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Continuous Functions as the Generators of T-norms

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Abstract: *T-norms are generalization of the usual two-valued logical conjunction, studied by classical logic, for fuzzy logics. T-norms are also used to construct the intersection of fuzzy sets or as a basis for aggregation operators. In probabilistic metric spaces, T-norms are used to generalize triangle inequality of ordinary metric spaces. Individual T-norms may of course frequently occur in further disciplines of mathematics, since the class contains many familiar functions. Firstly, we select some continuous function. Then we try to generate T-norms by using those functions.*
Keywords: *Algebraic Product T_P, Hamacher Product T_H, Lukasiewicz T_L*

I. Introduction

In 1942, K. Menger introduced the concept of triangular norm generalizing the classical triangular inequality. In 1960, B. Schweizer and A. Sklar after revision of this work redefined the concept of triangular norm as an associative and commutative binary operation on which is generally accepted today. Since, the T - norms have become important tools in different contexts. They play a fundamental role in probabilistic metric spaces, probabilistic norms and scalar products, multiple-valued logic, fuzzy sets theory. In the later field T-norms are used in order to generate the 'infimum' of fuzzy sets. But apart from the applications of these functions in the fields quoted above, there in an increasing interest in their intrinsic (theoretic) study, which no doubt is going to lead to new probabilities of application.

II. Generators of T-norms

Proposition 2.1: The T-norm Algebraic Product T_P is defined by $T_P(x,y) = xy$ is additively generated by the function $f(x) = -\ln x$.

Proof: Let, $y = f(x) = -\ln x$

$$\Rightarrow y = -\ln x$$
$$\Rightarrow e^{-y} = x$$
$$\Rightarrow x = e^{-y}$$

So, we have

$$f^{(-1)}(x) = \begin{cases} e^{-x} & ; \; x \epsilon [o, f(0)] \\ 0 & ; \; x \epsilon [f(0), \infty] \end{cases}$$

Now,

$$\begin{aligned} T_P(x,y) &= f^{(-1)}(f(x) + f(y)) \\ &= f^{(-1)}(-\ln x - \ln y) \\ &= f^{(-1)}(-\ln xy) \\ &= e^{-(-\ln xy)} \\ &= e^{\ln xy} \\ &= xy. \end{aligned}$$

Hence, the T-norm T_P is additively generated by the function $f(x) = -\ln x$.

Proposition 2.2: The T-norm Lukasiewicz T_L is additively generated by the function $f(x) = 1 - x$.

Proof: Let, $y = f(x) = 1 - x$

$$\Rightarrow x = 1 - y$$

So, we have

$$f^{(-1)}(x) = \begin{cases} 1 - x \, ; x \epsilon [0, f(0)] \\ 0 \quad ; x \epsilon [f(0), \infty] \end{cases}$$

Now,

$$\begin{aligned} T_L(x,y) &= f^{(-1)}\big(f(x)+f(y)\big) \\ &= f^{(-1)}(2-x-y) \\ &= 1-(2-x-y) \quad \text{if } (2-x-y)\epsilon[0,1] \\ &= 1-2+x+y \\ &= x+y-1. \end{aligned}$$

Therefore, the T-norm T_L is generated by the function $f(x) = 1 - x$.

Proposition 2.3: The T-norm Hamacher Product T_H is defined by $T_H(x,y) = \frac{xy}{x+y-xy}$ is additively generated by the function $f(x) = \frac{1-x}{x}$.

Proof: Let, $y = f(x) = \dfrac{1-x}{x}$

$$\Rightarrow y = \frac{1-x}{x}$$

$$\Rightarrow x = \frac{1}{y+1}.$$

So, we have

$$f^{(-1)}(x) = \begin{cases} \dfrac{1}{x+1} & ; \quad x\epsilon[0,1] \\ 0 & ; \quad x\epsilon[1,\infty] \end{cases}$$

Here,

$$\begin{aligned} T_H(x,y) &= f^{(-1)}(f(x)+f(y)) \\ &= f^{(-1)}\left(\frac{1-x}{x}+\frac{1-y}{y}\right) \\ &= f^{(-1)}\left(\frac{y-xy+x-xy}{xy}\right) \\ &= f^{(-1)}\left(\frac{y+x-2xy}{xy}\right) \\ &= \frac{1}{1+\frac{x+y-2xy}{xy}} \\ &= \frac{1}{\frac{xy+x+y-2xy}{xy}} \\ &= \frac{xy}{x+y-xy}. \end{aligned}$$

Therefore, the T-norm **Hamacher product**

$$T_H(x.y) = \frac{xy}{x+y-xy}$$

is additively generated by the function $f(x) = \frac{1-x}{x}$.

Proposition 2.4: The T-norm **Einstein Product** T_E is defined by

$$T_E(x,y) = \frac{xy}{1+(1-x)(1-y)}$$

is additively generated by the function $f(x) = \ln\frac{2-x}{x}$.

Proof: Let, $y = f(x) = \ln\frac{2-x}{x}$

$$\begin{aligned} &\Rightarrow y = \ln\frac{2-x}{x} \\ &\Rightarrow e^y = \frac{2-x}{x} \\ &\Rightarrow x + xe^y = 2 \\ &\Rightarrow x = \frac{2}{e^y+1}. \end{aligned}$$

So, we have $$f^{(-1)}(x) = \begin{cases} \frac{2}{e^x+1} & ; \; x\epsilon[0,f(0)] \\ 0 & ; \; x\epsilon[f(0),\infty] \end{cases}$$

Now,

$$T_E(x, y) = f^{(-1)}(f(x) + f(y))$$

$$= f^{(-1)}\left(\ln\frac{2-x}{x} + \ln\frac{2-y}{y}\right)$$

$$= f^{(-1)}\left(\ln\frac{(2-x)(2-y)}{xy}\right)$$

$$= \frac{2}{e^{\ln\frac{(2-x)(2-y)}{xy}} + 1}$$

$$= \frac{2}{\frac{(2-x)(2-y) + xy}{xy}}$$

$$= \frac{2xy}{(2-x)(2-y) + xy}$$

$$= \frac{2xy}{4 - 2y - 2x + xy + xy}$$

$$= \frac{xy}{2 - y - x + xy}$$

$$= \frac{xy}{1 + 1 - y - x + xy}$$

$$= \frac{xy}{1 + (1-x)(1-y)}.$$

Therefore, the T-norm T_E is additively generated by the function $f(x) = \ln\frac{2-x}{x}$.

III. Figures

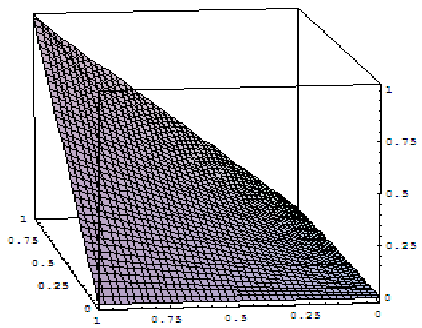

Fig: 2.1 3D Graph of Algebraic Product

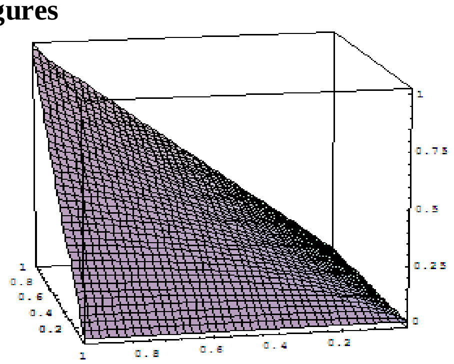

Fig: 2.3 3D Graph of Hamacher Product

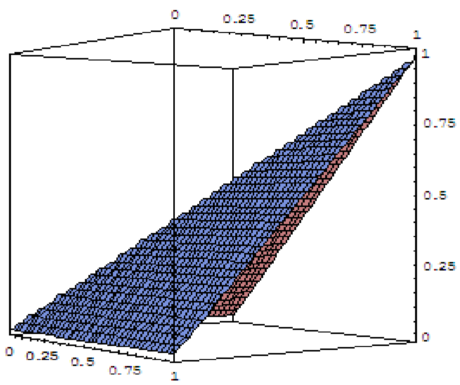

Fig: 2.4 3D Graph of Einstein Product

IV. Conclusion

In this paper, to find the different types of T-norm we have used only strictly decreasing function. The function which has no inverse we have used the definition of Pseudo-inverse. There are huge field for application of T-norm. We can apply T-norm for optimization under fuzzy constraints. There are vast field in business sectors for strategic management portfolio analysis such as growth strategy, leadership strategy, industry attractiveness, industry maturity etc.

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