IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 3 Ver. I (May - Jun. 2015), PP 82-89
www.iosrjournals.org

On N-Derivation in Prime near – Rings

Abdul Rahman H. Majeed , Enaam F.Adhab
Department of Mathematics, college of science, University of Baghdad . Baghdad , Iraq .

Abstract: *The main purpose of this paper is to show that zero symmetric prime left near-rings satisfying certain identities are commutative rings .*

I. Introduction

Let N be a zero symmetric left near – ring (i.e., a left near-ring N satisfying the property 0.x=0 for all x∈ N). we will denote the product of any two elements x and y in N ,i.e.; x.y by xy . The symbol Z will denote the multiplicative centre of N, that is Z={x ∈ N | xy = yx for all y ∈ N} . For any x, y ∈ N the symbol [x, y] = xy - yx stands for multiplicative commutator of x and y, while the symbol x∘y will denote xy+yx . N is called a prime near-ring if xNy = {0} implies either x = 0 or y = 0 . A nonempty subset U of N is called semigroup left ideal (resp. semigroup right ideal) if NU ⊆ U (resp.UN ⊆ U) and if U is both a semigroup left ideal and a semigroup right ideal, it will be called a semigroup ideal. Let I be a nonempty subset of N then a normal subgroup (I,+) of (N, +) is called aright ideal (resp. A left ideal) of N if (x+i)y–xy∈ I for all x,y∈ N and i ∈ I(resp. xi ∈ I for all i ∈ I and x ∈ N). I is called ideal of N if it is both a left ideal as well as a right ideal of N .For terminologies concerning near-rings ,we refer to Pilz [8].

An additive endomorphism d :N ⟶N is said to be a derivation of N if d(xy) = xd(y) + d(x)y , or equivalently , as noted in [5 , lemma 4] that d(xy) = d(x)y + xd(y) for all x,y ∈ N .

A map d: $\underbrace{N \times N \times \ldots \times N}_{n-times} \longrightarrow N$ is said to be permuting if the equation $d(x_1, x_2, \ldots, x_n) = d(x_{\pi(1)}, x_{\pi(2)}, \ldots, x_{\pi(n)})$ holds for all $x_1, x_2, \ldots, x_n \in N$ and for every permutation $\pi \in S_n$ where S_n is the permutation group on $\{1,2, \ldots, n\}$.

Let n be a fixed positive integer . An additive (i.e. ; additive in each argument) mapping d:$\underbrace{N \times N \times \ldots \times N}_{n-times} \longrightarrow N$ is said to be n-derivation if the relations

$$d(x_1 x_1', x_2, \ldots, x_n) = d(x_1, x_2, \ldots, x_n)x_1' + x_1 d(x_1', x_2, \ldots, x_n)$$

$$d(x_1, x_2 x_2', \ldots, x_n) = d(x_1, x_2, \ldots, x_n)x_2' + x_2 d(x_1, x_2', \ldots, x_n)$$

$$\vdots$$

$$d(x_1, x_2, \ldots, x_n x_n') = d(x_1, x_2, \ldots, x_n)x_n' + x_n d(x_1, x_2, \ldots, x_n')$$

Hold for all $x_1, x_1', x_2, x_2', \ldots, x_n, x_n' \in N$. If in addition d is a permuting map then d is called a permuting n-derivation of N .

Many authors studied the relationship between structure of near – ring N and the behaviour of special mapping on N. There are several results in the existing literature which assert that prime near-ring with certain constrained derivations have ring like behaviour . Recently several authors (see [1–6] for reference where further references can be found) have investigated commutativeity of near-rings satisfying certain identities. Motivated by these results now we shall consider n-derivation on a near-ring N and show that prime near-rings satisfying some identities involving n-derivations and semigroup ideals or ideals are commutative rings. In fact, our results generalize some known results viz. Theorems 1,2,3,4,5,6,7 [2] .

II. Preliminary Results

We begin with the following lemmas which are essential for developing the proofs of our main results. Proof of first lemma can be seen in [5, Lemma 3] while those of next three can be found in [4] and the last four can be found in [5] .

Lemma 2.1. Let N be a prime near-ring and U a nonzero semigroup ideal of N . If $x, y \in N$ and $xUy = \{0\}$ then $x = 0$ or $y = 0$.

Lemma 2.2. Let N be a prime near-ring . then d is permuting n-derivation of N if and only if
$d(x_1\, x_1{}', x_2, \dots, x_n) = x_1\, d(x_1{}', x_2, \dots, x_n) + d(x_1, x_2, \dots, x_n)x_1{}'$
for all $x_1, x_1{}', x_2, \dots, x_n \in$ N .

Lemma 2.3. Let N be a prime near-ring admitting a nonzero permuting n-derivation d such that d(N, N, . . ., N) $\subseteq$ Z then N is a commutative ring.

Lemma 2.4. Let N be a near-ring .Let d be a permuting n-derivation of N . Then for every $x_1, x_1{}', x_2, \dots, x_n, y \in$ N ,
(i) $(x_1\, d(x_1{}', x_2, \dots, x_n) + d(x_1, x_2, \dots, x_n)x_1{}')y = x_1\, d(x_1{}', x_2, \dots, x_n)y + d(x_1, x_2, \dots, x_n)x_1{}'y$,
(ii) $(d(x_1, x_2, \dots, x_n)x_1{}' + x_1\, d(x_1{}', x_2, \dots, x_n))y = d(x_1, x_2, \dots, x_n)x_1{}'y + x_1\, d(x_1{}', x_2, \dots, x_n)y$.

Remark 2.1. It can be easily shown that above lemmas (2.2 - 2.4) also hold if d is a nonzero n-derivation of near-ring N .

Lemma 2.5. Let d be an n-derivation of a near ring N . then d(Z,N,…,N) $\subseteq$ Z .

Lemma 2.6. Let N be a prime near ring , d a nonzero n-derivation of N , and $U_1,U_2,\dots,U_n$ be a nonzero semigroup left ideals of N . If $d(U_1,U_2,\dots,U_n) \subseteq Z$, then N is a commutative ring .

Lemma 2.7. Let N be a prime near ring ,d a nonzero n-derivation of N .and $U_1,U_2,\dots,U_n$ be a nonzero semigroup ideals of N such that $d([x, y],u_2,\dots,u_n) = 0$ for all $x, y \in U_1, u_2 \in U_2,\dots,u_n \in U_n$, then N is a commutative ring .

Lemma 2.8. Let N be a prime near-ring, d a nonzero n-derivation of N and U1, U2, . . ., Un be nonzero semigroup ideals of N.
(i) If x ∈ N and D(U1, U2, . . ., Un)x = {0}, then x = 0.
(ii) If x ∈ N and xD(U1, U2, . . ., Un) = {0}, then x = 0.

Main Result .

Theorem (2.1) Let N be a prime near ring which admits a nonzero n-derivation d , if $U_1,U_2,\dots,U_n$ are semigroup ideals of N ,then the following assertions are equivalent
(i) $d([x, y],u_2,\dots,u_n)=[d(x,u_2,\dots,u_n),y]$ for all $x,y \in U_1, u_2 \in U_2,\dots,u_n \in U_n$.
(ii) $[d(x,u_2,\dots,u_n),y]=[x, y]$ for all $x,y \in U_1, u_2 \in U_2,\dots,u_n \in U_n$
(iii) N is a commutative ring .

Proof. It is easy to verify that (iii)⇒(i) and (iii) ⇒ (ii) .
(i) ⇒ (iii) Assume that
$d([x, y],u_2,\dots,u_n)=[d(x,u_2,\dots,u_n),y]$ for all $x,y \in U_1,u_2 \in U_2,\dots,u_n \in U_n$ (1)

If we take y = x in (1) we get $[d(x,u_2,\dots,u_n),x]=0$, that is
$d(x,u_2,\dots,u_n)x = xd(x,u_2,\dots,u_n)$ for all $x \in U_1, u_2 \in U_2,\dots,u_n \in U_n$ (2) Replacing y by xy in (1) we get $d([x,xy],u_2,\dots,u_n)=[d(x,u_2,\dots,u_n),xy]$,then $d(x[x,y],u_2,\dots,u_n) = [d(x,u_2,\dots,u_n),xy]$, by definition of d we get $d(x,u_2,\dots,u_n)[x,y]+xd([x,y],u_2,\dots,u_n)=[d(x,u_2,\dots,u_n), xy]$, by using (1) again we get $d(x,u_2,\dots,u_n)[x,y]+x[d(x,u_2,\dots,u_n),y]=[d(x,u_2,\dots,u_n),xy]$, previous equation can be reduced to $xd(x,u_2,\dots,u_n)y = d(x,u_2,\dots,u_n)yx$, by (2) the previous equation yields
$d(x,u_2,\dots,u_n)xy = d(x,u_2,\dots,u_n)yx$ for all $x,y \in U_1, u_2 \in U_2,\dots,u_n \in U_n$ (3)

if we replace y by yr, where $r \in$ N , in (3) and using it again we get $d(x,u_2,\dots,u_n)y[x,r] = 0$, that is
$d(x,u_2,\dots,u_n)U_1[x,r] = 0$ for all $x \in U_1, u_2 \in U_2,\dots,u_n \in U_n$, $r \in$ N . (4)
By using lemma 2.1 ,we conclude that for each $x \in U_1$ either $x \in Z$ or $d(x,u_2,\dots,u_n) = 0$, but using lemma 2.5 lastly we get $d(x,u_2,\dots,u_n) \in Z$ for all $x \in U_1, u_2 \in U_2,\dots,u_n \in U_n$, i.e., $d(U_1,U_2,\dots,U_n) \subseteq Z$. Now by using lemma 2.6 we find that N is commutative ring .
(ii) ⇒ (iii) suppose that
$[d(x,u_2,\dots,u_n),y] = [x, y]$ for all $x,y \in U_1, u_2 \in U_2,\dots,u_n \in U_n$. (5)

If we take y = x in (5) , we get
$d(x,u_2,...,u_n)x = xd(x,u_2,...,u_n)$ for all $x \in U_1, u_2 \in U_2,...,u_n \in U_n$. (6)

Replacing x by yx in (5) and using it again , we get
$[d(yx,u_2,...,u_n),y] = [yx, y] = y[x, y] = y[d(x,u_2,...,u_n),y]$ for all $x,y \in U_1, u_2 \in U_2,...,u_n \in U_n$, so we have $d(yx,u_2,...,u_n)y - yd(yx,u_2,...,u_n) = yd(x,u_2,...,u_n)y - y^2d(x,u_2,...,u_n)$.

In view of lemmas 2.2 and 2.4 the last equation can be rewritten as
$yd(x,u_2,...,u_n)y + d(y,u_2,...,u_n)xy - (yd(y,u_2,...,u_n)x + y^2d(x,u_2,...,u_n)) = yd(x,u_2,...,u_n)y - y^2d(x,u_2,...,u_n)$, so we have $d(y,u_2,...,u_n)xy = yd(y,u_2,...,u_n)x$,by using (6) we have
$d(y,u_2,...,u_n)xy = d(y,u_2,...,u_n)yx$ for all $x,y \in U_1, u_2 \in U_2,...,u_n \in U_n$. (7)

Since equation (7) is the same as equation (3), arguing as in the proof of (i) ⇒ (iii) we find that N is a commutative ring .

Corollary (2.2) Let N be a prime near ring which admits a nonzero n-derivation d , then the following assertions are equivalent
(i) $d([x_1,y],x_2,...,x_n) = [d(x_1,x_2,...,x_n),y]$ for all $x_1, x_2,...,x_n, y \in N$.
(ii) $[d(x_1,x_2,...,x_n),y] = [x_1,y]$ for all $x_1, x_2,..., x_n, y \in N$.
(iii) N is a commutative ring .

Corollary (2.3) Let N be a prime near-ring .U is a nonzero semigroup ideal of N . If N admits a nonzero derivation d then the following assertions are equivalent
(i) $d([x, y]) = [d(x),y]$ for all $x, y \in U$.
(ii) $[d(x), y] = [x, y]$ for all $x, y \in U$.
(iii) N is commutative ring .

Corollary (2.4) ([2],theorem(1))Let N be a prime near-ring . If N admits a nonzero derivation d then the following assertions are equivalent
(i) $d([x, y]) = [d(x),y]$ for all $x, y \in N$.
(ii) $[d(x),y] = [x, y]$ for all $x, y \in N$.
(iii) N is commutative ring .

Theorem (2.5) Let N be a 2-torsion free prime near ring, if $U_1,U_2,...,U_n$ are nonzero ideals of N , d is a nonzero n-derivation.Then the following assertions are equivalent
(i) $d([x, y],u_2,...,u_n) \in Z$ for all $x,y \in U_1, u_2 \in U_2,...,u_n \in U_n$.
(ii) N is a commutative ring .

Proof. It is clear that (ii) ⇒ (i).
(i) ⇒ (ii) . $d([x, y],u_2,...,u_n) \in Z$ for all $x,y \in U_1, u_2 \in U_2,...,u_n \in U_n$. (8)
(1) If Z= {0} then $d([x,y],u_2,...,u_n) = 0$ for all $x,y \in U_1, u_2 \in U_2,...,u_n \in U_n$.

By lemma 2.7 , we conclude that N is a commutative ring .
(2) If Z ≠ {0} , replacing y by zy in (8) where $z \in Z$, we get $d([x, zy],u_2,...,u_n) = d(z[x, y],u_2,...,u_n) \in Z$ for all $x,y \in U_1, u_2 \in U_2,...,u_n \in U_n, z \in Z$. That is mean $d(z[x,y],u_2,...,u_n)\, r = rd(z[x, y],u_2,...,u_n)$ for all $x,y \in U_1, u_2 \in U_2,...,u_n \in U_n, z \in Z, r \in N$. By using lemma 2.4 we get
$d(z,u_2,...,u_n)[x, y]r + zd([x, y],u_2,...,u_n)r = rd(z,u_2,...,u_n)[x, y] + rzd([x, y],u_2,...,u_n)$
Using (8) the previous equation implies
$[d(z,u_2,...,u_n)[x, y] ,r] = 0$ for all $x,y \in U_1, u_2 \in U_2,...,u_n \in U_n, z \in Z, r \in N$.
Accordingly , $0 = [d(z,u_2,...,u_n)[x, y], r] = d(z,u_2,...,u_n)[[x, y], r]$ for all $r \in N$. Then we get $td(z,u_2,...,u_n)[[x, y], r] = 0$ for all $t \in N$, so by lemma 2.5 we get
$d(z,u_2,...,u_n)N[[x,y],r] = 0$ for all $x,y \in U_1, u_2 \in U_2,...,u_n \in U_n, z \in Z, r \in N$. (9)

Primeness of N yields either $d(Z,U_2,...,U_n)=0$ or $[[x, y], r] = 0$ for all $x,y \in U_1, r \in N$.
Assume that $[[x, y] , r] = 0$ for all $x,y \in U_1, r \in N$ (10)
Replacing y by xy in (10) yields

$[[x, xy], r] = 0$ and therefore $[x[x,y], r] = 0$, hence $[x,y][x, r] = 0$ for all $x,y \in U_1, r \in N$, so we get $[x, y]N[x, r] = 0$ for all $x,y \in U_1, r \in N$. (11)

Primeness of N implies that either $[x,y] = 0$ for all $x,y \in U_1$, or $x \in Z$ for all $x \in U_1$. If $[x,y] = 0$ for all $x,y \in U_1$ then we get $d([x, y],u_2,...,u_n) = 0$ for all $x,y \in U_1, u_2 \in U_2,...,u_n \in U_n$ and by lemma 2.7 we get the required result , now assume that $x \in Z$ for all $x \in U_1$, then by lemma 2.5 we obtain that $d(U_1,U_2,...,U_n) \subseteq Z$. Now by using lemma(1.16) we find that N is commutative ring .

On the other hand , if $d(Z,U_2,...,U_n) = 0$,then $d(d([x,y],u_2,...,u_n),u_2,...,u_n) = 0$ for all $x,y \in U_1, u_2 \in U_2,...,u_n \in U_n$, replace y by xy in the previous equation we get
$0 = d(d([x,xy],u_2,...,u_n),u_2,...,u_n) = d(d(x[x,y],u_2,...,u_n),u_2,...,u_n) = d(d(x,u_2,...,u_n)[x,y] + xd([x,y],u_2,...,u_n),u_2,...,u_n) = d(d(x,u_2,...,u_n)[x, y], u_2,...,u_n) + d(xd([x, y] ,u_2,...,u_n) ,u_2,...,u_n) = d(d(x,u_2,...,u_n), u_2,...,u_n)[x, y] + d(x,u_2,...,u_n)d([x, y], u_2,...,u_n) + d(x,u_2,...,u_n)d([x,y],u_2,...,u_n)+ xd(d([x,y],u_2,...,u_n),u_2,...,u_n))$, hence we get
$d(d(x,u_2,...,u_n) , u_2,...,u_n)[x,y] + 2d(x,u_2,...,u_n)d([x,y], u_2,...,u_n) = 0$ for all $x,y \in U_1, u_2 \in U_2,...,u_n \in U_n$. (12)

Replace x by $[x_1,y_1]$ in (12) ,where $x_1,y_1 \in U_1$, we get $2d([x_1,y_1],u_2,...,u_n)d([[x_1,y_1],y], u_2,...,u_n) = 0$ for all $x_1,y_1,y \in U_1, u_2 \in U_2,...,u_n \in U_n$, but N is 2-torsion free so we obtain $d([x_1,y_1],u_2,...,u_n)d([[x_1,y_1],y], u_2,...,u_n) = 0$ for all $x_1,y_1,y \in U_1, u_2 \in U_2,...,u_n \in U_n$.

From(8) we get
$d([x_1,y_1],u_2,...,u_n)Nd([[x_1,y_1],y], u_2,...,u_n) = 0$, primeness of N yields
either $d([x_1,y_1],u_2,...,u_n) = 0$ for all $x_1,y_1 \in U_1, u_2 \in U_2,...,u_n \in U_n$ and by lemma 2.7 we conclude that N is commutative ring .
or $d([[x_1,y_1],y],u_2,...,u_n)=0$ for all $x_1,y_1,y \in U_1,u_2 \in U_2,...,u_n \in U_n$, hence $0 = d(([x_1,y_1]y-y[x_1,y_1]) , u_2,...,u_n) = d([x_1,y_1]y,u_2,...,u_n) - d(y[x_1,y_1],u_2,...,u_n)=[x_1,y_1]d(y,u_2,...,u_n)+ d([x_1,y_1],u_2,...,u_n)y- (yd([x_1,y_1],u_2,...,u_n) + d(y, u_2,...,u_n)[x_1,y_1])$, using(8) in the last equation yields
$[x_1,y_1]d(y, u_2,...,u_n))=d(y, u_2,...,u_n)[x_1,y_1]$ for all $x_1,y_1,y \in U_1, u_2 \in U_2,...,u_n \in U_n$. (13)
Let $x_2,y_2,t \in U_1$, then $t[x_2,y_2] \in U_1$, hence we can taking $t[x_2,y_2]$ instead of y in (13) to get $[x_1,y_1]d(t[x_2,y_2],u_2,...,u_n)) = d(t[x_2,y_2], u_2,...,u_n)[x_1,y_1]$, hence $[x_2,y_2]d(t[x_2,y_2], u_2,...,u_n) = d(t[x_2,y_2], u_2,...,u_n) [x_2,y_2]$, therefore
$[x_2,y_2](d(t,u_2,...,u_n)[x_2,y_2]+ [x_2,y_2]td([x_2,y_2], u_2,...,u_n) = d(t,u_2,...,u_n)[x_2,y_2]^2 + td([x_2,y_2] , u_2,...,u_n) [x_2,y_2]$, using (12)and(8) implies

$d([x_2,y_2],u_2,...,u_n) [x_2,y_2]t= d([x_2,y_2],u_2,...,u_n)t[x_2,y_2]$, so we have
$d([x_2,y_2],u_2,...,u_n) [[x_2,y_2],t]=0$. i.e ; $d([x_2,y_2],u_2,...,u_n)N[[x_2,y_2],t]= \{0\}$ for all $t \in U$.
Primeness of N yields that
$d([x_2,y_2],u_2,...,u_n)=0$ or $[[x_2,y_2],t]=0$ for all $t \in U_1$, if $d([x_2,y_2],u_2,...,u_n)=0$ then by lemma 2.7 we conclude that N is commutative ring .

Now , when $[[x_2,y_2],t]=0$ for all $t \in U_1$, Replacing y_2 by x_2y_2 in previous equation yields
$[[x_2, x_2y_2], t] = 0$ and therefore $[x_2[x_2 , y_2], t] = 0$, hence $[x_2,y_2][x_2, t] = 0$ for all $x_2,y_2 ,t \in U_1$, so we get $[x_2, y_2] U_1 [x_2, t] = 0$,by lemma 2.1 we get $[x_2, y_2] = 0$ for all $x_2,y_2 \in U_1$ so we have $d([x_2,y_2],u_2,...,u_n)=0$ then by lemma 2.7 we find that N is commutative ring .

Corollary(2.6) Let N be a 2-torsion free prime near ring, if d is a nonzero n-derivation of N .Then the following assertions are equivalent
(i) $d([x_1,y],x_2,...,x_n) \in Z$ for all $x_1,x_2,..., x_n ,y \in N$.
(ii) N is a commutative ring

C**orollary(2.7)** Let N be a 2-torsion free prime near ring, U is a nonzero ideal of N. If d is a nonzero derivation of N .Then the following assertions are equivalent
(i) $d([x, y]) \in Z$ for all $x, y \in U$.
(ii) N is a commutative ring .

C**orollary(2.8)([2],Theorem 2)** Let N be a 2-torsion free prime near ring, if d is a nonzero derivation of N .Then the following assertions are equivalent
(i) $d([x, y]) \in Z$ for all $x, y \in N$.
(ii) N is a commutative ring .

Theorem(2.9) Let N be a prime near ring, if $U_1,U_2,...,U_n$ are nonzero semigroup ideals of N ,d is a nonzero n-derivation .Then the following assertions are equivalent

(i) $[d(u_1,u_2,...,u_n),y] \in Z$ for all $u_1 \in U_1, u_2 \in U_2,...,u_n \in U_n, y \in N$.
(ii) N is a commutative ring .

Proof. It is clear that (ii) ⇒ (i).
(i) ⇒ (ii) . $[d(u_1,u_2,...,u_n),y] \in Z$ for all $u_1 \in U_1, u_2 \in U_2,...,u_n \in U_n$, $y \in N$ (14)
Replacing y by $d(u_1,u_2,...,u_n)$ y in (14), we get $[d(u_1,u_2,...,u_n), d(u_1,u_2,...,u_n)y] \in Z$,
that is $[[d(u_1,u_2,...,u_n), d(u_1,u_2,...,u_n)y],t] = 0$ for all $u_1 \in U_1, u_2 \in U_2,...,u_n \in U_n$ and y, t $\in$ N.
Then we get $[d(u_1,u_2,...,u_n) [d(u_1,u_2,...,u_n),y],t] = 0$, hence
$d(u_1,u_2,...,u_n) [d(u_1,u_2,...,u_n),y]t = td(u_1,u_2,...,u_n) [d(u_1,u_2,...,u_n),y]$,by using (14) we get
$[d(u_1,u_2,...,u_n),y][d(u_1,u_2,...,u_n) ,t] = 0$
for all $u_1 \in U_1, u_2 \in U_2,...,u_n \in U_n$ and y, t $\in$ N (15)
In view of (14) , equation (15) assures that
$[d(u_1,u_2,...,u_n),y]N[d(u_1,u_2,...,u_n) ,y] = 0$
for all $u_1 \in U_1, u_2 \in U_2,...,u_n \in U_n, y \in N$ (16)
Primeness of N shows that $[d(u_1,u_2,...,u_n) ,y]= 0$ for all $u_1 \in U_1, u_2 \in U_2,...,u_n \in U_n$, $y \in N$,
Hence $d(U_1,U_2,...,U_n) \subseteq Z$ .Then by lemma 2.6 we conclude that N is a commutative ring .

Corollary(2.10) Let N be a prime near-ring, if d is a nonzero n-derivation of N. Then the following assertions are equivalent
(i) $[d(x_1,x_2,...,x_n),y] \in Z$ for all $x_1, x_2,...,x_n, y \in N$.
(ii) N is a commutative ring .

Corollary(2.11) Let N be a prime near ring , U is a nonzero semigroup ideal of N . If d is a nonzero derivation of N .Then the following assertions are equivalent
(i) $[d(x),y] \in Z$ for all $x ,y \in U$.
(ii) N is a commutative ring .

Corollary(2.12) ([2]Theorem 3) Let N be a prime near ring , if d is a nonzero derivation of N .Then the following assertions are equivalent
(i) $[d(x),y] \in Z$ for all $x ,y \in N$.
(ii) N is a commutative ring .

Theorem(2.13) Let N be a 2-torsion free prime near ring, then there exists no nonzero n-derivation d of N such that $d(x_1,x_2,...,x_n)\circ y = x_1\circ y$ for all $x_1, x_2,...,x_n, y \in N$.

Proof .
$d(x_1,x_2,...,x_n)\circ y = x_1\circ y$ for all $x_1, x_2,...,x_n, y \in N$ **.** (17)
replacing x_1 by yx_1 in(17),we get $d(yx_1,x_2,...,x_n)\circ y=(yx_1)\circ y= y(x_1\circ y) =y(d(x_1,x_2,...,x_n)\circ y)$
since $d(yx_1,x_2,...,x_n)\circ y= d(yx_1,x_2,...,x_n)y + yd(yx_1,x_2,...,x_n)$, by using lemma (1.13) , we obtain
$yd(x_1,x_2,...,x_n) y +d(y,x_2,...,x_n) x_1y + yd(y,x_2,...,x_n)x_1+ y^2 d(x_1,x_2,...,x_n) = yd(x_1,x_2,...,x_n)y + y^2d(x_1,x_2,...,x_n)$, hence we get
$d(y,x_2,...,x_n)x_1y + yd(y,x_2,...,x_n)x_1= 0$ for all $x_1,x_2,...,x_n,y \in N$. (18)

Replacing x_1 by zx_1 in (18),where $z \in N$, we get
$d(y,x_2,...,x_n) zx_1y + yd(y,x_2,...,x_n)zx_1= 0$, for all $x_1, x_2,...,x_n, y, z \in N$ that is
$d(y,x_2,...,x_n) zx_1y = - yd(y,x_2,...,x_n)zx_1 = (- yd(y,x_2,...,x_n)z)x_1 = d(y,x_2,...,x_n)zy x_1$, therefore $d(y,x_2,...,x_n) zx_1y - d(y,x_2,...,x_n)zy x_1 = 0$, hence $d(y,x_2,...,x_n) z(x_1y - y x_1)=0$ for all $x_1, x_2,...,x_n, y, z \in N$, so we obtain $d(y,x_2,...,x_n) N(x_1y - y x_1)=0$, primeness of N yields that $d(N,N,..N)=0$ or $y \in Z$, since d is a nonzero n-derivation of N we conclude $y \in Z$ for all $y \in N$,since N is 2-torsion free therefore (17) implies that $yd(x_1,x_2,...,x_n) =y x_1$ for all $x_1, x_2,...,x_n, y, \in N$, which implies that $yd(xx_1,x_2,...,x_n)=yx x_1$ for all $x_1, x_2,...,x_n, x,y \in N$.
hence $yd(x,x_2,...,x_n) x_1+ yxd(x_1,x_2,...,x_n) = yx x_1$,hence $yxd(x_1,x_2,...,x_n) = 0$ for all $x_1, x_2,...,x_n, x,y \in N$. i.e.; $yNd(x_1,x_2,...,x_n)=0$.By primeness of N and $d \neq 0$,we conclude that y=0 for all $y \in N$; a contradiction .

Theorem (2.14) Let N be a 2-torsion free prime near ring which admits a nonzero n-derivation , then the following assertions are equivalent
(i) $d(x\circ y,x_2,...,x_n) \in Z$ for all $x,y ,x_2,...,x_n \in N$.
(ii) N is a commutative ring .

Proof. It is easy to verify that (ii)$\Rightarrow$(i) .
(i) $\Rightarrow$(ii). Assume that $d(x\circ y, x_2,..., x_n) \square Z$ for all $x,y,x_2,...,x_n \square N$ (19)
(1) If $Z= \{0\}$ then $d(x\circ y,x_2,...,x_n) = 0$ for all $x,y ,x_2,...,x_n \square N$.
Replacing y by xy in (19) we obtain $0 = d(x\circ xy,x_2,...,x_n) = d(x(x\circ y),x_2,...,x_n) = xd(x\circ y,x_2,...,x_n) + d(x,x_2,...,x_n)(x\circ y)$, we get $d(x,x_2,...,x_n)(x\circ y) = 0$ for all $x,y ,x_2,...,x_n \square N$, thus $d(x,x_2,...,x_n)yx = - d(x,x_2,...,x_n)xy$ for all $x,y,x_2,...,x_n\square N$. (20)

Replacing y by zy in (20) and using (20) again , we get
$d(x,x_2,...,x_n)zyx = - d(x,x_2,...,x_n)xzy = (- d(x,x_2,...,x_n)xz) y = d(x,x_2,...,x_n)zxy$ for all $x ,y ,x_2,...,x_n \square N$.
That is $d(x,x_2,...,x_n)z[x, y] = 0$ for all $x,y,x_2,...,x_n ,z\square N$ .i.e.; $d(x,x_2,...,x_n)N[x, y] = 0$, primeness of N yields either $d(x,x_2,...,x_n) = 0$ or $[x,y]=0$, it follows that either $d(x,x_2,...,x_n) = 0$ or $x \square Z$ for all $x \square N$, but $x \square Z$ also implies $d(x,x_2,...,x_n) \square Z$, hence $d(N,N,..,N) \subseteq Z$ and using lemma 2.3 we conclude that N is a commutative ring .
(2)) If $Z \neq \{0\}$. Replacing y by zy in (18) where $z \square Z$,we get $d((x\circ zy),x_2,...,x_n) \square Z$, that is $d(z(x\circ y),x_2,...,x_n) \square Z$ for all $x,y ,x_2,...,x_n \square N ,z \square Z$, that is mean $d(z(x\circ y),x_2,...,x_n)r = rd(z(x\circ y),x_2,...,x_n)$ for all $r \square N$. then we have $d(z,x_2,...,x_n) (x\circ y)r+ zd((x\circ y),x_2,...,x_n)r = rd(z,x_2,...,x_n) (x\circ y) + rzd((x\circ y),x_2,...,x_n)$, by (18) we get
$d(z,x_2,...,x_n) (x\circ y) \square Z$ for all $x,y ,x_2,...,x_n \square N, z \square Z$. (21)
By lemma 2.5 we have $d(z,x_2,...,x_n) \square Z$ so (21) yields that $0 = [d(z,x_2,...,x_n) (x\circ y),t] = d(z,x_2,...,x_n)[(x\circ y),t]$, hence $d(z,x_2,...,x_n)N[(x\circ y),t] = 0$ for all $x,y ,x_2,...,x_n ,t \square N, z \square Z$. By primeness of N , the last equation forces either $d(Z,N,...,N) = \{0\}$ or $x\circ y \square Z$ for all $x,y \square N$.
Suppose that $d(Z,N,...,N)=\{0\}$, if $0 \neq y \square Z$, then $d(x\circ y,x_2,...,x_n) = d(xy + yx, x_2,...,x_n) = d(xy ,x_2,...,x_n) + d(yx, x_2,...,x_n) = d(x ,x_2,...,x_n)y +x d(y ,x_2,...,x_n)+ yd(x ,x_2,...,x_n) + d(y ,x_2,...,x_n)x = d(x ,x_2,...,x_n)y + d(x ,x_2,...,x_n)y$, since $d(x\circ y,x_2,...,x_n) \square Z$, hence $0 = d(d(x\circ y,x_2,...,x_n),x_2,...,x_n) = d((d(x ,x_2,...,x_n)y + d(x ,x_2,...,x_n)y), x_2,...,x_n) = d((d(x ,x_2,...,x_n)y,x_2,...,x_n) + d(d(x ,x_2,...,x_n)y, x_2,...,x_n)$, using the definition of d implies that $d((d(x ,x_2,...,x_n),x_2,...,x_n)y+ d(x ,x_2,...,x_n)d((y, x_2,...,x_n)+d((d(x ,x_2,...,x_n),x_2,...,x_n)y + d(x ,x_2,...,x_n) d((y, x_2,...,x_n) = 0$,hence

$d((d(x ,x_2,...,x_n),x_2,...,x_n)y+ d((d(x ,x_2,...,x_n),x_2,...,x_n)y = 0$, since $y \square Z$,then we get
$y(d((d(x ,x_2,...,x_n),x_2,...,x_n)+ d((d(x ,x_2,...,x_n),x_2,...,x_n)) = 0$, hence we get
$y N 2d((d(x ,x_2,...,x_n),x_2,...,x_n)$, since N is 2-torsion free prime and $y \neq 0$ then we get
$d((d(x ,x_2,...,x_n),x_2,...,x_n)= 0$ for all $x ,x_2,...,x_n \square N$,
$0=d((d(x^2,x_2,...,x_n),x_2,...,x_n)=d(xd(x,x_2,...,x_n),x_2,...,x_n)+d(d(x,x_2,...,x_n)x,x_2,...,x_n$
$=d(x ,x_2,...,x_n)d(x ,x_2,...,x_n)+xd(d(x,x_2,...,x_n),x_2,...,x_n) + d(x,x_2,...,x_n)d(x,x_2,...,x_n)+ d(d(x,x_2,...,x_n),x_2,...,x_n) x =2 d(x ,x_2,...,x_n)d(x ,x_2,...,x_n)$, but N is 2- torsion free , so we get $d(x ,x_2,...,x_n)d(x ,x_2,...,x_n)=0$ for all $x,x_2,...,x_n \square N$, hence get $d(x ,x_2,...,x_n)d(N ,N,...,N) = 0$ by lemma 2.8 we get $d(x ,x_2,...,x_n) = 0$,but $x, x_2,...,x_n$ are arbitrary element of N , thus we conclude that $d = 0$. This leads to a contradiction .Accordingly we have $x\circ y \square Z$ for all $x,y \square N$.

If $0 \neq y \square Z$, we have $x\circ y \square Z$,that is $x\circ y =y(x+x) \square Z$,it follows that $y(x+x)r = r y(x+x)$ for all $r \square N$ and. it follows that $y[x+x ,r] = 0$, so we get $yN[x+x,r] = 0$,since N is prime and $y \neq 0$ then we conclude that $x+x \square Z$ for all $x \square N$, since $x\circ y \square Z$ then $x\circ x \square Z$, hence $x^2+x^2 \square Z$ for all $x \square N$.
Thus $(x+x)tx = tx(x+x) = t(x^2+x^2) = (x^2+x^2)t =x(x+x)t = (x+x)xt$ for all $x,t \square N$ and therefore $(x+x)N[x,t] = 0$ for all $x,t \square N$,primeness of N yields $x \square Z$ or $2x = 0$,since N is 2-torsion free consequently , in both case we arrive at $x \square Z$ for all $x \square N$. Hence $d(N,N,...,N) \subseteq Z$ and lemma 2.3 assures that N is a commutative ring .

Corollary (2.15)([2] Theorem 5) Let N be a 2-torsion free prime near ring which admits a nonzero derivation d , then the following assertions are equivalent
(i) $d(x\circ y) \in Z$ for all $x,y\square N$.
(ii) N is a commutative ring .

Theorem (2.16) Let N be 2-torsion free a prime near ring which admits a nonzero n-derivation , then the following assertions are equivalent
(i) $d(x_1,x_2,...,x_n)\circ y \in Z$ for all $x_1 ,x_2,...,x_n ,y \square N$.
(ii) N is a commutative ring .

Proof . It is clear that (ii) $\Rightarrow$ (i) .
(i)$\Rightarrow$(ii).Assume that $d(x_1,x_2,...,x_n)\circ y \in Z$ for all $x_1 ,x_2,...,x_n ,y\square N$. (22)
(1) If $Z = \{0\}$, then equation (22) reduced to

$yd(x_1,x_2,...,x_n) = - d(x_1,x_2,...,x_n)y$ for all $x_1,x_2,...,x_n,y \in N$ (23)
Replacing y by zy in(23) we obtain
$zyd(x_1,x_2,...,x_n) = - d(x_1,x_2,...,x_n)zy = (- d(x_1,x_2,...,x_n)z)y = zd(x_1,x_2,...,x_n)y$ for all $x_1 ,x_2,...,x_n ,y , z \in N$, hence $z[d(x_1,x_2,...,x_n),y] = 0$ for all $x_1 ,x_2,...,x_n ,y , z \in N$, primeness of N yields $[d(x_1,x_2,...,x_n),y] = 0$, thus we have $d(N,N,..,N) \subseteq Z$ and from lemma 2.3 it follows that N is commutative .
(2) Suppose that $Z \neq \{0\}$, if $0 \neq z \in Z$, since $d(x_1,x_2,...,x_n)\circ y \in Z$ for all $x_1 ,x_2,...,x_n ,y \in N$ then $d(x_1,x_2,...,x_n)\circ z \in Z$, hence we get $d(x_1,x_2,...,x_n)z+ zd(x_1,x_2,...,x_n) \in Z$,hence $z(d(x_1,x_2,...,x_n)+d(x_1,x_2,...,x_n)) \in Z$, by lemma (1.18) we find that
$d(x_1,x_2,...,x_n)+d(x_1,x_2,...,x_n) \in Z$ for all $x_1,x_2,...,x_n \in N$. (24)
Moreover from (22) it follows that
$d((x_1,x_2,...,x_n)+(x_1,x_2,...,x_n))y+yd((x_1,x_2,...,x_n)+ (x_1,x_2,...,x_n)) \in Z$ for all $x_1 ,x_2,...,x_n ,y \in N$, and by (23) we obtain $(d((x_1,x_2,...,x_n)+ (x_1,x_2,...,x_n))+ d((x_1,x_2,...,x_n)+ (x_1,x_2,...,x_n)))y \in Z$ for all $x_1 ,x_2,...,x_n ,y \in N$ and therefore we have
$(d((x_1,x_2,...,x_n)+ (x_1,x_2,...,x_n))+ d((x_1,x_2,...,x_n)+ (x_1,x_2,...,x_n)))t\ y = y(d((x_1,x_2,...,x_n)+ (x_1 ,x_2 ,..., x_n)) +d((x_1,x_2,...,x_n)+ (x_1,x_2,...,x_n)))t = (d((x_1,x_2,...,x_n)+ (x_1,x_2,...,x_n))+ d((x_1,x_2,...,x_n)+ (x_1,x_2,...,x_n)))yt$ for all $x_1 ,x_2,...,x_n ,y,t \in N$. So that
$(d((x_1,x_2,...,x_n)+ (x_1,x_2,...,x_n))+ d((x_1,x_2,...,x_n)+ (x_1,x_2,...,x_n)))N[t, y] = \{0\}$ for all $x_1 ,x_2,...,x_n ,y,t \in N$.
In view of the primeness of N , the previous equation yields
either $d((x_1,x_2,...,x_n)+ (x_1,x_2,...,x_n))+ d((x_1,x_2,...,x_n)+ (x_1,x_2,...,x_n)) = 0$ and thus $d = 0$,a contradiction ,or $N \subseteq Z$ in which case $d(N,N,...,N) \subseteq Z$,hence by lemma 2.3 we conclude that N is a commutative ring .

Theorem (2.17) Let N be a 2-torsion free prime near-ring .Then there exists no nonzero n- derivation d of N satisfying one of the following conditions
(i) $d(x\circ y,x_2,...,x_n) = [x,y]$
(ii) $d([x,y],x_2,...,x_n) = x\circ y$

Proof .(i) We have $d(x\circ y,x_2,...,x_n) = [x, y]$. (25)
Replacing y by xy in(25) we get $d(x\circ xy,x_2,...,x_n) = [x,xy]$,so we have
$d(x(x\circ y),x_2,...,x_n) = x [x,y]$, hence by def of d we obtain $d(x,x_2,...,x_n) (x\circ y)+ xd((x\circ y),x_2,...,x_n) = x [x,y]$, using (25) in previous equation yields $d(x,x_2,...,x_n) (x\circ y)+x[x,y]=x[x,y]$ and we obtain
$d(x,x_2,...,x_n) (x\circ y) = 0$ for all $x ,y ,x_2,...,x_n \in N$. (26)
Replacing y by yz in (26) we get $d(x,x_2,...,x_n) (xyz +yzx) = 0$, hence $0 = d(x,x_2,...,x_n) xyz + d(x,x_2,...,x_n) yzx = - d(x,x_2,...,x_n)yxz + d(x,x_2,...,x_n) yzx$, so we have
$d(x,x_2,...,x_n)y((-x)z + xz) = 0$, but N is prime so we obtain for any fixed $x \in N$ either $d(x,x_2,...,x_n) = 0$ or $x \in Z$. (27)
But $x \in Z$ also implies that $d(x,x_2,...,x_n) \in Z(N)$ and (24)forces $d(x,x_2,...,x_n)\in Z$ for all $x \in N$, hence $d(N,N,...,N) \subset Z$ and using Lemma 2.3 , we conclude that N is a commutative ring . In this case (25) and 2-torsion freeness implies that
$d(xy,x_2,...,x_n) = 0$ for all $x,y,x_2,...,x_n \in N$ (28)
This mean $d(x,x_2,...,x_n) y + xd(y,x_2,...,x_n) = 0$, replacing x by zx in previous theorem yields $d(zx,x_2,...,x_n) y + zxd(y,x_2,...,x_n) = 0$,using (28) implies $zxd(y,x_2,...,x_n) = 0$ for all $x ,y ,x_2,...,x_n ,z \in N$. that is mean $xNd(y,x_2,...,x_n) = 0$ for all $x ,y ,x_2,...,x_n \in N$. Since N is prime and $d \neq 0$, we conclude that $x = 0$ for all $x \in N$, a contradiction .
(ii) If N satisfies $d([x,y],x_2,...,x_n) = x\circ y$ for all $x ,y ,x_2,...,x_n \in N$, then again using the same arguments we get the required result .
The following example proves that the hypothesis of primness in various theorems is not superfluous.

$$N = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & z \end{pmatrix}, x,y,z,0 \in S \right\}$$ is zero symmetric near-ring with regard to matrix addition and matrix multiplication . Define $d: \underbrace{N \times N \times \ldots \times N}_{n-times} \longrightarrow N$ such that

$$d\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & 0 & z_1 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & 0 & z_2 \end{pmatrix}, \ldots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & 0 \\ 0 & 0 & z_n \end{pmatrix} \right) = \begin{pmatrix} 0 & x_1x_2 \ldots x_n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

It is easy to verify that d is a nonzero derivation of N satisfying the following conditions for all $A,B,A_1,A_2,...,A_n \in N$,
(i) $d([A,B],A_2,...,A_n)=[d(A,A_2,...,A_n),B]$
(ii) $[d(A,A_2,...,A_n),B]=[A,B]$

(iii) $d([A, B],A_{2,\ldots,}A_n) \in Z$
(iv) $[d(A_1,A_2,\ldots,A_n),B] \in Z$ for all $A_1 ,A_2,\ldots,A_n ,B \in N$.
(v) $d(A_1,A_2,\ldots,A_n)\circ B = A_1\circ B$
(vi) $d(A\circ B,A_2,\ldots,A_n) \in Z$
(vii) $d(A_1,A_2,\ldots,A_n)\circ B \in Z$
(viii) $d(A\circ B,A_2,\ldots,A_n) = [A, B]$
(iX) $d([A,B],A_2,\ldots,A_n) = A\circ B$
However, N is not a commutative ring.

Reference

[1]. ArgaÇ.N. "On Prime And Semiprime Near-Rings With Derivations. Int .J. & Math .J.Sci.Vol.20 , No. 4 (1997)737-740 .
[2]. Ashraf.M. , Bouab .A. and Raji .A " On derivations and commutativity in prime near-rings" Science Direct Journal of Taibah University for Science ,V.8 ,(2014) 301–306
[3]. Asraf.M. and Siddeeque.M.A , "On permuting n-derivations in near-rings , Commun. Kor. Math. Soc. 28 (4) (2013)697–707.
[4]. Ashraf.M. , Sideeque.M.A and Parveen.N " On semigroup ideals and n-derivations in near-rings " Science Direct Journal of Taibah University for Science,V. 9 (2015) 126–132.
[5]. Bell.H.E. , "On Derivations in Near-Rings II ", vol. 426, Kluwer Aca-demic Publishers, Dordrecht, 1997, pp. 191–197.
[6]. Bell.H.E . and mason. G. "on derivations in near-rings , near-rings and near fields". (G.Betsch , ed.) North – Holland , Amsterdam (1987), 31 – 35 .
[7]. Boua.A. and Oukhtite.L. ," Derivations on prime near-rings", Int. J. Open Problems Comput. Math. 4 (2) (2011) 162–167.
[8]. Pilz . G. "Near–Rings". North Holland Publishing Company Amsterdam .New York , Oxford. (1983) .