,...$$

And by using $\alpha = 0.114$ and $\beta = 0.860$ we get

$$f_{00}^{(n)} = 0.114(1-860)^{n-2} \times 0.860 = 0.114(0.14)^{n-2}(0.860) = 0.098(0.14)^{n-2}$$

$$n = 2,3,...$$

$$f_{00}^{(1)} = 1-\alpha, n = 1$$

$$= 1-0.114 = 0.886, \qquad n = 1$$

Then the probability distribution for first recurrence to case (0) is

$$f_{00}^{n} = \begin{cases} 0.098(0.14)^{n-2} & n = 2,3,... \\ 0.886 & n = 1 \end{cases}$$

Then the day of first recurrence time to case (0) is

$$E(T) = \sum_{n=1}^{\infty} n f_{\infty}^{(n)} = \frac{\alpha+\beta}{\beta} = \frac{1}{0.883} = 1.133$$

Also for case (1) the mean

$$\frac{\alpha+\beta}{\beta} = \frac{1}{0.117} = 8.5470$$

Then the probability distribution for R (rainy days) is $\frac{1}{0.860} = 1.163$ (Geometric distribution) and the probability distribution D (dry days) is $\frac{1}{0.114} = 8.772$

That means the circulation of weather is 10 days, nine are dry and one is rainy.

Testing the model:

Here we try to test the efficiency of the model by obtaining the (4) step transition probability (actual) and comparing by the transition probability model that obtained from the model.

The below is (T.P.M.) with 4 steps (dry and rainy) we compare the case in each day and coming day for four days

281	36	317
37	7	281

$$p^{*4} = \begin{bmatrix} 0.886 & 0.114 \\ 0.841 & 0.159 \end{bmatrix}$$ and compared with

$$p^{4} = \begin{bmatrix} 0.883 & 0.11 \\ 0.883 & 0.11 \end{bmatrix}$$

The variations are not differ, this means the model is with high efficiency for representing the rainy and dry days.

VII. Conclusions and recommendations:

By applying this model with (TPM) it appears 88% from days in the season of rain is rainy.

We get the first recommendation is 1.133 for dry days and 8.547 for rainy days also 9 days are rainy and one day is dry; the model compared by the actual and it explains for comparing of high the forecasting values do not differ from the actual and that means the model is efficiency and represents the data with high prediction so we highly recommend to apply this model for other cities.

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